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ABOUT ERRORS IN USING A SIMPLIFIED EQUIVALENT CIRCUIT OF INDUCTION MACHINE IN DYNAMIC CONDITIONS

BY

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Abstract. The paper presents an analytical and numerical study regarding the errors arrising by use of the simplified equivalent circuit of the induction machine. The dynamic behavior of an induction machine operating as a generator or motor has been considered. A comparative evaluation of how using the simplified equivalent circuit affects the currents, fluxes and electromagnetic torque has been carried out.

Key words: induction machine; rotor circuit; equivalent circuit; PSpice.

1. Introduction

The study of the induction machine operating as a motor or generator is usually carried out by using the two-phase model or spatial phasors. In terms of information provided, the two models are perfectly equivalent; the only difference between them is the mathematical form in which the equations describing machine operation are presented.

As known, starting from the mathematical model described by the operating equations, an equivalent electrical circuit can be achieved. In case the

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spatial phasors model is used, the mathematical form of the equations is more compact, and the equivalent circuit is shown in Fig. 1.

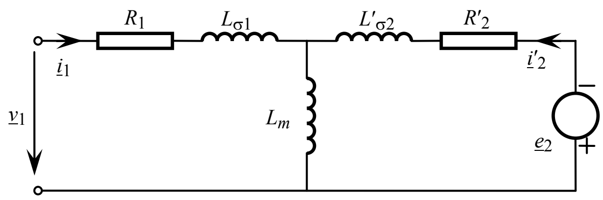

Fig. 1 – Equivalent circuit.

This paper aims to study the conditions in which the induction machine operating as a motor or generator close to the synchronism can be represented by a simplified equivalent electrical circuit without electromotive force in rotor, as shown in Fig. 2. The electromotive force source has been replaced by a resistor, which imposes null phase angle between the voltage across its terminals, $\underline{v}_2$, and the current which floes through it, $\underline{i}'_2$.

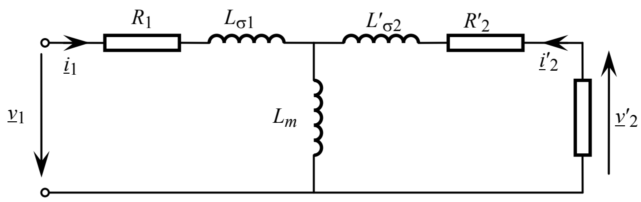

Fig. 2 – Simplified equivalent circuit.

This leads to an important simplification of the equations describing the operation of the induction machine, but limits their application area.

2. Rotor Voltage Equation Analysis

For shortening, in what follows will be assumed the spatial phasors model. Because the aim of this study concerns only the equivalent rotor circuit, it is necessary to consider the rotor voltage equation

$$0 = R'_2 \underline{i}'_2 + \frac{\mathrm{d}\underline{\psi}'_2}{\mathrm{d}t} - \mathrm{j}\omega_r \underline{\psi}'_2 \tag{1}$$

$$0 = R'_2 \underline{i}'_2 + \underbrace{\mathrm{j}\omega_{\psi_2}\underline{\Psi}'_2}_{-\underline{e}_{2\mathrm{din}}} + \underbrace{\frac{\underline{\Psi}'_2}{\Psi'_2}\cdot\frac{\mathrm{d}\Psi'_2}{\mathrm{d}t}}_{\underline{e}_{2\mathrm{st}}} \underbrace{-\mathrm{j}\omega_r\underline{\Psi}'_2}_{\underline{e}_{2\mathrm{ref}}} \tag{2}$$

$$0 = R_2^{'} \underline{i}_2^{'} - \underline{e}_{2\,\mathrm{din}} + \underline{e}_{2\,\mathrm{st}} + \underline{e}_{2\,\mathrm{ref}}\,, \tag{3}$$

where $\underline{e}_{r\,\mathrm{din}}$ is the dynamic electromotive force, and $\underline{e}_{r\,\mathrm{st}}$ is the statically-induced one.

As it is known, in steady-state the magnitude of the rotor flux is a constant so its derivative has zero value. Therefore, the $\underline{e}_{2\mathrm{st}}$ component no longer exists. Consequently, the voltage equation simplifies to

$$0 = R_2^{'} \underline{i}_2^{'} + \mathrm{j}\Omega_{\psi 2} \underline{\psi}_2^{'} - \mathrm{j}\Omega_r \underline{\psi}_2^{'} \tag{4}$$

and

$$\underline{i}_2^{'} = -\mathrm{j}\frac{1}{R_2^{'}}\left(\Omega_{\psi 2} - \Omega_r\right)\underline{\psi}_2^{'}\,. \tag{5}$$

A well known result is thus obtained: the rotor current phasor is perpendicular to the rotor flux and in phase (or opposition) with the induced electromotive force, $\underline{e}_{2\mathrm{ref}}$. Therefore, the spatial phasor of the rotor current is perpendicular to the spatial phasor of the rotor flux, fact already known. Moreover, $\underline{i}_r$ is collinear to $\underline{e}_r$**.** These results are underlined in Fig. 3 for both motor ($\Omega_{\psi_2} < \Omega_r$) and generator ($\Omega_{\psi_2} > \Omega_r$) working. In steady-state, the simplified model in Fig. 2 can be used and the numerical results would still hold.

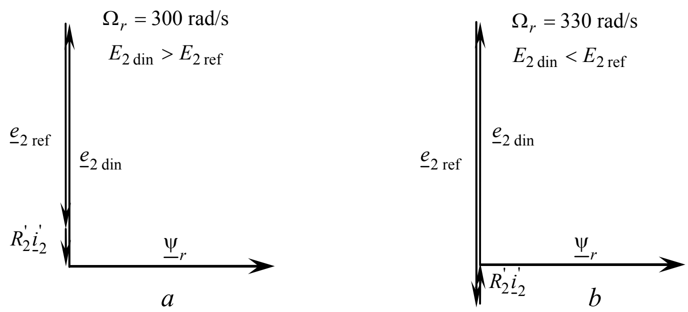

Fig. 3 – Rotor voltages phasors diagram in steady-state:
a – motor; *b* – generator.

In order to confirm and exemplify the analytical methods, the behavior of an induction machine has been simulated in PSpice, rated as follows:

$P_n = 5$ kW; $R_1 = 1.4\,\Omega$; $L_{\sigma 1} = 7.5$ mH;
$U_{1\mathrm{ln}} = 380$ V; $R_2^{'} = 1.4\,\Omega$; $L_{\sigma 2}^{'} = 7.5$ mH;
Y connection; $L_m = 117.5$ mH.

The numerical results relating to the amplitude of rotor voltages are shown in Fig. 4. It can be seen that around synchronism in both motor and generator operating, the value of electromotive force, $\underline{e}_{2x}$ dominates the voltage across resistor, $R_2 \underline{i}_2'$.

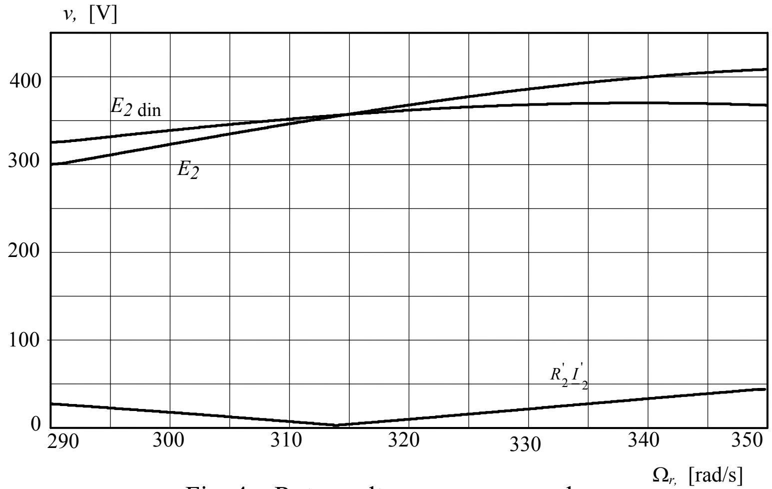

Fig. 4 – Rotor voltages *versus* speed.

3. About Transient

In transient, the phenomena taking place in the rotor are more complex. The following assumes only the variation of rotor quantities due to changes in angular speed, therefore a dynamic operating. We are not interested in what causes the rotor speed change; we are interested only in the acceleration value.

When the speed changes the rotor flux changes as well, as shown in Fig. 5. If we consider a steady or quasi-steady state, an increase of the rotor flux clearly occurs when the rotor angular speed increases. In dynamical operating, the value of acceleration does have a light influence on the instantaneous flux amplitude, but it does not alter the general tendency of increasing with the increase of angular speed.

Under these conditions, according to eq.(2), there is an additional term which heavily alters the phasor diagram. As it can be seen from Fig. 5, the value of the derivative of the rotor flux depends on the rotor acceleration approximately proportional.

The greater the latter is, the more altered the phasor diagram of rotor voltages is. Fig. 6 illustrates the four possible situations for the phasor diagram, depending on operating as a motor or generator and on the increase or decrease of angular rotor speed as well, which implies an increase or decrease of rotor flux.

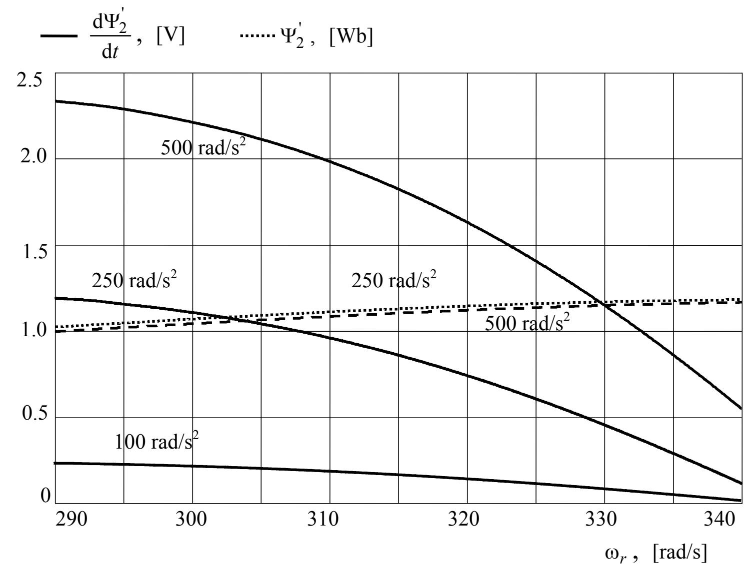

Fig. 5 – Rotor voltages phasors in transient *vs.* speed, for motor and generator working.

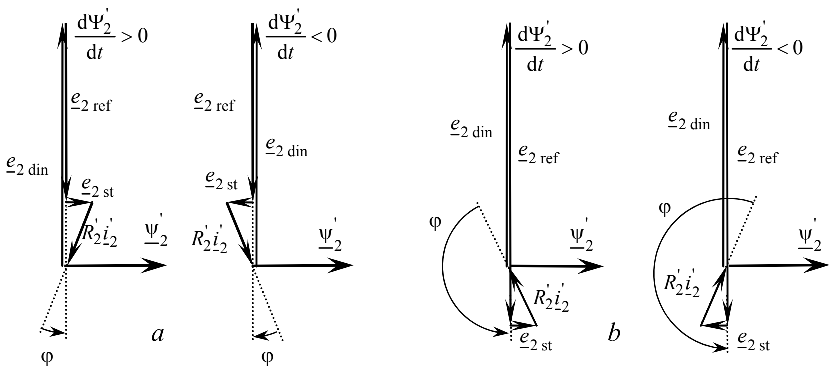

Fig. 6 – Rotor voltages phasors in transient: *a* – motor; *b* – generator.

The presence of statically induced electromotive force always leads to a change in the phase angle between the $\underline{v}_2$ voltage and the $\underline{i}_2^{'}$ rotor current. The phase angle, φ, is determined by the ratio

$$\mathrm{tg}\varphi = \frac{\mathrm{d}\Psi_2^{'} / \mathrm{d}t}{R_2^{'} I_2^{'}} . \tag{6}$$

The derivative of the flux is proportional to the rotor acceleration, while the voltage across resistance $R_2^{'}$ depends on the angular speed as it decreases towards zero when close to synchronism.

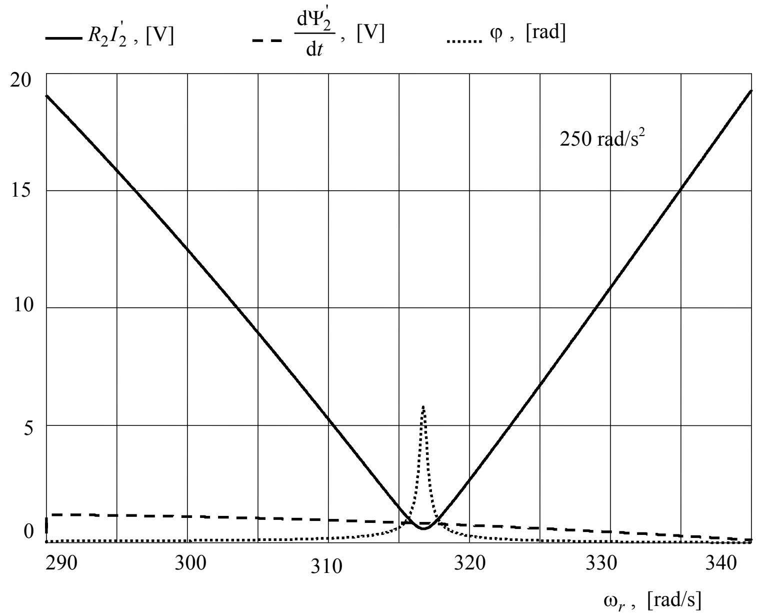

Fig. 7 – Rotor quantities *vs*. speed.

Fig. 7 shows the manner in which the two voltages that determine the value of the phase angle shift change when varying angular speed at constant acceleration. For the voltage across $R_2^{'}$ the dependence obtained is almost proportional to the slip, with a minimum corresponding to the synchronism speed. A decrease across the entire range of values for angular speed can be seen on the component related to the derivative of rotor flux. The ratio between the two components shows a maximum around the synchronism value, not because of the derivative increase, but mostly due to the sharp drop of the value of rotor current at slow loads, around the synchronism, respectively.

Based on these results two value ranges for the angular rotor speed are revealed in which the weight of each component is different. For angular speeds close to synchronism, the value of the phase angle, φ, is high, but the value of each particular component is reduced. We expect a relatively small influence over the quantities directly influenced, such as stator and rotor fluxes. In the meantime, however, highly sensitive quantities such as stator current and electromagnetic torque are more influenced. For angular speeds further away from synchronism, low influences on all quantities of interest are expected.

4. Numerical Simulation

The analytical results obtained by analyzing the rotor voltage equation have been compared to the numerical ones obtained by PSpice simulation of induction machine behavior. A machine characterized by the parameters previously presented has been considered; main quantities such as the currents through the coils (Figs. 8 and 10), fluxes (Fig. 9) and electromagnetic torque (Figs. 11 and 12) have been targeted.

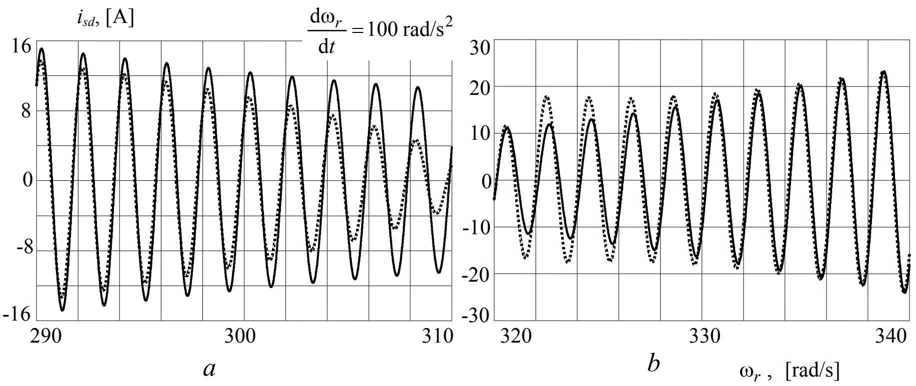

Fig. 8 – Stator currents comparison:
a – motor; *b* – generator.

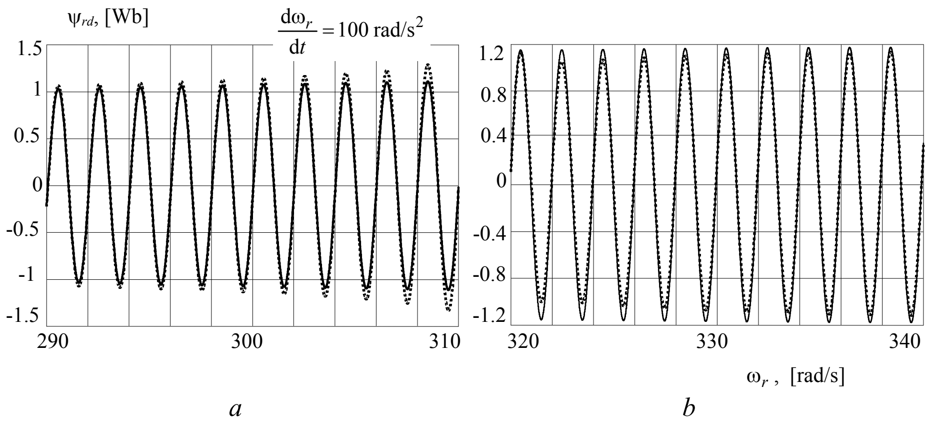

Fig. 9 – Comparing rotor fluxes:
a – motor; *b* – generator.

In all cases, a continuous line marks the results obtained by simulation using the equivalent circuit of Fig. 1. These results have been considered as a reference. The dashed line has been used for the results obtained using the simplified circuit of Fig. 2.

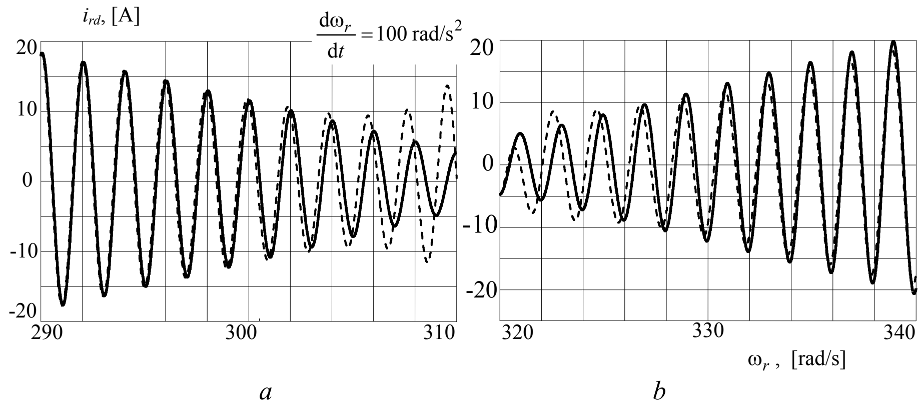

Fig. 10 – Comparing rotor currents: *a* – motor; *b* – generator.

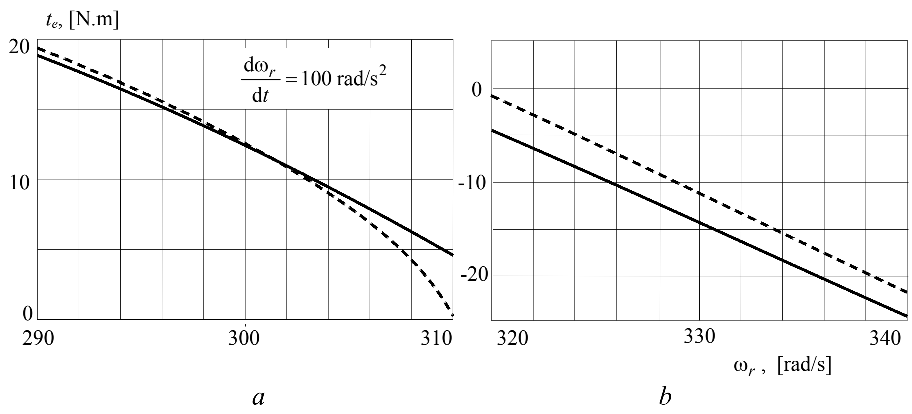

Fig. 11 – Comparing electromagnetic torques: *a* – motor; *b* – generator.

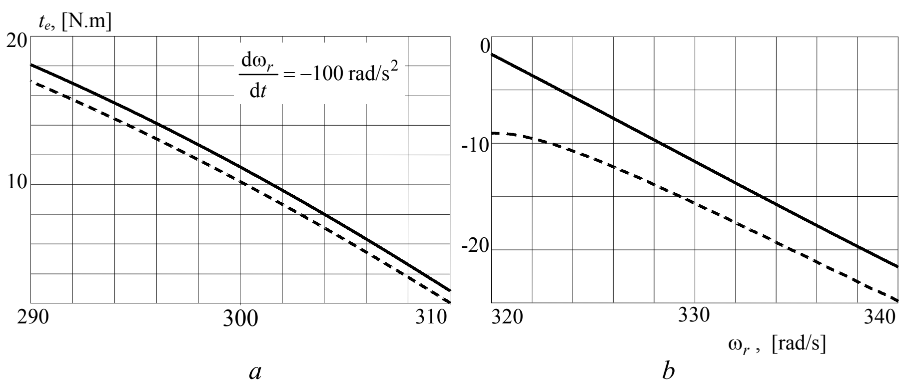

Fig. 12 – Comparing electromagnetic torques: *a* – motor; *b* – generator.

5. Conclusions

The study of the induction machine operating as motor or generator, with positive or negative slip around the synchronism, can be considerable simplified if the mathematical model corresponding to the equivalent simplified circuit of Fig. 2 is used. This can be done without restrictions in a steady-state characterized by a constant supply voltage and a constant rotor speed as well.

In case that, whatever the cause, the rotor changes its speed, a specific transient state occurs, distorted by the use of the simplified equivalent circuit. The value of the error introduced depends on the value of rotor acceleration. In the range of acceleration values that may be obtained in practical situations for a given induction machine, if the operation is close to synchronism, the errors introduced by using the simplified equivalent circuit are great. All quantities of interest are affected: currents, fluxes, electromagnetic torque. In such conditions, the only valid results are those based on the model corresponding to the complete equivalent circuit represented in Fig. 1.

For high slips or low rotor acceleration, the model based on the simplified equivalent circuit can be used in determining currents and fluxes. Even so, for the electromagnetic torque there are some errors that must be taken into account.

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CONSIDERAŢII PRIVID CIRCUITUL ECHIVALENT ROTORIC AL MAŞINII ASINCRONE ÎN REGIM DINAMIC

(Rezumat)

Se prezintă rezultatele unui studiu analitic şi numeric privind erorile rezultate prin utilizarea modelului matematic bazat pe circuitul echivalent simplificat al maşinii

asincrone. S-a avut în vedere regimul dinamic în care rotorul îşi modifică viteza de rotaţie cu o anumită acceleraţie, în regim de generator sau motor. S-a evaluat comparativ efectul utilizării circuitului echivalent simplificat asupra curenţilor, fluxurilor şi cuplului electromagnetic.