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THE CALCULATION OF KINETIC/ANGULAR MOMENTUM OF MERCURY RING IN THE TOROIDAL GYROMOTOR

BY

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Abstract. The results of a study concerning the toroidal mercury ring gyromotor are presented; the calculation has been made at different supply voltages for mean angular speed of mercury and for the speeds in the electroconducting fluid layers. Interpreting the tests results it has been found that the mercury revolves at different speeds in the layers. From the analysis of the obtained results it can be found that the resulted angular moments are comparable to the usual angular moments of classical solid ring gyroscopes.

Key words: kinetic moment; toroidal gyromotor.

1. Generalities

The most important parameter of mercury ring gyroscope is the *motion quantity moment* or the kinetic moment. To determine the expression for kinetic moment calculation we consider a point, A_i, of mass m_i, in the gyroscope placed to the distance $\mathbf{r}_i$ from the origin of fixed co-ordinates system *Oxyz* (Fig. 1).

The linear rate point, A_i, is

$$\mathbf{v}_i = \frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t}. \qquad (1)$$

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In this case, the motion quantity of point A_i will be

$$m_i \mathbf{v}_i = m_i \frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t}. \tag{2}$$

The motion quantity of point A_i is a vector collinear with the linear rate vector $\mathbf{v}_i$, and, as a result, it is perpendicular to the position vector $\mathbf{r}_i$; consequently

$$\mathbf{r}_i \times m_i \mathbf{v}_i = \mathbf{r}_i \times m_i \frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t}. \tag{3}$$

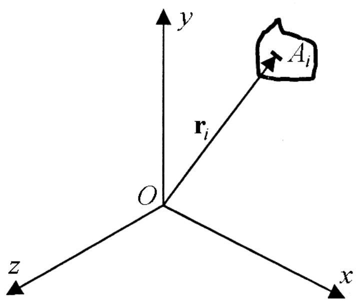

Fig. 1 – Determination of kinetic moment.

It is noted with $\mathbf{k}_i = \mathbf{r}_i \times m_i \mathbf{v}_i$ the motion quantity moment of point A_i. The motion quantity moment of the mercury care will be

$$\sum_{i=1}^{n} \mathbf{k}_i \sum_{i=1}^{n} \left(\mathbf{r}_i \times m_i \mathbf{v}_i\right) = \mathbf{K}, \tag{4}$$

where

$$\mathbf{K} = \sum_{i=1}^{n} \left(\mathbf{r}_i \times m_i \mathbf{v}_i\right) = \iiint_V \left(\mathbf{r}_i \times r \mathbf{v}_i\right) \mathrm{d}V \tag{5}$$

is the angular (kinetic) moment of gyroscope's ring, V – the volume of gyroscope's ring;

$$\mathrm{d}m_i = r\,\mathrm{d}V, \tag{6}$$ $$\mathrm{d}V = \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z. \tag{7}$$

The linear rate of point A_i may be written as a function of the angular speed

$$\mathbf{v}_i = \boldsymbol{\omega} \times \mathbf{r}_i , \tag{8}$$

where $\boldsymbol{\omega}$ is the angular speed of ring.

Taking into account that the vectors $\boldsymbol{\omega}$ and $\mathbf{r}_i$ are perpendicular

$$v_i = w r_i . \tag{9}$$

The modulus of motion quantity moment of point A_i will be

$$k_i = r_i\ m_i v_i = r_i\ m_i w r_i = m_i w r_i^2 \tag{10}$$

and for all points

$$K = w\sum_{i=1}^{n} m_i\, r_i^2 = w\, J_0\,, \tag{11}$$

where $J_0 = \sum_{i=1}^{n} m_i\, r_i^2$ is the polar moment of inertia.

2. Calculation of Inertia Moment

The axial moment of inertia, J_x, of a cylinder (Fig. 2), is calculated with the relation

$$J_x = mr_i^2 = m\left(y_i^2 + z_i^2\right) = m\frac{R^2}{2}, \tag{12}$$

where

$$m = rV = rp\,R^2 l \tag{13}$$

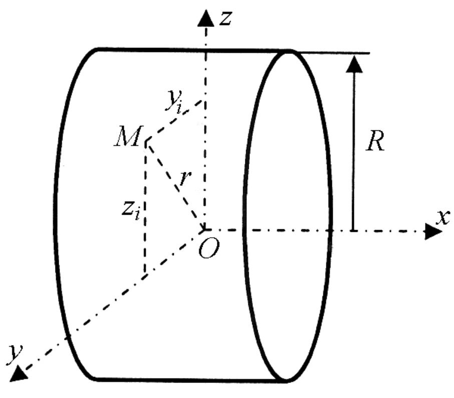

Fig. 2 – The inertia moment of cylinder.

and consequently

$$J_x = rp\,l\frac{R^4}{2}. \tag{14}$$

Here m is the mass of cylinder; r_i – the distance to the mass point M_i; R – the cylinder's radius; V – the volume; l – the length of cylinder; ρ – the density.

3. The Kinetic Moment Based on the Mean Speed

The calculation of axial inertia moment of mercury ring (Fig. 3) is based on the following expression:

$$J = r_{\text{Hg}} p \Delta \frac{R_2^4 - R_1^4}{2}, \tag{15}$$

where: $\rho_{\text{Hg}} = 13.55$ g/cm^2 = 13.55×10^{-3} kg/m^3 – the specific density of mercury; $R_2 = 80 \times \times 10^{-3}$ m – the exterior radius of the ring; $R_1 = = 40 \times 10^{-3}$ m – the interior radius of the ring; $\Delta = 6 \times 10^{-3}$ m – the thickness of the ring; $J = = 0.52249$ kg.m^2.

The kinetic (angular) moment is calculated with the expression

$$K = J w_{\text{med}}. \tag{16}$$

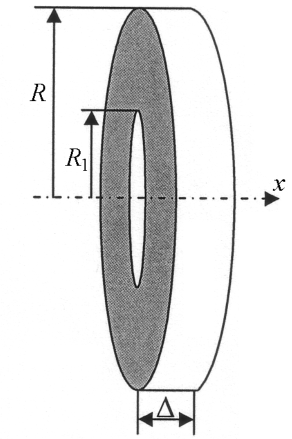

Fig. 3 – The mercury ring.

The Table 1 was established based on the obtained experimental results, which represents the values of the angular moment as a function of the supply voltages values for the frequencies of 330 Hz and 500 Hz.

Ω_m is the mean angular speed resulted from the mediation of angular speed in the mercury layers proper to six radii measured (s. Tables 2 and 3).

Table 1
Experimental Results

U_1, [V]	f, [Hz]	J, [kg.m^2]	ω_{med}, [rad/sec]	K, [kg.m^2.rad/sec]
40	330	0.004914144	22.708	0.111590382
	500	0.004914144	25.4	0.12485857
60	330	0.004914144	27.08	0.133114332
	500	0.004914144	32.76	0.16102667
80	330	0.004914144	34.03	0.167267633
	500	0.004914144	44.42	0.218325589
100	330	0.004914144	44.825	0.220276504
	500	0.004914144	60.62	0.29791998
110	330	0.004914144	61.124	0.300372137
	500	0.004914144	72.58	0.356688228
130	330	0.004914144	79.818	0.387323001
	500	0.004914144	86.4	0.424670496

4. The Kinetic Moment Based on the Layers Speed

Taking into account that the liquid armature rotates in layers at different speeds it results the necessity to calculate the kinetic (angular) moment considering the variation of these speeds.

The calculation of layers inertia moment is performed (Fig. 4) based on the following expression:

$$J_{\text{strat}} = r_{\text{Hg}} \Delta p \,\frac{R_{2\text{strat}}^4 - R_{1\text{strat}}^4}{2}. \tag{17}$$

The angular (kinetic) moment has been calculated based on the expression

$$K = J w_{\text{strat}}. \tag{18}$$

In this way it was considered that the armature can be divided into six layers which correspond, to a great extent, to the layers experimentally obtained.

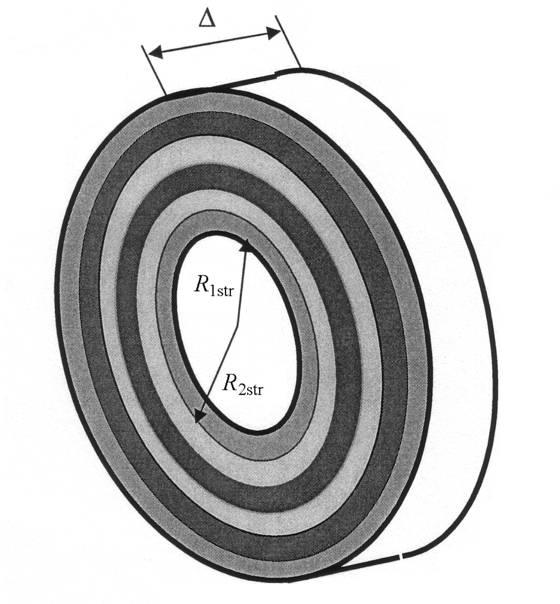

Fig. 4 – The mercury layers with different speeds.

So, in the Tables 2 and 3 they are indicated the kinetic moments of the six layers considered for different supply voltages of the inductors.

Table 2
The Kinetic Moments at the Frequency of 330 Hz

Str. No	U V	$R_2 \times 10^{-3}$ m	$R_1 \times 10^{-3}$ m	J kg.m^2	Ω rad/s	K_{strat} kg.m^2.rad/s	K kg.m^2.rad/s
1	40	45	40	0.0002	26.67	0.005254	0.0984
2		52	45	0.0004	24.42	0.010012	
3		60	52	0.0007	22.33	0.0161	
4		67	60	0.0009	21.19	0.019452	
5		75	67	0.0015	18.93	0.027761	
6		80	75	0.0012	16.67	0.019829	
1	60	45	40	0.0002	32.93	0.006487	0.1147
2		52	45	0.0004	29.59	0.012132	
3		60	52	0.0007	26.58	0.019164	
4		67	60	0.0009	23.91	0.021949	
5		75	67	0.0015	21.59	0.031662	
6		80	75	0.0012	19.63	0.02335	
1	80	45	40	0.0002	42.29	0.008331	0.1421
2		52	45	0.0004	37.44	0.01535	
3		60	52	0.0007	33.18	0.023923	
4		67	60	0.0009	29.52	0.027099	
5		75	67	0.0015	26.47	0.038818	
6		80	75	0.0012	24.04	0.0286	
1	100	45	40	0.0002	56.56	0.011142	0.18624
2		52	45	0.0004	49.58	0.020328	
3		60	52	0.0007	43.51	0.031371	
4		67	60	0.0009	38.26	0.035123	
5		75	67	0.0015	34.84	0.051093	
6		80	75	0.0012	31.26	0.037184	
1	110	45	40	0.0002	77.31	0.01523	0.24617
2		52	45	0.0004	67.65	0.027736	
3		60	52	0.0007	59.05	0.042575	
4		67	60	0.0009	51.52	0.047295	
5		75	67	0.0015	45.07	0.066095	
6		80	75	0.0012	39.71	0.047235	
1	130	45	40	0.0002	100.9	0.019875	0.32136
2		52	45	0.0004	88.08	0.036113	
3		60	52	0.0007	77	0.055553	
4		67	60	0.0009	70.45	0.064673	
5		75	67	0.0015	62.67	0.09191	
6		80	75	0.0012	44.76	0.053242	

Table 3
The Kinetic Moments at the Frequency of 500 Hz

Str. No	U V	$R_2 \times 10^{-3}$ m	$R_1 \times 10^{-3}$ m	J kg.m^2	Ω rad/s	K_{strat} kg.m^2.rad/s	K kg.m^2.rad/s
1	40	45	40	0.0002	30.6	0.006028	0.10954
2		52	45	0.0004	27.7	0.012546	
3		60	52	0.0007	25	0.018025	
4		67	60	0.0009	23.1	0.021206	
5		75	67	0.0015	20.6	0.03021	
6		80	75	0.0012	18.1	0.021653	
1	60	45	40	0.0002	40.4	0.007967	0.13865
2		52	45	0.0004	35.9	0.014719	
3		60	52	0.0007	31.8	0.022928	
4		67	60	0.0009	29.4	0.026989	
5		75	67	0.0015	26.3	0.038569	
6		80	75	0.0012	23.1	0.027477	
1	80	45	40	0.0002	55.3	0.010902	0.18745
2		52	45	0.0004	48.6	0.019926	
3		60	52	0.0007	43.3	0.031219	
4		67	60	0.0009	39.6	0.036353	
5		75	67	0.0015	35.3	0.051767	
6		80	75	0.0012	31.3	0.037279	
1	100	45	40	0.0002	74.7	0.014724	0.24833
2		52	45	0.0004	65.4	0.026814	
3		60	52	0.0007	57.5	0.041457	
4		67	60	0.0009	51.9	0.047644	
5		75	67	0.0015	46.4	0.068046	
6		80	75	0.0012	41.7	0.04965	
1	110	45	40	0.0002	91.8	0.018093	0.31016
2		52	45	0.0004	80.4	0.032964	
3		60	52	0.0007	70.3	0.050686	
4		67	60	0.0009	63.6	0.058385	
5		75	67	0.0015	58.8	0.08623	
6		80	75	0.0012	53.6	0.063805	
1	130	45	40	0.0002	110	0.02167	0.35953
2		52	45	0.0004	95.8	0.039278	
3		60	52	0.0007	83.5	0.060203	
4		67	60	0.0009	75.4	0.069217	
5		75	67	0.0015	67.3	0.098695	
6		80	75	0.0012	59.2	0.070466	

5. Conclusions

It has been found that the kinetic (angular) moment of the mercury ring determined taking into account the mean speed is a little higher than the one calculated by considering the angular speeds in layers

The kinetic moments obtained are comparable to the kinetic moments of classical solid ring gyroscopes at which $K = 0.1\ldots0.5$ kg.m^2.rad.s^{-1}.

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CALCULUL MOMENTULUI CINETIC AL TORULUI DIN MERCUR LA GIROMOTORUL TOROIDAL

(Rezumat)

Se prezintă calculul momentelor cinetice pentru giromotorul cu torul din mercur; calculul a fost efectuat pentru diferite tensiuni de alimentare, pentru viteza unghiulară medie a mercurului şi pentru diferite viteze ale straturilor de fluid electroconductor. Pe timpul experimentărilor s-a constatat că mercurul se roteşte cu viteze diferite in straturi. Din analiza rezultatelor obţinute se poate constata că momentele cinetice sunt compatibile cu cele uzuale ale giroscoapelor clasice cu torul solid.