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AN OPTIMIZED PSPICE IMPLEMENTATION OF THE THREE-PHASE INDUCTION MACHINE MODEL

BY

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Abstract. The paper presents an improved version of implementing the three-phase model of the induction machine in PSpice program. The voltage equations and the flux expressions in the three-phase mathematical model are rewritten in a convenient form. Optimizing the implementation is therefore carried out: the circuit the Pspice simulation is based on has a lower complexity than that of the classical situation.

The proposed implementation can be used in a PSpice program, designed as a subcircuit which can be called when required by the main program.

The proposed code proved to be fast and easy to use, extremely useful in a wide range of matters regarding the simulation of induction machine behavior. No particular convergence problems occur due to the proposed implementation of the three-phase model.

Key words: induction machine; three-phase model; PSpice.

1. Introduction

In the study of the three-phase asynchronous motor, the *d*-*q* model is mainly used in steady state and transient conditions because it is simpler, more

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easily to implement and includes less equations. Using the *d-q* model is clearly advantageous especially if three-phase balanced voltage supply and machine parameters are identical for all three phases.

The limitations of the model emerge when, caused by whatever reasons, the parameters characterizing the three phases are not identical. In this situation, the only model that leads to correct results is the natural model for the asynchronous three-phase machine – the three-phase model.

In case the three-phase asynchronous machine is supplied with non-sinusoidal, unbalanced or asymmetrical three-phase voltage or current source, the use of the *d-q* model in the classic manner is no longer possible. In these cases was suggested by the authors in a previous paper (2010), to use the bidirectional Park transformation, which allows actually achieving both the direct and inverse voltage and current transformations simultaneously through a single processing circuit. But the advantage of using the *d-q* model reduces. The three-phase model can be used without restriction and can be employed as a standard one.

The three-phase model is a natural mathematical model that takes into account the phenomena in each phase and the interactions between stator and rotor phases as well. The number of equations is high, the parameters which characterize phases interactions are variable in time and the equation system is nonlinear. All these disadvantages led to an unpopular model, difficult to implement. However, in some circumstances using this model is mandatory.

The paper shows a PSpice optimized implementation of the three-phase model, starting from the possibilities and specifics of the software.

2. Analysing the Three-Phase Model Equations

We assume a stationary reference frame for the stator quantities and a rotating reference frame for the rotor quantities as shown in Fig. 1.

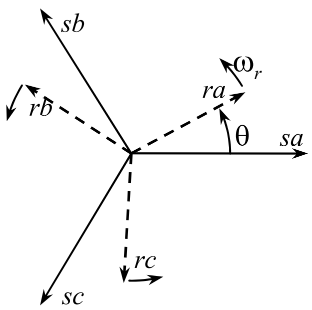

Fig. 1 – Reference frames.

The voltage eqs. of the stator and rotor circuits of induction machine, expressed in a matrix form, are

$$[v]=[R][i]+\frac{\mathrm{d}}{\mathrm{d}t}\{[L][i]\}, \tag{1}$$

where the inductor matrix is

$$[L]=\begin{bmatrix}[L_{ss}] & [M_{sr}]\\ [M_{rs}] & [L_{rr}]\end{bmatrix}.. \tag{2}$$

The stator and rotor inductor matrices are

$$[L_{ss}]=\begin{bmatrix} L_{\sigma s}+L_{os} & \frac{-L_{os}}{2} & \frac{-L_{os}}{2}\\ \frac{-L_{os}}{2} & L_{\sigma s}+L_{os} & \frac{-L_{os}}{2}\\ \frac{-L_{os}}{2} & \frac{-L_{os}}{2} & L_{\sigma s}+L_{os}\end{bmatrix}, \tag{3}$$

respectively

$$[L_{rr}]=\begin{bmatrix} L_{\sigma r}+L_{or} & \frac{-L_{or}}{2} & \frac{-L_{or}}{2}\\ \frac{-L_{or}}{2} & L_{\sigma r}+L_{or} & \frac{-L_{or}}{2}\\ \frac{-L_{or}}{2} & \frac{-L_{or}}{2} & L_{\sigma r}+L_{or}\end{bmatrix}. \tag{4}$$

The mutual inductor matrices, depending on angular position, θ, are

$$[M_{sr}]=\begin{bmatrix} M_{sr}\cos\theta & M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right)\\ M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right) & M_{sr}\cos\theta & M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right)\\ M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right) & M_{sr}\cos\theta\end{bmatrix}, \tag{5}$$

$$[M_{rs}]=\begin{bmatrix} M_{sr}\cos\theta & M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right) & M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right)\\ M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & M_{sr}\cos\theta & M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right)\\ M_{sr}\cos\left(\theta-\frac{2\pi}{3}\right) & M_{sr}\cos\left(\theta+\frac{2\pi}{3}\right) & M_{sr}\cos\theta\end{bmatrix}. \tag{6}$$

Considering

$$L_{os}=L_{or}=M_{sr}=M_{rs}=L_o \tag{7}$$

the stator and rotor inductor matrices become:

$$[L_{ss}]=\begin{bmatrix} L_{\sigma s}+L_o & \frac{-L_o}{2} & \frac{-L_o}{2} \\ \frac{-L_o}{2} & L_{\sigma s}+L_o & \frac{-L_o}{2} \\ \frac{-L_o}{2} & \frac{-L_o}{2} & L_{\sigma s}+L_o \end{bmatrix}, \tag{8}$$

respectively

$$[L_{rr}]=\begin{bmatrix} L_{\sigma r}+L_o & \frac{-L_o}{2} & \frac{-L_o}{2} \\ \frac{-L_o}{2} & L_{\sigma r}+L_o & \frac{-L_o}{2} \\ \frac{-L_o}{2} & \frac{-L_o}{2} & L_{\sigma r}+L_o \end{bmatrix}, \tag{9}$$

while, the mutual inductor matrices become

$$[M_{sr}]=[M_{sr}]^T=L_o\begin{bmatrix} \cos\theta & \cos\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) \\ \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\theta & \cos\left(\theta+\frac{2\pi}{3}\right) \\ \cos\left(\theta+\frac{2\pi}{3}\right) & \cos\left(\theta-\frac{2\pi}{3}\right) & \cos\theta \end{bmatrix}. \tag{10}$$

So, we can express the stator voltage equations as

$$\begin{cases} v_{sa}=R_s i_{sa}+L_{\sigma s}\dfrac{\mathrm{d}i_{sa}}{\mathrm{d}t}+v_{i\,sa}, \\ v_{sb}=R_s i_{sb}+L_{\sigma s}\dfrac{\mathrm{d}i_{sb}}{\mathrm{d}t}+v_{i\,sb}, \\ v_{sc}=R_s i_{sc}+L_{\sigma s}\dfrac{\mathrm{d}i_{sc}}{\mathrm{d}t}+v_{i\,sc}. \end{cases} \tag{11}$$

The induced voltage expressions, $v_{i\,sx}$, are intricate because the windings interaction is strongly depending on the angular position, θ

$$\begin{cases} v_{i\,sa}=L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{i_{sa}-\frac{1}{2}i_{sb}-\frac{1}{2}i_{sc}+\cos\theta i_{ra}+\cos\left(\theta+\frac{2\pi}{3}\right)i_{rb}+\cos\left(\theta-\frac{2\pi}{3}\right)i_{rc}\right\}, \\ v_{i\,sb}=L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{-\frac{1}{2}i_{sa}+i_{sb}-\frac{1}{2}i_{sc}+\cos\left(\theta-\frac{2\pi}{3}\right)i_{ra}+\cos\theta i_{rb}+\cos\left(\theta+\frac{2\pi}{3}\right)i_{rc}\right\}, \\ v_{i\,sc}=L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{-\frac{1}{2}i_{sa}-\frac{1}{2}i_{sb}+i_{sc}+\cos\left(\theta+\frac{2\pi}{3}\right)i_{ra}+\cos\left(\theta-\frac{2\pi}{3}\right)i_{rb}+\cos\theta i_{rc}\right\}. \end{cases} \tag{12}$$

Similarly, for the rotor quantities we can write

$$\begin{cases} v_{ra} = R_r i_{ra} + L_{\sigma r}\dfrac{\mathrm{d}i_{ra}}{\mathrm{d}t} + v_{i\,ra}, \\ v_{rb} = R_r i_{rb} + L_{\sigma r}\dfrac{\mathrm{d}i_{rb}}{\mathrm{d}t} + v_{i\,rb}, \\ v_{rc} = R_r i_{rc} + L_{\sigma r}\dfrac{\mathrm{d}i_{rc}}{\mathrm{d}t} + v_{i\,rc}. \end{cases} \tag{13}$$

$$\begin{cases} v_{i\,ra} = L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{ i_{ra} - \dfrac{1}{2}i_{rb} - \dfrac{1}{2}i_{rc} + \cos\theta i_{sa} + \cos\left(\theta - \dfrac{2\pi}{3}\right)i_{sb} + \cos\left(\theta + \dfrac{2\pi}{3}\right)i_{sc} \right\}, \\ v_{i\,rb} = L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{ -\dfrac{1}{2}i_{ra} + i_{rb} - \dfrac{1}{2}i_{rc} + \cos\left(\theta + \dfrac{2\pi}{3}\right)i_{sa} + \cos\theta i_{sb} + \cos\left(\theta - \dfrac{2\pi}{3}\right)i_{sc} \right\}, \\ v_{i\,ra} = L_0\dfrac{\mathrm{d}}{\mathrm{d}t}\left\{ -\dfrac{1}{2}i_{ra} - \dfrac{1}{2}i_{rb} + i_{rc} + \cos\left(\theta - \dfrac{2\pi}{3}\right)i_{sa} + \cos\left(\theta + \dfrac{2\pi}{3}\right)i_{sb} + \cos\theta i_{sc} \right\}. \end{cases} \tag{14}$$

3. PSpice Implementation of the Voltage Equations

Stator voltage eqs. (11) are easy to implement in PSpice since they are the mathematical expression of the behavior of some ordinary circuit elements, that is, resistor and inductor. An *E*-type controlled voltage source E_{sx} is added in Fig 2, corresponding to the induced voltage, $v_{i\,rx}$.

Its value is calculated separately (Fig. 2 *b*) because, as (12) points out, the current flowing through L_0 is made of six components. The *G*-type controlled current source, G_{sx}, performs the weighted sum of the six currents according to (12). The only meaning of the E_{sx} source is to copy the voltage value obtained in the right circuit to the left one (Fig. 2).

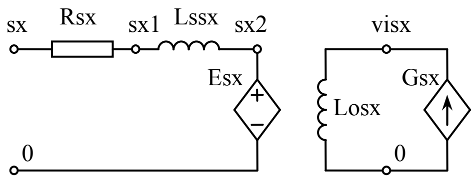

Fig. 2 – Stator voltage equation equivalent.

The correspondence between the electrical quantities in the eqs. and the quantities calculated in PSpice is

$$\begin{cases} v_{sx} & \leftrightarrow & \mathrm{V(sx)}, \\ i_{sx} & \leftrightarrow & \mathrm{I(Rsx)}, \\ v_{i\,sx} & \leftrightarrow & \mathrm{V(sx2) = V(visx)}. \end{cases} \tag{15}$$

Starting from eqs. (13) and (14) the implementation of the rotor voltage equations is similarly achieved. The result is shown in Fig. 3 and the correspondence between variables is indicated by

$$\begin{cases} v_{rx} & \leftrightarrow \quad \mathrm{V(rx)}, \\ i_{rx} & \leftrightarrow \quad \mathrm{I(Rrx)}, \\ v_{i\,rx} & \leftrightarrow \quad \mathrm{V(rx2)} = \mathrm{V(virx)}. \end{cases} \tag{16}$$

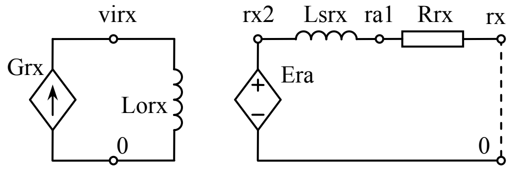

Fig. 3 – Rotor voltage equation equivalent.

4. PSpice Implementation of the Mechanical Quantities

The eqs. describing the mechanical behavior of the induction machine, part of the three-phase model, are the expression of electromagnetic torque and the movement equation

$$t_e = [i_s]^T \frac{\mathrm{d}}{\mathrm{d}\theta}\{[m_{sr}]\}[i_r], \tag{17}$$

$$J\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = t_e - t_r - F_\alpha \omega_r, \tag{18}$$

where the derivative of the mutual inductor matrix is

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\{[m_{sr}]\} = -L_o \begin{bmatrix} \sin\theta & \sin\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) \\ \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\theta & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \sin\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\theta \end{bmatrix}. \tag{19}$$

Implementing these in PSpice is presented in more detail by the authors in previous papers (1997, 2005) for the case of the *d*-*q* model. The generic integrator of the mechanical equations is represented in Fig. 4 *a*. A characteristic of implementing the three-phase model is the more complex expression of electromagnetic torque, the calculations being carried out by the controlled current source, Gte. In addition it has to be calculated the position angle of the rotor, θ, which appends to the quantities already calculated, t_e and ω_r,

$$\theta(t) = \theta_0 + \int_0^t \omega_r \mathrm{d}t, , \tag{20}$$

where the corresponding generic integrator is shown in Fig. 4 *b*.

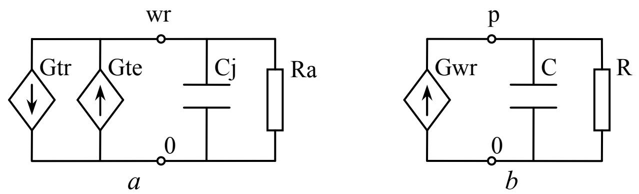

Fig. 4 – Mechanical equivalent circuits: *a* – angular speed generic integrator; *b* – angular position generic integrator.

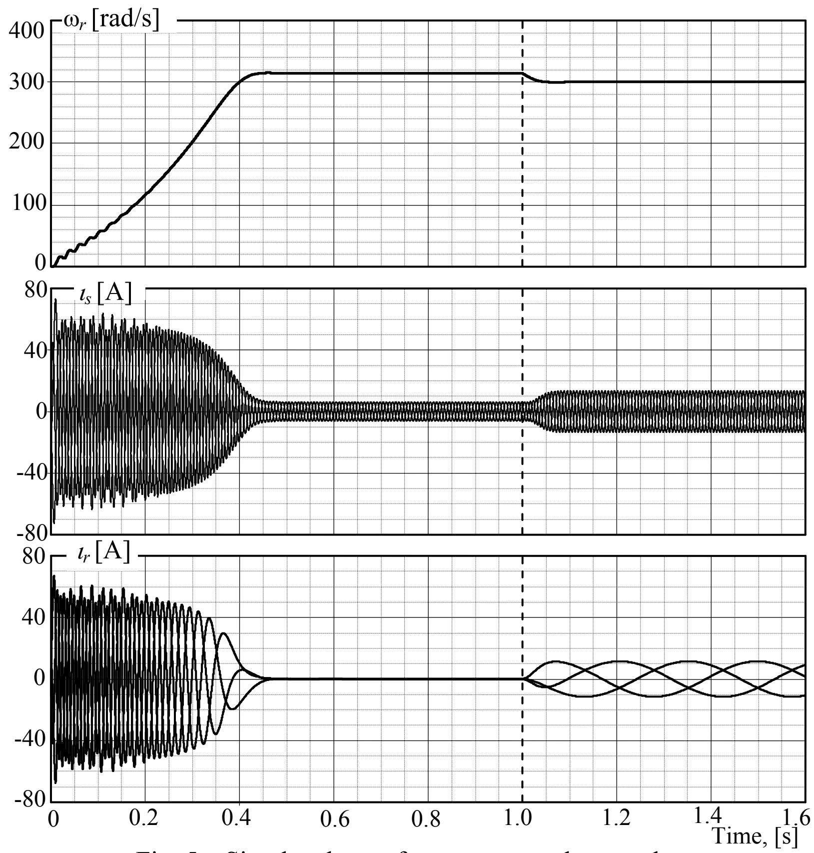

Fig. 5 – Simulated waveforms: *a* – angular speed; *b* – stator currents; *c* – rotor currents.

The correspondence between the mechanical quantities in the equations and the PSpice quantities is:

$$\begin{array}{llllllll} t_e & \leftrightarrow & \text{I(Gte)}; & \omega_r & \leftrightarrow & \text{V(wr)}; & \theta & \leftrightarrow \text{V(p)}; \\ t_r & \leftrightarrow & \text{I(Gtr)}; & \omega_{r0} & \leftrightarrow & \text{IC for Cj}; & \theta_0 & \leftrightarrow \text{IC for C.} \end{array} \qquad (21)$$

5. Experimental Results

Fig. 5 shows some of the results obtained by PSpice simulating the behavior of a 5 kW three-phase induction machine. The waveforms obtained for the angular speed and stator and rotor currents are shown in a typical situation. The motor starts in up to 0.5 s. Notice that at t =1 s the machine is abruptly loaded. The code proposed proved to be fast and easy to use, extremely useful in a wide range of issues regarding the simulation of induction machines behavior. No special convergence problems occurred due to the proposed three-phase model.

6. Conclusions

The implementation of the induction machine three-phase model proposed in this paper is optimized for PSpice. The implementation takes into account program specifics and its own capabilities of calculating and simulating the electrical circuits operation. Rewriting the voltage eqs. in the form of (11) and (13) together with expressions (12) and (14) allowed the use of the controlled current sources, G_{sx} and G_{rx}, which perform a large amount of calculations based on the values of the six windings currents

The voltage eqs. and flux expressions have been implemented by means of six equivalent circuits, each having four nodes. There are 24 nodes resulting in terms of electromagnetic phenomena simulation. Two more nodes are added to simulate mechanical phenomena. The total of 26 + 1 nodes is not little. However, it is reasonably large taking into account the complexity of the process to be simulated.

The implementation shown above can be used in a PSpice program, designed as a sub-circuit which can be called when required by the main program.

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O IMPLEMENTARE OPTIMIZATĂ ÎN PSPICE A MODELULUI TRIFAZAT AL MAŞINII ASINCRONE

(Rezumat)

Se prezintă o variantă imbunătăţită de implementare a modelului trifazat al maşinii asincrone în programul PSpice. Ecuaţiile de tensiuni şi expresiile fluxurilor se rescriu într-o formă convenabilă. Se realizează în acest fel o optimizare a implementării, circuitul pe baza căruia se efectuează simularea în PSpice având o complexitate mai redusă decât în situaţiile clasice. Implementarea propusă poate fi folosită în programul PSpice sub forma unui subcircuit ce poate fi apelat la nevoie de către programul principal.

Utilizarea implementării propuse s-a dovedit facilă si relativ rapidă, fiind extrem de utilă într-o gamă foarte largă de probleme de simulare a comportării maşinilor asincrone. In nici o situaţie nu au apărut probleme deosebite de convergenţă datorate implementării particulare a modelului trifazat propus.