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SKIN EFFECT IN A STRAIGHT CYLINDRICAL CONDUCTOR, HAVING A CIRCULAR SECTION, IN PERIODICAL NON-HARMONIC STEADY-STATE

BY

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Abstract. Utilizing a symbolic method which permits to represent periodical non-harmonic signals through hypercomplex "images", the skin effect in a straight cylindrical conductor, through which flows a periodical non-harmonic current, is studied. Having in view that the low current's harmonics generate a low skin effect, the high frequencies – a net one and the medium harmonics produce a intermediate skin effect, the hypercomplex vectors moduli $\hat{\mathbf{E}}_{\text{int}}(x)$, $\hat{\mathbf{H}}_{\text{int}}(x)$ and $\hat{\mathbf{S}}_{l}(a)$ are determined, in each of these frequency domains. Since the conductor is considered to have a linear behavior in the electromagnetic field, applying the superposition theorem the above hypercomplex vectors moduli corresponding to the global skin effect are determined. In the same time are obtained the active power, $P_l(a)$, and the reactive power, $Q_l(a)$, in the conductor's length unit.

Key words: skin effect; straight cylindrical conductor with circular section; symbolic method to represent periodical non-harmonic signals through hypercomplex "images"; hypercomplex vectors moduli $\hat{\mathbf{E}}_{\text{int}}(x)$, $\hat{\mathbf{H}}_{\text{int}}(x)$, $\hat{\mathbf{S}}_{l}(a)$; active power, $P_l(a)$, and reactive power, $Q_l(a)$, corresponding to conductor's length unit.

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1. Introduction

In a previous work (Rosman, submitted) the skin effect in a conducting plate with a rectangular section through which flows a periodical non-harmonic current was studied. The method used to study this case was based on the utilization of a symbolic proceeding to represent the periodical non-harmonic signals through hypercomplex "images" (Rozenfeld, 1979).

In what follows, using the same proceeding as in the previous paper, the skin effect in a straight cylindrical conductor, having a circular section, was studied when through this conductor flows a periodical non-harmonic current. It is supposed that the conductor has a linear behavior in the electromagnetic field.

Let be such a conductor, infinitely long, having the straight section radius *a* (Fig. 1), situated at a sufficiently great distance from other conductors through which flow currents, so that the proximity effect may be neglected. When through the cylindrical conductor flows a *harmonical* current, $i=\sqrt{2}I\sin\omega t$, the complex effective values of the electromagnetic field's vectors are (Mocanu, 1981)

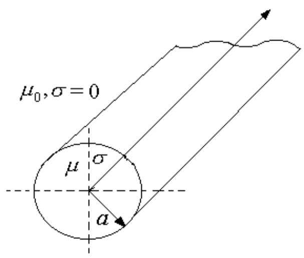

Fig. 1 – Transversal section through the cylindrical conductor.

$$\underline{E}_{\text{int}}(r)=\frac{\underline{\gamma}I}{d\pi a\sigma}\cdot\frac{\mathrm{I}_0(\underline{\gamma}r)}{\mathrm{I}_1(\underline{\gamma}a)},\ \underline{H}_{\text{int}}(r)=\frac{I}{2\pi a}\cdot\frac{\mathrm{I}_0(\underline{\gamma}r)}{\mathrm{I}_0(\underline{\gamma}a)},\quad(1)$$

with $r\in[0,a]$, where

$$\underline{\gamma}=(1+\mathrm{j})\alpha=\sqrt{\mathrm{j}\omega\mu\sigma},\ \alpha=\sqrt{\frac{\omega\mu\sigma}{2}},\quad(2)$$

$\underline{\gamma}$ being the complex propagation constant, α – the attenuation (phase) constant, σ – the conductor's conductibility, μ – his magnetic permeability, and I_0, I_1 – modified Bassel's functions of first species, and of zero, respectively one order.

The complex effective value of Poynting's vector modulus in a point situated on the conductor's surface ($r = a$) and corresponding to his length unit is, in this case,

$$\underline{S}_l(a)=\frac{\underline{\gamma}I^2}{(2\pi a)^2\sigma}\cdot\frac{\mathrm{I}_0(\underline{\gamma}a)}{\mathrm{I}_1(\underline{\gamma}a)}\cdot\quad(3)$$

The functions $\mathrm{I}_0(m)$ allow the series developments (Gray & Mathews, 1958)

$$\mathrm{I}_0(m) = 1 + \frac{m^2}{2\cdot 2} + \frac{m^4}{2^2\cdot 4^2} + \ldots, \mathrm{I}_1(m) = \frac{m}{2}\left[1 + \frac{m^2}{2(2+2)} + \frac{m^4}{2\cdot 4(2+2)(2+4)} + \ldots\right], \quad (4)$$

with $m = \underline{\gamma} r$, $\underline{\gamma} a$; so, retaining from these series only the two first terms expressions (1) become

$$\underline{E}_{\mathrm{int}}(r) = \frac{2\underline{I}}{\pi a^2\sigma}\cdot\frac{4+\underline{\gamma}^2 r^2}{8+\underline{\gamma}^2 a^2}, \underline{H}_{\mathrm{int}}(r) = \frac{\underline{I}}{2\pi a}\cdot\frac{4+\underline{\gamma}^2 r^2}{4+\underline{\gamma}^2 a^2}, \quad (5)$$

specific for the *weak skin effect*. Also the complex value of Poynting vector in a point situated on the conductor's surface ($r = a$) and corresponding to his length unit is, in this case,

$$\underline{S}_l(a) = \frac{I^2}{\pi^2 a^3\sigma}\cdot\frac{4+\underline{\gamma}^2 a^2}{8+\underline{\gamma}^2 a^2}. \quad (6)$$

In case of *net skin effect*, *i.e.* at high frequencies, the functions $\mathrm{I}_0(m)$, $\mathrm{I}_1(m)$ may be approximated with theirs asymptotical values (Gray & Mathews, 1958)

$$\mathrm{I}_0(m) \approx \mathrm{I}_1(m) = \frac{\mathrm{e}^m}{\sqrt{2\pi m}}, \quad (7)$$

so that expressions (1) become

$$\underline{E}_{\mathrm{int}}(r) = \frac{\underline{\gamma}\underline{I}}{2\pi a\sigma}\mathrm{e}^{-\underline{\gamma}(a-r)},\ \underline{H}_{\mathrm{int}}(r) = \frac{\underline{I}}{2\pi a}\mathrm{e}^{-\underline{\gamma}(a-r)}. \quad (8)$$

and consequently

$$\underline{S}_l(a) = \frac{\underline{\gamma} I^2}{4\pi^2 a^2\sigma}. \quad (9)$$

In what follows the dispersion with respect to time of material constants ε, μ is neglected.

2. Study Method

Since the studied regime is a periodical non-harmonic steady-state, it is usefully to utilize the symbolic method which permits to represent periodical non-harmonic signals through hypercomplex “images” (Rozenfeld, 1949). The basic principles of this method are exposed in many previous papers so that in this section are presented briefly only the most significant elements of this method. Thus, considering a periodical non-harmonic signal which admits the Fourier series development

$$a(t)=\sum_{k=0}^{\infty}A_k^{'}\cos k\omega t+\sum_{k=0}^{\infty}A_k^{''}\sin k\omega t=\sqrt{2}\sum_{k=0}^{\infty}A_k\sin(k\omega t+\alpha_k), \quad (10)$$

the hypercomplex “image”

$$\hat{A}=\sum_{k=0}^{\infty}1_k A_k^{'}+\sum_{k=0}^{\infty}\mathrm{j}_k A_k^{''} \quad (11)$$

or

$$\hat{a}=\sum_{k=0}^{\infty}1_k A_k\cos(k\omega t+\alpha_k)+\sum_{k=0}^{\infty}\mathrm{j}_k A_k\sin(k\omega t+\alpha_k) \quad (12)$$

may be attached to the considered signal, where

$$\sum_{k=0}^{\infty}1_k=1 \quad (13)$$

and

$$1_k^2=1_k,\ \mathrm{j}_k^2=-1_k,\ 1_k\mathrm{j}_k=\mathrm{j}_k 1_k=\mathrm{j}_k,\ 1_p 1_q=\mathrm{j}_p\mathrm{j}_q=1_p\mathrm{j}_q=1_q\mathrm{j}_p=0,\ (p\neq q). \quad (14)$$

The advantages of this method are the consequences of the symbolical identity

$$\frac{\mathrm{d}^m}{\mathrm{d}t^m}\equiv\sum_{k=0}^{\infty}(\mathrm{j}_k k\omega)^m,\ m\in\mathbb{N}, \quad (15)$$

satisfied by the hypercomplex elements $\hat{a}$ or $\hat{A}$.

3. Hypercomplex Vectors $\hat{\mathbf{E}}_{\mathrm{int}}(r)$, $\hat{\mathbf{H}}_{\mathrm{int}}(r)$, $\hat{\mathbf{S}}(a)$

$\hat{\mathbf{E}}_{\mathrm{int}}(x)$ and $\hat{\mathbf{H}}_{\mathrm{int}}(x)$ are the hypercomplex “images” of vectors $\mathbf{E}_{\mathrm{int}}(r,t)$, $\mathbf{H}_{\mathrm{int}}(r,t)$ which are periodical but non-harmonic functions of time when through the cylindrical conductor flows the current

$$i = \sqrt{2}\sum_{k=0}^{\infty} I_k \sin(k\omega t + \gamma_{i_k}). \tag{16}$$

Theirs complex effective values may be deduced from relations (1) substituting $\underline{\gamma}$ with expression

$$\hat{\gamma} = \sum_{k=0}^{\infty}(1_k + \mathrm{j}_k)\alpha_k = \sqrt{\sum_{k=0}^{\infty}\mathrm{j}_k k\omega\sigma\mu},\ \alpha_k = \sqrt{\frac{k\omega\sigma\mu}{2}}, \tag{17}$$

established in a previous paper (Rosman, 1979), where $\hat{\gamma}$ is *the hypercomplex constant of propagation* and α_k – the attenuation (phase) constant of k-rank harmonic. Consequently

$$\hat{E}_{\text{int}}(r) = \frac{\hat{\gamma}\hat{I}}{2\pi a\sigma}\cdot\frac{\mathrm{I}_0(\hat{\gamma}r)}{\mathrm{I}_1(\hat{\gamma}a)},\ \hat{H}_{\text{int}}(r) = \frac{\hat{I}}{2\pi a}\cdot\frac{\mathrm{I}_0(\hat{\gamma}r)}{\mathrm{I}_0(\hat{\gamma}a)}, r \in [0,a], \tag{18}$$

with

$$\hat{I} = \sum_{k=0}^{\infty}(1_k I_k^{'} + \mathrm{j}_k I_k^{''}) \tag{19}$$

– the hypercomplex “image” of the current (16).

The hupercomplex modulus of Poynting vector in a point situated on the conductor’s surface ($r = a$) is, in this case,

$$\hat{S}(a) = \hat{E}_{\text{int}}(a)\hat{H}^{*}_{\text{int}}(a), \tag{20}$$

where $H^{*}_{\text{int}}(a)$ is the hypercomplex conjugate of $\hat{H}_{\text{int}}(a)$. Having in view relations (18) the hypercomplex modulus of Poynting vector (20), corresponding to conductor’s length unit, is

$$\hat{S}_l(a) = \frac{\hat{\gamma}I^2}{4\pi^2 a^2\sigma}\cdot\frac{\mathrm{I}_0(\hat{\gamma}a)}{\mathrm{I}_1(\hat{\gamma}a)}; \tag{21}$$

here $I^2 = \hat{I}\hat{I}^{*}$.

The hupercomplex apparent power defined in a previous paper (Rosman, 1960) is given by relation

$$\hat{s} = \hat{U}\hat{I}^{*} = \sum_{k=0}^{\infty}1_k P_k + \sum_{k=0}^{\infty}\mathrm{j}_k Q_k, \tag{22}$$

where

$$P=\sum_{k=0}^{\infty}P_k\,,\quad Q=\sum_{k=0}^{\infty}Q_k\,,\tag{23}$$

represent the active, respectively the reactive power. But the hypercomplex apparent power on the cylindrical conductor's length unit is equal with the hypercomplex modulus (21) of Poynting vector in a point situated on the conductor's surface multiplied with $2\pi a$ namely

$$\hat{s}=2\pi a\hat{S}(a).\tag{24}$$

Consequently it is possible to estimate the active power dissipated on the conductor's length unit and then the conductor's resistance increasing coefficient in ac,

$$k_R=\frac{P_l}{P_{0l}},\tag{25}$$

where

$$P_l=\frac{P}{2\pi a}\ \text{and}\ P_{0l}=R_{0l}I^2=\frac{1}{\pi a^2\sigma}I^2,\tag{26}$$

P_{0l} being the dissipated power in the cylindrical conductor in dc, on his length unit.

If the current's fundamental frequency is not to high, in the frequencies spectrum of this one may be identified three different domains in which the skin effect produced by the respectively current's harmonics is qualitatively different namely: a) the domain $[0, pf]$, where the frequencies are sufficiently low so that the skin effect may be considered as being a *weak* one; b) the domain $[pf, qf]$, in which the current's harmonics frequencies are medium and as such the skin effect is, in his turn, *medium*; c) the domain $[qf, \infty)$ where the skin effect is *net*. The frequency of current's fundamental term is f and $p, q \in$ □, with $p < q$.

3.1. The Skin Effect Produced by Low Frequencies

In this case expressions (1) of hypercomplex state vectors moduli, $\hat{E}_{\text{int}}(r)$, $\hat{H}_{\text{int}}(r)$, may be approximated with expressions (5), which result retaining from series developments (4) only the first two terms. So

$$\hat{E}_{\text{int}\,p}(r)=\frac{2\hat{I}_p}{\pi a^2\sigma}\cdot\frac{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu r^2}{8+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu a^2},\quad \hat{H}_{\text{int}\,p}(r)=\frac{\hat{I}_p}{2\pi a}\cdot\frac{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu r^2}{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu a^2},\tag{27}$$

where

$$\hat{I}_p = \sum_{k=0}^{p} \left(1_k I_k^{'} + \mathrm{j}_k I_k^{''} \right) \tag{28}$$

and

$$\hat{\gamma}_p = \sqrt{\sum_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu} = \sum_{k=0}^{p} \left(1_k + \mathrm{j}_k \right)\alpha_k . \tag{29}$$

Taking into account relations (13) expressions (27) become

$$\begin{cases} \hat{E}_{\mathrm{int}\,p}(r) = \dfrac{2\hat{I}_p}{\pi a^2 \sigma} \cdot \dfrac{\sum\limits_{k=0}^{p} \left(1_k 4 + \mathrm{j}_k k\omega\sigma\mu r^2 \right)}{\sum\limits_{k=0}^{p} \left(1_k 8 + \mathrm{j}_k k\omega\sigma\mu a^2 \right)}, \\ \hat{H}_{\mathrm{int}\,p}(r) = \dfrac{\hat{I}_p}{2\pi a} \cdot \dfrac{\sum\limits_{k=0}^{p} \left(1_k 4 + \mathrm{j}_k k\omega\sigma\mu r^2 \right)}{\sum\limits_{k=0}^{p} \left(1_k 4 + \mathrm{j}_k k\omega\sigma\mu a^2 \right)}. \end{cases} \tag{30}$$

Amplifying the fractions from the right side of relations (30) with the hypercomplex conjugates of theirs denominators, $\sum_{k=0}^{p} \left(1_k 8 - \mathrm{j}_k k\omega\sigma\mu a^2 \right)$, respectively $\sum_{k=0}^{p} \left(1_k 4 - \mathrm{j}_k k\omega\sigma\mu a^2 \right)$, relations (30) become

$$\begin{cases} \hat{E}_{\mathrm{int}\,p}(r) = \dfrac{2\hat{I}_p}{\pi a^2 \sigma} \cdot \dfrac{\sum\limits_{k=0}^{p} \left[1_k \left(32 + k^2\omega^2\sigma^2\mu^2 r^2 \right) + \mathrm{j}_k 4\left(r^2 - a^2 \right) k\omega\sigma\mu \right]}{64 + \omega^2\sigma^2\mu^2 a^4 \sum\limits_{k=0}^{p} k^2}, \\ \hat{H}_{\mathrm{int}\,p}(r) = \dfrac{\hat{I}_p}{2\pi a} \cdot \dfrac{\sum\limits_{k=0}^{p} \left[1_k \left(16 + k^2\omega^2\sigma^2\mu^2 r^2 \right) + \mathrm{j}_k 4\left(r^2 - a^2 \right) k\omega\sigma\mu \right]}{16 + \omega^2\sigma^2\mu^2 a^4 \sum\limits_{k=0}^{p} k^2}. \end{cases} \tag{31}$$

As regards expression (20) of Poynting vector hypercomplex modulus, in a point situated on the conductor's lateral surface, this one becomes

$$\hat{S}_{l_p}(a) = \frac{2I_p^2}{\pi a^2 \sigma} \cdot \frac{\sum_{k=0}^{p}\left[1_k\left(32 + k^2\omega^2\sigma^2\mu^2 a^4\right) + \mathrm{j}_k 4k\omega\sigma\mu a^2\right]}{64 + \omega^2\sigma^2\mu^2 a^4 \sum_{k=0}^{p} k^2}. \tag{32}$$

Having in view relations (21),…,(24) it results the expressions of active and reactive power on conductor's length unit

$$P_{l_p} = \frac{2I_p^2}{\pi a^2 \sigma} \cdot \frac{32 + \omega^2\sigma^2\mu^2 a^4 \sum_{k=0}^{p} k^2}{64 + \omega^2\sigma^2\mu^2 a^4 \sum_{k=0}^{p} k^2}, \tag{33}$$

respectively

$$Q_{l_p} = \frac{8I_p^2}{\pi} \cdot \frac{\omega\mu a \sum_{k=0}^{p} k}{64 + \omega^2\sigma^2\mu^2 a^4 \sum_{k=0}^{p} k^2}. \tag{34}$$

But (Ryžik & Gradshtein, 1961)

$$\sum_{k=0}^{p} k = \frac{p(p+1)}{2}, \quad \sum_{k=0}^{p} k^2 = \frac{p(p+1)(p+2)}{6}, \tag{35}$$

so relations (33) and (34) become

$$P_{l_p} = \frac{2I_p^2}{\pi a^2 \sigma} \cdot \frac{192 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4}{384 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4}, \tag{36}$$

respectively

$$Q_{l_p} = \frac{24I_p^2}{\pi} \cdot \frac{p(p+1)\omega\mu a}{384 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4}. \tag{37}$$

3.2. Skin Effect Produced by Medium Frequencies

The medium rank currents harmonics, (*pf*,...,*qf*), produce at their turn a medium skin effect, characterized by hypercomplex modulus of electromagnetic field state vectors given by relations (18), where current's hypercomplex "images", $\hat{I}$, must be substituted with

$$\hat{I}_{pq} = \sum_{k=p}^{q} (1_k I_k^{'} + \mathrm{j}_k I_k^{''}) \tag{38}$$

and the hypercomplex propagation constant, $\hat{\gamma}$, with (s. rel. (17))

$$\hat{\gamma}_{pq} = \sqrt{\sum_{k=p}^{q} \mathrm{j}_k k\omega\sigma\mu} = \sum_{k=p}^{q} (1_k + \mathrm{j}_k)\alpha_k, \tag{39}$$

obtaining

$$\hat{E}_{\mathrm{int}\,pq}(r) = \frac{\hat{\gamma}_{pq}\hat{I}_{pq}}{2\pi a\sigma} \cdot \frac{\mathrm{I}_0(\hat{\gamma}_{pq}r)}{\mathrm{I}_1(\hat{\gamma}_{pq}a)},\ \hat{H}_{\mathrm{int}\,pq}(r) = \frac{\hat{I}_{pq}}{2\pi a} \cdot \frac{\mathrm{I}_0(\hat{\gamma}_{pq}r)}{\mathrm{I}_0(\hat{\gamma}_{pq}a)}, r \in [0, a]. \tag{40}$$

In the same time

$$\hat{S}_{l_{pq}}(r) = \frac{\hat{\gamma}_{pq} I_{pq}^2}{4\pi^2 a^2 \sigma} \cdot \frac{\mathrm{I}_0(\hat{\gamma}_{pq}a)}{\mathrm{I}_1(\hat{\gamma}_{pq}a)}. \tag{41}$$

The explicit calculus of hypercomplex moduli $\hat{E}_{\mathrm{int}\,pq}(r)$, $\hat{H}_{\mathrm{int}\,pq}(r)$ as well as of active power, $P_{l_{pq}}$, and reactive power, $Q_{l_{pq}}$, on the conductor's length unit is quite difficult. As regards the powers the following relation

$$2\pi a\hat{S}_{l_{pq}}(a) = \sum_{k=p}^{q} 1_k P_{l_{pqk}} + \sum_{k=p}^{q} \mathrm{j}_k Q_{l_{pqk}} \tag{42}$$

may be written, with

$$P_{l_{pq}} = \sum_{k=p}^{q} P_{l_{pqk}},\ Q_{l_{pq}} = \sum_{k=p}^{q} Q_{l_{pqk}}. \tag{43}$$

3.3. Skin Effect Produced by High Harmonics

The current's harmonics having the frequencies greater as qf generate a net skin effect. In such situation the hypercomplex modulus of electromagnetic fields state vectors may be performed using relations similar to those utilized in harmonic steady-state, that is relations (8), namely

$$\hat{E}_{\mathrm{int}\,q}(r)=\frac{\hat{\gamma}_q\hat{I}_q}{2\pi a\sigma}\mathrm{e}^{-\hat{\gamma}_q(a-r)},\ \hat{H}_{\mathrm{int}\,q}(r)=\frac{\hat{I}_q}{2\pi a}\mathrm{e}^{-\hat{\gamma}_q(a-r)}, \tag{44}$$

where

$$\hat{I}_q=\sum_{k=q}^{\infty}\left(1_k I_k^{'}+\mathrm{j}_k I_k^{''}\right) \tag{45}$$

and

$$\hat{\gamma}_q=\sqrt{\sum_{k=q}^{\infty}\mathrm{j}_k k\omega\sigma\mu}=\sum_{k=q}^{\infty}\left(1_k+\mathrm{j}_k\right)\alpha_k. \tag{46}$$

As well the Poynting vector hypercomplex modulus in a point situated on the cylindrical conductor's surface is, having in view relation (9),

$$\hat{S}_{l_q}(a)=\frac{\hat{\gamma}_q I_q^2}{4\pi^2a^2\sigma}=\frac{I_q^2}{4\pi^2a^2\sigma}\sum_{k=q}^{\infty}\left(1_k+\mathrm{j}_k\right)\alpha_k=\sqrt{\frac{\omega\mu}{2\sigma}}\cdot\frac{I_q^2}{4\pi^2a^2}\sum_{k=q}^{\infty}\left(1_k+\mathrm{j}_k\right)\sqrt{k}, \tag{47}$$

where relation (17_2) was taken into account too.

Having in view relations (21),...,(23) it is possible to establish the expressions of active and reactive powers corresponding to cylindrical conductor's length unit namely

$$P_{l_q}=\sqrt{\frac{\omega\mu}{2\sigma}}\cdot\frac{I_q^2}{4\pi}\sum_{k=p}^{q}\sqrt{k},\ Q_{l_q}=\sqrt{\frac{\omega\mu}{2\sigma}}\cdot\frac{I_q^2}{4\pi}\sum_{k=p}^{q}k. \tag{48}$$

3.4. The Global Effect

The cylindrical conductor's material having a linear behavior in electromagnetic field, the determination of quantities ($\hat{E}_{\mathrm{int}}(r)$, $\hat{H}_{\mathrm{int}}(r)$, $\hat{S}_l(a)$, P_l,

Q_l) which characterize the global skin effect may be realized utilizing the superposition theorem namely

$$\begin{cases} \hat{E}_{\text{int}}(r) = \hat{E}_{\text{int}\,p}(r) + \hat{E}_{\text{int}\,pq}(r) + \hat{E}_{\text{int}\,q}(r), \\ \hat{H}_{\text{int}}(r) = \hat{H}_{\text{int}\,p}(r) + \hat{H}_{\text{int}\,pq}(r) + \hat{H}_{\text{int}\,q}(r); \end{cases} \tag{49}$$

$$\hat{S}_l(a) = \hat{S}_{l_p}(a) + \hat{S}_{l_{pq}}(r) + \hat{S}_{l_q}(a); \tag{50}$$

$$P_l = P_{l_p} + P_{l_{pq}} + P_{l_q},\ Q_l = Q_{l_p} + Q_{l_{pq}} + Q_{l_q}. \tag{51}$$

Having in view relations (31), (40) and (44) expressions (49) become

$$\begin{aligned} \hat{E}_{\text{int}}(r) = {} & \frac{2\hat{I}_p}{\pi a^2\sigma} \cdot \frac{\sum_{k=0}^{p}\left[1_k\left(32 + k^2\omega^2\sigma^2\mu^2 r^2\right) + \mathrm{j}_k 4\left(2r^2 - a^2\right)k\omega\sigma\mu\right]}{384 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4} + \\ & + \frac{\hat{\gamma}_{pq} I_{pq}}{2\pi a\sigma} \cdot \frac{\mathrm{I}_0\left(\hat{\gamma}_{pq} r\right)}{\mathrm{I}_1\left(\hat{\gamma}_{pq} a\right)} + \frac{\hat{\gamma}_q \hat{I}_q}{2\pi a\sigma} \mathrm{e}^{-\hat{\gamma}_q (a-r)}, \end{aligned} \tag{52}$$

$$\begin{aligned} \hat{H}_{\text{int}}(r) = {} & \frac{3\hat{I}_p}{\pi a} \cdot \frac{\sum_{k=0}^{p}\left[1_k\left(16 + k^2\omega^2\sigma^2\mu^2 r^2\right) + \mathrm{j}_k 4\left(r^2 - a^2\right)k\omega\sigma\mu\right]}{96 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4} + \\ & + \frac{\hat{I}_{pq}}{2\pi a} \cdot \frac{\mathrm{I}_0\left(\hat{\gamma}_p r\right)}{\mathrm{I}_0\left(\hat{\gamma}_p a\right)} + \frac{\hat{I}_q}{2\pi a} \mathrm{e}^{-\hat{\gamma}_q (a-r)}. \end{aligned} \tag{53}$$

As well taking into account relations (21), (41) and (47) expression (50) becomes

$$\begin{aligned} \hat{S}_l(a) = {} & \frac{6I_p^2}{\pi^2 a^3\sigma} \cdot \frac{\sum_{k=0}^{p}\left[1_k\left(32 + k^2\omega^2\sigma^2\mu^2 r^4\right) + \mathrm{j}_k k\omega\sigma\mu\right]}{384 + \omega^2\sigma^2\mu^2 a^4} + \\ & + \frac{\hat{\gamma}_{pq} I_{pq}^2}{4\pi^2 a^2\sigma} \cdot \frac{\mathrm{I}_0\left(\hat{\gamma}_{pq} a\right)}{\mathrm{I}_1\left(\hat{\gamma}_{pq} a\right)} + \sqrt{\frac{\omega\mu}{2\sigma}} \cdot \frac{I_q^2}{4\pi} \sum_{k=q}^{\infty}\left(1_k + \mathrm{j}_k\right)\sqrt{k}. \end{aligned} \tag{54}$$

At the same time

$$P_l = \frac{2I_p^2}{\pi a^2 \sigma} \cdot \frac{192 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4}{384 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4} + P_{l_{pq}} + \sqrt{\frac{\omega\mu}{2\sigma}} \cdot \frac{I_q^2}{4\pi} \sum_{k=q}^{\infty} \sqrt{k}, \quad (55)$$

respectively

$$Q_l = \frac{24 I_p^2}{\pi} \cdot \frac{p(p+1)\omega\mu a}{384 + p(p+1)(p+2)\omega^2\sigma^2\mu^2 a^4} + Q_{l_{pq}} + \sqrt{\frac{\omega\mu}{2\sigma}} \cdot \frac{I_q^2}{4\pi} \sum_{k=q}^{\infty} \sqrt{k}, \quad (56)$$

where relations (36), (37), (43) and (48) were taked into account too.

Knowing the expressions of powers P_l, (rel. (55)), and P_{ol}, (rel. (26)), the conductor's resistance increasing coefficient in ac may be determined using relation (25).

3.4. Particular Case

If the current's fundamental term frequency is enough high, the skin effect, in his assembly, may be considered as being a *net* one. The electromagnetic field's state vectors hypercomplex moduli may be determined with relations of type (8), where $\underline{\gamma}$ is substituted with $\hat{\gamma}$ and $\underline{I}$ with $\hat{I}$ namely

$$\hat{E}_{\text{int}}(r) = \frac{\hat{\gamma}\hat{I}}{2\pi a \sigma} \mathrm{e}^{-\hat{\gamma}(a-r)}, \quad \hat{H}_{\text{int}}(r) = \frac{\hat{I}}{2\pi a} \mathrm{e}^{-\hat{\gamma}(a-r)}. \quad (57)$$

In the same time utilizing a proceeding similar to that used in (9) it is possible to obtain the Poynting vector's hypercomplex modulus (refered to conductor's length unit)

$$\hat{S}_l(a) = \frac{\hat{\gamma} I^2}{4\pi^2 a^2 \sigma}. \quad (58)$$

Having in view relations (22),...,(24) it is possible to establish the relations of active and reactive powers corresponding to conductor's length unit

$$P_l = \frac{1}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}} I^2 \sum_{k=0}^{n} \sqrt{k}, \quad Q_l = \frac{1}{2\pi a}\sqrt{\frac{\omega\mu}{2\sigma}} I^2 \sum_{k=0}^{n} \sqrt{k}, \quad (59)$$

where only the first n current's harmonics were considered.

In this particular case is possible to calculate the resistance increasing coefficient in ac, given by relation (25). Taking into account relations (26) and (59_1) it results

$$k_R = \frac{a}{2}\sqrt{\frac{\omega\mu\sigma}{2}}\sum_{k=0}^{n}\sqrt{k}. \tag{60}$$

In the same time it is possible, in this particular case, to establish the expression of the *waves hypercomplex impedance* (Rosman, 2010)

$$\hat{\zeta}_0 = \frac{\hat{E}_{\text{int}}(r)}{\hat{H}_{\text{int}}(r)} = \frac{\hat{\gamma}}{\sigma} = \sum_{k=0}^{\infty}\left(1_k + \mathrm{j}_k\right)\sqrt{\frac{k\omega\mu}{2\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\mathrm{e}^{-\sum_{k=0}^{\infty}\mathrm{e}^{\mathrm{j}_k\pi/4}}, \tag{61}$$

which is independent with respect to r.

4. Conclusions

The skin effect in a strainght cylindrical conductor, having a circular section, through which flows a periodical non-harmonic current, is studied. The utilized study method is based on a symbolic proceeding which attaches to each periodical non-harmonic signal a hypercomplex “image”.

Having in view that the law order currents harmonics produce a weak skin effect and the high order ones create a net skin effect, the electromagnetic field’s hypercomplex state vectors are determined separately, corresponding to the two types of current’s harmonics as well as for the medium current’s harmonics which produce, at their term, a medium skin effect.

The expressions of hypercomplex moduli, $\hat{E}_{\text{int}}(r)$ and $\hat{H}_{\text{int}}(r)$, as well as $\hat{S}(a)$ are determined, which correspond to the global skin effect applying the superposition theorem, supposing that the cylindrical conductor’s material has a linear behavior in the electromagnetic field.

At the same time the active and reactive powers corresponding to the conductor’s length unit are determined.

The particular case when the current’s fundamental frequency is sufficient high is studied too; in this case the global skin effect is a net one.

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EFECTUL PELICULAR ÎNTR-UN CONDUCTOR CILINDRIC DREPT, DE SECȚIUNE CIRCULARĂ, ÎN REGIM PERMANENT PERIODIC NEARMONIC

(Rezumat)

Studiul efectului pelicular într-un conductor cilindric drept, de secțiune circulară, în regim permanent periodic nearmonic, a fost efectuat cu ajutorul unei metode simbolice care permite să se ataşeze fiecărui semnal periodic nearmonic o „imagine” hipercomplexă. Acest procedeu „algebrizează” operațiile diferențiale în raport cu timpul. Principalele rezultate ale studiului întreprins sunt următoarele:

1° S-au determinat valorile hipercomplexe ale vectorilor de stare ai câmpului electromagnetic ($\hat{E}_{int}(r)$, $\hat{H}_{int}(r)$, $\hat{J}(r)$) precum şi valoarea hipercomplexă a vectorului lui Poynting ($\hat{S}(a)$), produse, separat, de armonicile joase, medii şi înalte ale curentului care circulă prin conductor.

2° Considerând că materialul din care este confecționat conductorul se comportă liniar în câmpul electromagnetic, aplicând teorema suprapunerii se determină valorile hipercomplexe ale vectorilor de stare ai câmpului precum şi a vectorului lui Poynting.

3° Se determină valorile hipercomplexe ale aceloraşi vectori în cazul particular în care frecvența termenului fundamental al curentului (periodic, nearmonic) care circulă prin conductorul cilindric este înaltă.

4° În acest din urmă caz particular se determină coeficientul de creştere a rezistenței în curent alternativ precum şi impedanța hipercomplexă a undelor.