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THE RESONANCE AT THE ACCESS GATES OF A LINEAR AND NON-AUTONOMOUS GENERAL TWO-PORT, IN HARMONIC STEADY-STATE

BY

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Abstract. The resonance realization possibilities at the access gates of a linear and non-autonomous general two-port, in harmonic steady-state, are studied. The conditions assuring the simple, double or triple resonance were established.

Key words: linear and non-autonomous general two-ports; harmonic steady-state; simple, double or triple resonances.

1. Introduction

The resonance, in harmonic steady-state, at the input gate of a linear and non-autonomous general two-port (LNGT), was studied in some previous works (Rosman, 2004, 2005), considering only the case when the studied two-ports are of “soft” type (Rosman, 2008). The utilized proceeding was based on the equivalence between a linear and non-autonomous general two-port of “soft” type (Rosman, 2005) and a restricted sense two-port, as well as on the

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results concerning the resonance at the access gates of a restricted sense two-port established in the author's Ph. D. dissertation (Rosman, 1968).

The goal of this paper is to extend the resonance study in harmonic steady-state at the gates of an LNGT of "hard" type (Rosman, 2008), that is at those general two-ports which have access to the outside at all three gates.

Let be an LNGT (Fig. 1) having the eqs. (Sigorsky, 1956)

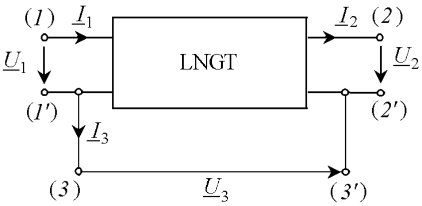

Fig. 1

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \\ \underline{U}_3 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} & \underline{A}_{13} \\ \underline{A}_{21} & \underline{A}_{22} & \underline{A}_{23} \\ \underline{A}_{31} & \underline{A}_{32} & \underline{A}_{33} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix}, \quad (1)$$

where $\underline{A}_{ij}$, (i, j = 1, 2, 3), represent the fundamental parameters. As well

$$\underline{Z}_2 = \frac{\underline{U}_2}{\underline{I}_2} = R_2 + \mathrm{j}X_2, \quad \underline{Z}_2 = \frac{\underline{U}_3}{\underline{I}_3} = R_3 + \mathrm{j}X_3 \quad (2)$$

while

$$\underline{Z}_{e1} = \frac{\underline{U}_1}{\underline{I}_1} = R_{e1} + \mathrm{j}X_{e1} \quad (3)$$

is the equivalent complex impedance at the LNGT's input gate.

If the signals $\underline{U}_2$, $\underline{I}_2$, $\underline{U}_3$, $\underline{I}_3$ are eliminated through eqs. (1),…, (3) it results the expression

$$\underline{Z}_{e1} = \frac{\left(\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{22}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}. \quad (4)$$

2. Simple Resonances

2.1. Resonance at the (*1*), (*1*') Gate

Such a regime is realized if

$$X_{e1} = 0. \quad (5)$$

Having in view (4), as well as expressions (2) and (3), condition (5) is verified if either parameters R_2, X_2 satisfy eq.

$$a\left(R_2^2+X_2^2\right)+b\,R_2+g\,X_2+d=0 \tag{6}$$

or parameters R_3, X_3 verify eq.

$$a'\left(R_3^2+X_3^2\right)+b'R_3+g'X_3+d'=0 \tag{7}$$

The expressions of α, β, γ, δ, α', β', γ', δ' coefficients are given in Appendix.

It is possible to ascertain that in case of an LNGT having a certain structure and if the complex impedance $\underline{Z}_3 = R_3 + \mathrm{j}X_3$ is constant relation (6) represents, in the plane (R_2, X_2), the equation of a circle, which constitutes, in fact, the geometrical-locus diagram equation of the complex impedance $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ which assures the resonance's realization at the LNGT's input gate. A similar conclusion may be drawn as regards eq. (7), when an LNGT with a certain structure is considered, the complex impedance $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ being constant. In this case eq. (7) represents the geometrical-locus diagram equation of the complex impedance $\underline{Z}_3 = R_3 + \mathrm{j}X_3$ corresponding to resonance regimes at the LNGT's input gate. Eq. (7), represents a circle too.

In the particular case when LNGT's gate (*3*), (*3'*) is in short-circuit ($R_3 = 0$, $X_3 = 0$) relation (7) becomes

$$d'=0 \tag{8}$$

and having in view the last relation (A.2) it results

$$\begin{gathered}
\Im m\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]\left(R_2^2+X_2^2\right)+\\
+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)+\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\times\right.\\
\left.\times\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]R_2+\Re e\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)-\right.\\
\left.-\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]X_2+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\times\right.\\
\left.\times\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]=0.
\end{gathered} \tag{9}$$

This last relation represents the equation of a circle which represents the geometrical-locus diagram equation of the complex impedance $\underline{Z}_2$ in working regimes which realize the resonance at the input gate, (*1*), (*1'*) of an LNGT, when the gate (*3*), (*3'*) is in short-circuit. It is necessary to observe that, properly, in this case is realized a double resonance, at the gates (*1*), (*1'*) and (*3*), (*3'*), having in view that simultaneously relations $X_{e1} = 0$ and $X_3 = 0$ are satisfied.

In case of a passive LNGT, the complex impedances $\underline{Z}_2$, $\underline{Z}_3$ being passive too, the geometrical-locus diagrams (6), (7) and (9) must be limited to the circles arcs situated in the plane $R_2 = \Re e\left(\underline{Z}_2\right) \geq 0$, respectively $R_2 = \Re e\left(\underline{Z}_2\right) \geq 0$.

2.2. Resonances at the Gates (*2*), (*2'*) and (*3*), (*3'*)

These resonances represent banal cases when either $X_2 = 0$, or $X_3 = 0$.

3. Double Resonances

3.1. Double Resonance at the Gates (*1*), (*1'*) or (*2*), (*2'*)

In this case must be satisfied simultaneously relations (5) and

$$X_2 = 0 \tag{9}$$

eq. (6) becoming

$$a\,R_2^2 + b\,R_2 + d = 0\,. \tag{10}$$

It is obvious that such a double resonance regime may be realized only if the resistance R_2 is one of eq.'s (10) roots, namely

$$R_2^{'}, R_2^{''} = \frac{-b \pm \sqrt{b^2 - 4ad}}{2a}\,; \tag{11}$$

these roots are real if

$$b > \pm 2\sqrt{ad}\;. \tag{12}$$

They are positive if

$$a > 0,\; b < 0,\; d > 0 \text{ or } a < 0,\; b > 0,\; d < 0\;. \tag{13}$$

Taking into account that $\alpha = 0$, $\beta = 0$, $\delta = 0$ represent, in plane (R_3, X_3), circles eqs., it results that: a) in the first case the complex impedance $\underline{Z}_3$ affix must be situated outside of circles $\alpha = 0$, $\delta = 0$ and inside of circle $\beta = 0$; b) in the second case be must be situated, in the contrary, inside of circles $\alpha = 0$, $\delta = 0$ and outside of circle $\beta = 0$. Both conditions may be satisfied only if fundamental parameters A_{ij}, (i, j = 1, 2, 3), satisfy certain relations quite difficult to determine.

It results that, generally speaking, may be realized by an LNGT at the most two distinct regimes of double resonance at the gates $(1),\ (1')$ and $(2),\ (2')$.

If the LNGT's gate $(3),\ (3')$ is in short-circuit relation (7) comes to (8), which when the resonance at gate $(2),\ (2')$ is realized leads to the algebraic eq. of second order

$$\begin{gathered}\Im m\left[\left(\underline{A}_{23}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]R_2^2+\Im m\left[\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\times\right.\\ \left.\times\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\right]R_2+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]=0,\end{gathered} \tag{14}$$

having the roots

$$R_2^{*},R_2^{**}=\frac{-n\pm\sqrt{m^2-4np}}{2m}, \tag{15}$$

where

$$\begin{cases} m=\Im m\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right],\\ n=\Im m\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right],\\ p=\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]. \end{cases} \tag{16}$$

The roots $R_2^{*},\ R_2^{**}$ are real if

$$m\geq\pm2\sqrt{np} \tag{17}$$

being positive if

$$m>0,\ n<0,\ p>0 \text{ or } m<0,\ n>0,\ p<0. \tag{18}$$

In fact such regimes are, strictly speaking, triple resonances since the gate $(3),\ (3')$ being in short-circuit, $X_3 = 0$. It is easy to deduce that in this particular case an LNGT may realize at most two distinct double resonance regimes at the gates $(1),\ (1')$ and $(2),\ (2')$ when the gate $(3),\ (3')$ is in short-circuit, which are, as was before showed, triple resonances.

3.2. Double Resonance at the Gates (1), $(1')$ and (3), $(3')$

The realization of such a regime implies to satisfy, except relation (5), and relation

$$X_3 = 0. \tag{19}$$

In these conditions eq. (7) becomes

$$a' R_3^2 + b' R_3 + d' = 0 \tag{20}$$

having the roots

$$R_3', R_3'' = \frac{-b' \pm \sqrt{b'^2 - 4a'd'}}{2a'}. \tag{21}$$

These roots are real if

$$b' > \pm 2\sqrt{a'b'} \tag{22}$$

and positive when

$$a' > 0,\ b' < 0,\ d' > 0 \text{ or } a' < 0,\ b' > 0,\ d' < 0. \tag{23}$$

If conditions (22) and (23) are satisfied is realized a double resonance regime at the gates (1), $(1')$ and (3), $(3')$. It results that at most two double resonance regimes at the gates (1), $(1')$ and (3), $(3')$ may be realized.

Having in view that $\alpha' = 0$, $\beta' = 0$ and $\delta' = 0$ represent circles eqs. in plane (R_2, X_2) (s. rel. (A.2)) it results that: a) when $\alpha' > 0$, $\beta' < 0$ and $\delta' < 0$ the complex impedance $\underline{Z}_2$ affix must be situated outside the circle $\beta' = 0$ and inside the circles $\alpha' = 0$, $\delta' = 0$; b) in the opposite case this affix must be situated inside the circle $\beta' = 0$ and outside the circles $\alpha' = 0$, $\delta' = 0$.

Conditions (13) are satisfied only if LNGT's fundamental parameters, $\underline{A}_{ij}$, $(i, j = 1, 2, 3)$, satisfy certain conditions quite difficult to be determined.

When the gate (3), $(3')$ is in short-circuit, the realization of double resonance at the gates (1), $(1')$ and (3), $(3')$ was studied in § **3.1**.

3.3. Double Resonance at the Gates (2), $(2')$ and (3), $(3')$

Such a regime is realized when are satisfied simultaneously relations (9) and (19). In this case the equivalent complex impedance at the LNGT's gate (1), $(1')$ is, according to (4),

$$\underline{Z}_{e1r} = \frac{\left(\underline{A}_{11}R_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)R_2 + \underline{A}_{12}R_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}R_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)R_2 + \underline{A}_{22}R_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}. \quad (24)$$

4. Triple Resonance at the LNGT's Gates

To realize the triple resonance at the three gates of an LNGT it is necessary to satisfy, except relations (9) and (19), the condition

$$X_{e1r} = 0 \quad (25)$$

too, where $X_{e1r} = \Im m\left(\underline{Z}_{e1r}\right)$. Having in view relation (24) condition (25) may be written

$$\begin{aligned}
&\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)R_2^2R_3^2 + \Im m\left[\underline{A}_{11}\left(\underline{A}_{23}^*\underline{A}_{31}^* - \underline{A}_{21}^*\underline{A}_{33}^*\right) + \underline{A}_{21}^*\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\right]R_2^2R_3 + \\
&+\Im m\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*\right)R_2R_3^2 + \Im m\left[\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^*\underline{A}_{31}^* - \underline{A}_{21}^*\underline{A}_{33}^*\right)\right]R_2^2 + \\
&+\Im m\left(\underline{A}_{12}\underline{A}_{22}^*\right)R_3^2 + \Im m\left[\underline{A}_{11}\left(\underline{A}_{23}^*\underline{A}_{32}^* - \underline{A}_{23}^*\underline{A}_{32}^*\right) + \underline{A}_{12}\left(\underline{A}_{23}^*\underline{A}_{31}^* - \underline{A}_{21}^*\underline{A}_{33}^*\right) + \right. \\
&\left. +\underline{A}_{21}^*\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right) + \underline{A}_{22}^*\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\right]R_2R_3 + \quad (26) \\
&+\Im m\left[\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^*\underline{A}_{32}^* - \underline{A}_{22}^*\underline{A}_{33}^*\right) + \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\times\right. \\
&\left.\times\left(\underline{A}_{23}^*\underline{A}_{31}^* - \underline{A}_{21}^*\underline{A}_{33}^*\right)\right]R_2 + \Im m\left[\underline{A}_{12}\left(\underline{A}_{23}^*\underline{A}_{32}^* - \underline{A}_{22}^*\underline{A}_{33}^*\right) + \underline{A}_{22}^*\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right]R_3 + \\
&+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^*\underline{A}_{32}^* - \underline{A}_{22}^*\underline{A}_{33}^*\right)\right] = 0.
\end{aligned}$$

It results that for an LNGT having a certain structure the triple resonance may be realized only if resistances R_2 and R_3 satisfy relation (26).

As regards the LNGT having the gate (3), $(3')$ in short-circuit the triple resonance was studied in § **3.1**.

The triple resonance regime at the gates of an LNGT coincides with the regime named in a previous paper (Rosman, 2005) *regime of global resonance*, which is characterized by relation.

$$Q_1 + Q_2 + Q_3 = 0 \quad (27)$$

because in the triple resonance regime $Q_1 = 0$, $Q_2 = 0$, $Q_3 = 0$.

5. Conclusions

1. At the access gates of a linear and non-autonomous general two port, working in harmonic steady-state, it is possible to realize the resonance: a) individually, at each of the three access gates, named *simple resonances*; b) simultaneously at two gates each, named *double resonances* and c) simultaneously at all the three gates, named *triple resonance*.

2. The conditions which assure the realization of simple resonances at the input gate, (1), $(1')$, are given by relations (6) or (7).

3. The realization of double resonance is possible only if are satisfied certain conditions by the complex impedance at the gate where is not realized the resonance.

4. The triple resonance may be realized only when the resistances at the gates (2), $(2')$ and (3), $(3')$ satisfy condition (26).

Appendix

Parameters α, β, γ, δ from eq. (6) have the expressions

$$
\begin{aligned}
a &= \Im m\left(\underline{A}_{11}\underline{A}^*_{21}\right)\left(R_3^2+X_3^2\right)+\Im m\Big[\underline{A}_{11}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)+\underline{A}^*_{21}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\Big]R_3+\\
&+\Re e\Big[\underline{A}_{11}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)-\underline{A}^*_{21}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\Big]X_3+\\
&+\Im m\Big[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)\Big],\\
b &= \Im m\left(\underline{A}_{11}\underline{A}^*_{22}-\underline{A}^*_{12}\underline{A}_{21}\right)\left(R_3^2+X_3^2\right)+\Im m\Big[\underline{A}_{12}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)+\\
&+\underline{A}^*_{21}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}^*_{22}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}_{11}\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big]R_3+\\
&+\Re e\Big[\underline{A}_{12}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)-\underline{A}^*_{21}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)-\underline{A}^*_{22}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)+\\
&+\underline{A}^*_{11}\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big]X_3+\Im m\Big[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)+\\
&+\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big],\\
g &= \Re e\left(\underline{A}_{11}\underline{A}^*_{22}-\underline{A}^*_{12}\underline{A}_{21}\right)\left(R_3^2+X_3^2\right)+\Re e\Big[\underline{A}^*_{22}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)-\\
&-\underline{A}_{12}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{12}\underline{A}^*_{33}\right)+\underline{A}_{11}\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big]R_3+\Im m\Big[\underline{A}_{12}\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)-\\
&-\underline{A}^*_{21}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}^*_{22}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)-\underline{A}_{11}\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big]X_3-\\
&-\Re e\Big[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}^*_{23}\underline{A}^*_{31}-\underline{A}^*_{21}\underline{A}^*_{33}\right)-\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\times\\
&+\left(\underline{A}^*_{23}\underline{A}^*_{32}-\underline{A}^*_{22}\underline{A}^*_{33}\right)\Big],
\end{aligned}
\tag{A.1}
$$

$$d=\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)\left(R_3^2+X_3^2\right)+\Im m\left[\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]R_3-$$
$$-\Re e\left[\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]X_3+ \tag{A.1}$$
$$+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right].$$

In the same time parameters α', β', γ', δ' from eq (7) are

$$a'=\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left(R_3^2+X_3^2\right)+\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}\right)R_2+\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}\right)X_2+$$
$$+\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right),$$

$$b'=\Im m\left[\underline{A}_{21}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)+\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]\left(R_3^2+X_3^2\right)+$$
$$+\Im m\left[\underline{A}_{21}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)+\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{12}\underline{A}_{33}\right)+\right.$$
$$\left.+\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]R_2+\Re e\left[-\underline{A}_{21}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)-\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)+\right.$$
$$\left.+\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)+\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]X_2+\Im m\left[\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\right.$$
$$\left.+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right],$$

$$g'=\Re e\left[-\underline{A}_{21}^{*}\left(\underline{A}_{23}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)+\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]\left(R_3^2+X_3^2\right)+$$
$$+\Re e\left[-\underline{A}_{21}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)-\right. \tag{A.2}$$
$$\left.-\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\right]R_2+\Im m\left[-\underline{A}_{21}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)+\right.$$
$$\left.+\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)-\underline{A}_{11}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]X_2+\Re e\left[-\underline{A}_{22}^{*}\left(\underline{A}_{23}\underline{A}_{31}-\underline{A}_{12}\underline{A}_{33}\right)+\right.$$
$$\left.+\underline{A}_{12}\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right],$$

$$d'=\Im m\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]\left(R_3^2+X_3^2\right)+\Im m\left[\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\times\right.$$
$$\left.\times\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)+\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\right]R_2+$$
$$+\Re e\left[\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)\left(\underline{A}_{23}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)-\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\times\right.$$
$$\left.\times\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right]X_2+\Im m\left[\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right].$$

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REZONANŢA LA PORŢILE DE ACCES ALE UNUI CUADRIPOL GENERAL LINIAR ŞI NEAUTONOM, ÎN REGIM PERMANENT ARMONIC

(Rezumat)

Se studiază posibilităţile de realizare a rezonanţei, în regim permanent armonic, la porţile unui cuadripol general, liniar şi neautonom, de tip „hard" (Rosman, 2008). Se stabilesc condiţiile de realizare a rezonanţei (simple) la poarta de intrare a unui astfel de cuadripol precum şi cele de realizare simultană, a rezonanţelor duble, la oricare pereche de porţi ale cuadripolului şi ale rezonanţei triple, la toate cele trei porţi de acces ale acestuia.