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HYPERCOMPLEX INSTANTANEOUS POWERS IN LINEAR AND PASSIVE ELECTRICAL NETWORKS IN PERIODICAL NON-HARMONIC STEADY-STATE

BY

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Abstract. The method proposed by V.N. Nedelcu to characterize the energy regime of a linear and passive electrical network, excited by a periodical harmonic signal, utilizing the symbolic method, based on representation of harmonic signals of same frequency through complex “images”, is extended to the case when the network in excited by a periodical, non-harmonic signal utilizing the symbolic method based on representation of such signals (“originals”), through “hypercomplex images”.

Key words: instantaneous hypercomplex powers; periodical non-harmonic steady-state; symbolic hypercomplex method.

1. Introduction

Let be a linear and passive one-port (LPOP), working in harmonic steady-state, excited by the voltage

$$u_b = \sqrt{2}U_b \cos\left(wt + g_{u_b}\right); \qquad (1)$$

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the current which flows through the LPOP being

$$i = \sqrt{2} I \cos\left(wt + g_i\right). \tag{2}$$

The instantaneous power exchanged by the LPOP with the outside is

$$p = u_b i = U_b I \cos j + U_b I \cos\left(2wt + g_u + g_i\right), \tag{3}$$

where

$$j = g_u - g_i. \tag{4}$$

It is well known that in this case, the LPOP’s energy regime is characterized, in main, by the active and reactive powers. Having in view that relation (3) may be written

$$p = U_b I \cos j \left[1 + \cos 2\left(wt + g_i\right)\right] + U_b I \sin j \sin\left(wt + g_i\right), \tag{5}$$

the active power, $P = UI\cos j$, represents the ***average value*** of the instantaneous power while the reactive power, $Q = UI\sin j$, may be defined as the *amplitude of his oscillating component*. With the view to eliminate this existent disparity in the definition manner of active and reactive power, V.N. Nedelcu (for instance, 1963) has adopted an artifice which may be rendered evident if the symbolic complex method to represent the harmonic signals is utilized. Namely to signals (1) and (2) are attached theirs complex instantaneous “images”

$$\underline{u}_b = \sqrt{2} U_b \mathrm{e}^{\mathrm{j}\left(wt + g_{u_b}\right)},\ \underline{i} = \sqrt{2} I \mathrm{e}^{\mathrm{j}\left(wt + g_i\right)}, \tag{6}$$

or theirs complex effective “images”

$$\underline{U}_b = U_b \mathrm{e}^{\mathrm{j} g_{u_b}},\ \underline{I}_b = I_b \mathrm{e}^{\mathrm{j} g_{\mathrm{i_b}}}; \tag{7}$$

to the instantaneous power exchanged by the LPOP with the outside it is possible to attach the *complex instantaneous apparent power*

$$\underline{s} = \frac{1}{2} \underline{u}_b \left(\underline{i} + \underline{i}^*\right) = \underline{U}_b \underline{I}^* + U_b I \mathrm{e}^{\mathrm{j}2wt} = s(t) \mathrm{e}^{\mathrm{j}s(t)}, \tag{8}$$

which is different from the expression of complex apparent power,

$$\underline{S} = \frac{1}{2} \underline{u}_b \underline{i}^* = \underline{U}_b \underline{I}^* = P + \mathrm{j}Q. \tag{9}$$

In these conditions the instantaneous power (3) represents the real part of the complex instantaneous apparent power

$$p = \Re e(\underline{s}) \,. \tag{10}$$

V.N. Nedelcu (*op. cit.*) has proposed for the instantaneous power, as real part of the complex instantaneous apparent power, the denomination of *instantaneous active power*. Consequently the imaginary part of the complex instantaneous apparent power

$$q = \Im m(\underline{s}) = -\frac{1}{2}\left(\underline{s} - \underline{s}^{*}\right) = U_b I \sin j \; + U_b I \sin\left(2wt + g_{u_b} + g_i\right) \tag{11}$$

represents the *instantaneous reactive power*.

It results that the complex apparent power is the average value of the complex instantaneous power

$$\underline{S} = \left\langle \underline{s} \right\rangle = \frac{1}{T}\int_0^T \underline{s}\ \mathrm{d}t = \frac{1}{2}\underline{u}_b\,\underline{i}^{*} = U_b I \mathrm{e}^{\mathrm{j}j} \; = P + \mathrm{j}Q \,, \tag{12}$$

where the active power, P, and the reactive power, Q, are defined as average values of the instantaneous active power (3),

$$P = \left\langle p \right\rangle = \frac{1}{T}\int_0^T p\ \mathrm{d}t = U_b I \cos j \;\;, \tag{13}$$

respectively the average value of the instantaneous reactive power

$$Q = \left\langle q \right\rangle = \frac{1}{T}\int_0^T q\ \mathrm{d}t = U_b I \sin j \;\;, \tag{14}$$

$T = 2p/w$ being the period of the harmonic signals $u_b(t)$, $i(t)$.

Relations (8) and (12) lead to expression

$$\underline{s} = \underline{S} + \underline{s}_f \,, \tag{15}$$

where

$$\underline{s}_f = \frac{1}{2}\underline{u}_b\,\underline{i} = U_b I \mathrm{e}^{\mathrm{j}2wt} = \underline{S}_f\,\mathrm{e}^{\mathrm{j}2wt} = p_f + \mathrm{j}q_f \,. \tag{16}$$

with

$$\underline{S}_f = \underline{U}_b \underline{I},\ p_f = U_b I \cos\left(2wt + g_{u_b} + g_i\right),\ q_f = U_b I \sin\left(2wt + g_{u_b} + g_i\right). \quad (17)$$

Here $\underline{S}_f$ represents the *complex apparent fluctuating power*, p_f – *the active instantaneous fluctuating power*, while q_f – *the reactive instantaneous fluctuating power*.

The aim of this paper is to extend the powers study proceeding proposed by V.N. Nedelcu, to the more generally case of the periodical, *non-harmonic* steady-state. In this case is useful to utilize the hypercomplex symbolic method of representation of periodic, non-harmonic signals through hypercomplex “images”, elaborated by B.A. Rozenfeld (1949). In a previous paper (Rosman, 2010) an LPOP in periodical non-harmonicsteady-state was considered, supplied by the voltage

$$u_b(t) = \sqrt{2}\sum_{k=0}^{\infty} U_{b_k} \cos\left(kwt + g_{u_k}\right), \quad (18)$$

the current which flows through the LPOP being

$$i(t) = \sqrt{2}\sum_{k=0}^{\infty} I_k \cos\left(kwt + g_{i_k}\right), \quad (19)$$

with

$$g_{u_k} - g_{i_k} = j_k. \quad (20)$$

Utilizing the “polar” hypercomplex symbolic representation of periodic non-harmonic signals through hypercomplex “images”, it is useful to attach to the signals (18) and (19) such “images” (Rosman, *op. cit*) namely

$$\hat{u}_b = \sqrt{2}\sum_{k=0}^{\infty} 1_k U_{b_k} \cos\left(kwt + g_{u_{b_k}}\right) + \sqrt{2}\sum_{k=0}^{\infty} \mathrm{j}_k U_{b_k} \sin\left(kwt + g_{u_{b_k}}\right), \quad (21)$$

respectively

$$\hat{i} = \sqrt{2}\sum_{k=0}^{\infty} 1_k I_k \cos\left(kwt + g_{i_k}\right) + \sqrt{2}\sum_{k=0}^{\infty} \mathrm{j}_k I_k \sin\left(kwt + g_{i_k}\right). \quad (22)$$

Here functions 1_k , j_k are orthonormalized. If in the vector space of “images” $\hat{u}$, respectively $\hat{i}$, the vectorial product is introduced, associative and distributive with respect to addition, the so obtained vector space is a Hilbert one. The so defined algebra is commutative representing a real sum, of real numbers field (generated by 1_0) and the numberable set of complex number fields (generated by the pair of elements 1_k , j_k). The unity element of this algebra is

$$\sum_{k=0}^{\infty} 1_k = 1 . \tag{23}$$

Also

$$1_k^2 = 1_k,\ \mathrm{j}_k^2 = -1_k,\ 1_k \mathrm{j}_k = \mathrm{j}_k 1_k = \mathrm{j}_k,\ 1_p 1_q = 1_p \mathrm{j}_q = 1_q 1_p = 1_q \mathrm{j}_p = 0,\ (p \neq q) . \tag{24}$$

The symbolic relation

$$\frac{\mathrm{d}^m}{\mathrm{d}t^m} \equiv \sum_{k=0}^{\infty} \left(\mathrm{j}_m k w \right)^m, \quad m \in \mathbb{N}, \tag{25}$$

is valid too, rendering evident the advantage of this symbolic method to “algebrize” the differential operations with respect the time.

2. The Hypercomplex Instantaneous Apparent Power

Utilizing relation (8) as model which is valid in periodical, harmonic steady-state, it is possible to define a *hypercomplex instantaneous apparent power*, in periodical, non-harmonic steady-state namely

$$\hat{s} = \frac{1}{2} \hat{u}_b \left(\hat{\imath} + \hat{\imath}^* \right) . \tag{26}$$

Having in view that

$$\hat{\imath}^* = \sqrt{2} \sum_{k=0}^{\infty} 1_k I_k \left(kwt + g_{\mathrm{i}_k} \right) - \sqrt{2} \sum_{k=0}^{\infty} \mathrm{j}_k I_k \sin\left(kwt + g_{\mathrm{i}_k} \right) \tag{27}$$

and substituting in (26) expressions (21), (22) and (27) it results

$$\begin{aligned} \hat{s} = 2 &\left[\sum_{k=0}^{\infty} 1_k U_{b_k} \cos\left(kwt + g_{u_{b_k}} \right) + \sum_{k=0}^{\infty} \mathrm{j}_k U_{b_k} \sin\left(kwt + g_{u_{b_k}} \right) \right] \times \\ &\times \sum_{k=0}^{\infty} 1_k I_k \cos\left(kwt + g_{\mathrm{i}_k} \right). \end{aligned} \tag{28}$$

Taking into account relations (24) expression (28) becomes finally

$$\hat{s}=\sum_{k=0}^{\infty}1_k U_{b_k} I_k\left[\cos j_k+\cos\left(2kwt+g_{u_{b_k}}+g_{i_k}\right)\right]+ \\ +\sum_{k=0}^{\infty}\mathrm{j}_k U_{b_k} I_k\left[\sin j_k+\sin\left(2kwt+g_{u_{b_k}}+g_{i_k}\right)\right]. \tag{29}$$

It is reasonable to consider that the average value

$$\hat{S}=\langle\hat{s}\rangle=\frac{1}{T}\int_0^T\hat{s}\ \mathrm{d}t=\sum_{k=0}^{\infty}1_k U_{b_k} I_k\cos j_k+\sum_{k=0}^{\infty}\mathrm{j}_k U_{b_k} I_k\sin j_k \tag{30}$$

represents the *hypercomplex apparent power*, which may be written as

$$\hat{S}=\sum_{k=0}^{\infty}1_k P_k+\sum_{k=0}^{\infty}\mathrm{j}_k Q_k\ , \tag{31}$$

with

$$P_k=U_{b_k} I_k\cos j_k\ ,\quad Q_k=U_{b_k} I_k\sin j_k\ , \tag{32}$$

representing the active, respectively the reactive power corresponding to the harmonic of order *k*.

It is also reasonable to consider that

a) $\sum_{k=0}^{\infty}1_k U_{b_k} I_k\left[\cos j_k+\cos\left(2kwt+g_{u_{b_k}}+g_{i_k}\right)\right]$ represents the *hyper-complex active instantaneous power* and

b) $\sum_{k=0}^{\infty}\mathrm{j}_k U_{b_k} I_k\left[\sin j_k+\sin\left(2kwt+g_{u_{b_k}}+g_{i_k}\right)\right]$ represents the *hyper-complex reactive instantaneous power*.

The average values of these powers are

$$\hat{P}=\sum_{k=0}^{\infty}1_k U_{b_k} I_k\cos j_k\ ,\quad \hat{Q}=\sum_{k=0}^{\infty}\mathrm{j}_k U_{b_k} I_k\sin j_k\ , \tag{33}$$

with

$$\hat{P}=\sum_{k=0}^{\infty}1_k P_k\ ,\quad \hat{Q}=\sum_{k=0}^{\infty}\mathrm{j}_k Q_k\ , \tag{34}$$

so that relation (31) becomes

$$\hat{S} = \hat{P} + \hat{Q}\,. \tag{35}$$

Theirs moduli,

$$P = \sum_{k=0}^{\infty} U_k I_k \cos j_k\,, \quad Q = \sum_{k=0}^{\infty} U_k I_k \sin j_k\,, \tag{36}$$

represent the well-known expressions of active, respectively reactive power in periodical, non-harmonic steady-state.

3. The Hypercomplex Instantaneous Apparent Fluctuating Power

Expressions (29) and (30) lead to relation

$$\hat{s} = \hat{S} + \hat{s}_f\,, \tag{37}$$

where

$$\hat{s}_f = \sum_{k=0}^{\infty} 1_k U_{b_k} I_k \cos\left(2kwt + g_{u_{b_k}} + g_{i_k}\right) + \sum_{k=0}^{\infty} \mathrm{j}_k U_{b_k} I_k \sin\left(2kwt + g_{u_{b_k}} + g_{i_k}\right) \tag{38}$$

represents the *hypercomplex instantaneous apparent fluctuating power*. It is easy to observe that

$$\hat{s}_f = \frac{1}{2}\hat{u}_b \hat{i}\,, \tag{39}$$

where relations (21) and (22) were taken into account. In relation (33)

a) $\sum_{k=0}^{\infty} 1_k U_{b_k} I_k \cos\left(2kwt + g_{u_{b_k}} + g_{i_k}\right)$ represents the *hypercomplex instantaneous fluctuating active power* and

b) $\sum_{k=0}^{\infty} \mathrm{j}_k U_{b_k} I_k \sin\left(2kwt + g_{u_{b_k}} + g_{i_k}\right)$ represents the *hypercomplex instantaneous fluctuating reactive power*.

The moduli of these powers may be considered as representing

$$p_f = \sum_{k=0}^{\infty} U_{b_k} I_k \cos\left(2kwt + g_{u_{b_k}} + g_{i_k}\right) \tag{40}$$

– the *instantaneous fluctuating active power* and

$$q_f = \sum_{k=0}^{\infty} U_{b_k} I_k \sin\left(2kwt + g_{u_{b_k}} + g_{i_k}\right) \tag{41}$$

– the *instantaneous fluctuating reactive power*.

4. Conclusions

Using a variant of the hypercomplex symbolic representation of periodical, non-harmonic signals, proposed by B.A. Rozenfeld, the concepts of: hypercomplex instantaneous apparent power, hypercomplex apparent power, hypercomplex instantaneous active power, hypercomplex instantaneous reactive power, hypercomplex instantaneous apparent fluctuating power, hypercomplex instantaneous active fluctuating power, hypercomplex instantaneous reactive fluctuating power, which characterize the energy regime of a linear and passive one-port working in periodical, non-harmonic steady-state, are introduced. These powers represent a generalization of those proposed by V.N. Nedelcu, in harmonic steady-state, utilizing, with this view, the hypercomplex symbolic method of representation of periodical, non-harmonic signals.

REFERENCES

Nedelcu V.N., *Die einheitliche Leistungstheorie der unsymmetrischen und mehriwelligen Mehrphasensystem*. ETZ-A, **84**, *5*, 153-157 (1963).

Rozenfeld B.A., *Symbolic Method and Vectorial Diagrams for Non-Sinusoidal Currents* (in Russian). Tr. sem. vekt. i tenz. anal., **7**, 381-387 (1949).

Rosman H., *About a Symbolic Representation Method of Periodical Non-Harmonic Signals*. Proc. of the 6th Internat. Conf. On Electr. a. Power Engng. EPE 2010, Oct. 28-30, "Gh. Asachi" Techn. Univ., Jassy. Vol. **I**, 2010, 227-229.

PUTERILE INSTANTANEE HIPERCOMPLEXE ÎN CIRCUITE ELECTRICE LINIARE ŞI PASIVE, ÎN REGIM PERMANENT PERIODIC NEARMONIC

(Rezumat)

Se extinde propunerea lui V.N. Nedelcu de a caracteriza regimul energetic al unui uniport liniar şi pasiv, excitat de un semnal armonic, la cazul în care uniportul este excitat de un semnal periodic nearmonic. Se utilizează, în acest scop, metoda simbolică de reprezentare a semnalelor periodice nearmonice prin „imagini" hipercomplexe.