BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVII (LXI), Fasc. 5, 2011
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

THE TRANSFER OF MAXIMUM ACTIVE POWER, IN HARMONIC STEADY-STATE, THROUGH A LINEAR, NON-AUTONOMOUS AND PASSIVE GENERAL TWO-PORT, HAVING THE COUPLING BRANCH BETWEEN THE GATES (*1*), (*1′*) AND (*2*), (*2′*) NONLINEAR INERTIAL AND PASSIVE (II)

BY

HUGO ROSMAN[*]

“Gheorghe Asachi” Technical University of Iaşi,
Faculty of Electrical Engineering, Energetics
and Applied Informatics

Received, February 2, 2011
Accepted for publication: June 8, 2011

Abstract. The nonlinear differential equation of first order satisfied by the function $X_3(R_3)$ is established, where $\underline{Z}_3(I_m) = R_3(I_m) + \mathrm{j}X_3(I_m)$ represents the equivalent complex impedance of the nonlinear inertial connection branch between the gates (*1*), (*1′*) and (*2*), (*2′*) of a general, linear, non-autonomous two-port, in harmonic steady-state, so that the maximum active power transferred through the two-port have an extreme value, I_m representing the amplitude of an arbitrary harmonic current.

Key words: linear, non-autonomous and passive general two-port; non-linear inertial coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*); maximum transferred active power; extreme value.

[*] *e-mail*: adi_rotaru2005@yahoo.com

1. Introduction

In the first part of this work (Rosman, 2007), the transfer of maximum active power, in harmonic steady-state, through a general linear, non-autonomous and passive two-port was studied when the coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*) of the two-port is constituted by the serial connexion of a resistor, a coil and a condenser, all three nonlinear inertial.

In what follows the same problem is studied considering the more generally case when the coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*) is constituted by some connexion of nonlinear inertial resistors, coils and condensers, supposing that the equivalent resistance and reactance of this coupling branch are functions of a same value of the amplitude of a harmonic certain current.

Beforehand it is necessary to specify that the eqs, of a general linear, non-autonomous and passive two-port (GLNPT – Fig. 1) are, in harmonic steady-state (Sigorsky, 1955),

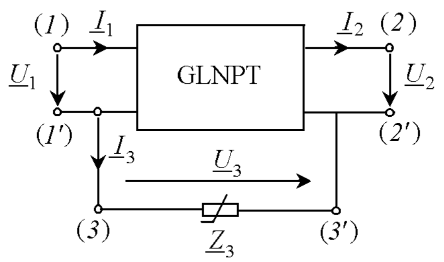

Fig. 1

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \\ \underline{U}_3 \end{bmatrix} = \left[\underline{A}\right] \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix}, \tag{1}$$

where

$$\left[\underline{A}\right] = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} & \underline{A}_{13} \\ \underline{A}_{21} & \underline{A}_{22} & \underline{A}_{23} \\ \underline{A}_{31} & \underline{A}_{32} & \underline{A}_{33} \end{bmatrix} \tag{2}$$

represents the two-port’s fundamental parameters matrix and

$$\underline{Z}_3 = \frac{\underline{U}_3}{\underline{I}_3} = R_3 + \mathrm{j}X_3 \tag{3}$$

is the complex impedance of the coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*).

In a previous paper (Rosman, 2003) the problem concerning the transfer of active power through a GLNPT having a *linear* coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*), in harmonic steady-state, was studied. It was established that this power can reach a maximum value having the expression

$$P_{2\max} = \frac{U_1^2}{4} \cdot \frac{R_3^2 + X_3^2 + aR_3 + bX_3 + c}{d\left(R_3^2 + X_3^2\right) + eR_3 + fX_3 + g}, \tag{4}$$

where: U_1 is the RMS value of the applied voltage at the input gate (*1*), (*1′*) and

a, b,…, g – coefficients which depend only on the two-port's fundamental parameters, $\underline{A}_{ij}$, (i, j = 1, 2, 3); theirs expressions are given in Appendix 1.

2. Case of General, Linear, Non-Autonomous and Passive Two-Ports, in Harmonic Steady-State, Having the Coupling Branch between (*1*), (*1′*) and (*2*), (*2′*) Gates Nonlinear Inertial

Having in view the nonlinear inertial elements property concerning the response *vs*. excitation characteristic, which is linear in instantaneous values and nonlinear in RMS values (Philippow, 1963) it results that if at the input gate of a linear (on nonlinear inertial) two-port, having the coupling branch between the (*1*), (*1′*) and (*2*), (*2′*) gates nonlinear inertial, is applied a harmonic voltage, the steady-state which is established is a harmonic one. Consequently the stady of two-ports in such regimes may be performed using the complex symbolic method. In these conditions it is obvious that relation (4) must be reconsidered becoming

$$P_{2\max} = \frac{U_1^2}{4}\cdot\frac{R_3^2(I_m) + X_3^2(I_m) + aR_3(I_m) + bX_3(I_m) + c}{d\left[R_3^2(I_m) + X_3^2(I_m)\right] + eR_3(I_m) + fX_3(I_m) + g}, \tag{5}$$

because

$$\underline{Z}_3 = \frac{\underline{U}_3}{\underline{I}_3} = \underline{Z}_3(I_m) = R_3(I_m) + \mathrm{j}X_3(I_m). \tag{6}$$

Consequently, in the studied case in this section the maximum active power may be considered as a function of a single independent variable, $P_{2\ \max}(I_m)$. Here I_m represents the amplitude of a harmonic arbitrary current.

2.1. Differential Equation Satisfied by the Function $X_3(R_3)$ in the Studied Case

In view to determine the extreme value of function $P_{2\max}(I_m)$, when these ones exist, the derivative

$$\begin{aligned}\frac{\mathrm{d}P_{2\max}}{\mathrm{d}I_m} &= \frac{U_1^2}{4}\cdot\frac{1}{\left\{d\left[R_3^2(I_m) + X_3^2(I_m)\right] + eR_3(I_m) + fX_3(I_m) + g\right\}^2}\times\\ &\times\Bigg(\left\{A\left[R_3^2(I_m) - X_3^2(I_m)\right] + 2BR_3(I_m)X_3(I_m) + CR_3(I_m) + DX_3(I_m) + E\right\}\frac{\mathrm{d}R_3}{\mathrm{d}I_m} + \\ &+\left\{-B\left[R_3^2(I_m) - X_3^2(I_m)\right] + 2AR_3(I_m)X_3(I_m) - DR_3(I_m) + CX_3(I_m) + F\right\}\frac{\mathrm{d}X_3}{\mathrm{d}I_m}\Bigg)\end{aligned} \tag{7}$$

is performed with

$$\begin{cases} A = e - ad,\ B = f - bd,\ C = 2(g - cd), \\ D = af - bc,\ E = ag - ce,\ F = bg - cf. \end{cases} \tag{8}$$

The expressions of coefficients A, B,…, F, which depend only on fundamental parameters, $\underline{A}_{ij}$, (i, j = 1, 2, 3), of the GLNPT, are given in Appendix 2.

If the derivative (7) is annulled it results the eq.

$$\frac{\mathrm{d}X_3(I_m)}{\mathrm{d}R_3(I_m)} + \frac{A\left[R_3^2(I_m) - X_3^2(I_m)\right] + 2BR_3(I_m)X_3(I_m) + CR_3(I_m) + DX_3(I_m) + E}{-B\left[R_3^2(I_m) - X_3^2(I_m)\right] + 2AR_3(I_m)X_3(I_m) - DR_3(I_m) + CX_3(I_m) + F}. \tag{9}$$

Introducing the notations

$$R_3(I_m) = x,\ \ X_3(I_m) = y \tag{10}$$

eq. (9) becomes

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{A(x^2 - y^2) + 2Bxy + Cx + Dy + E}{-B(x^2 - y^2) + 2Axy - Dx + Cy + F} = 0. \tag{11}$$

It is easy to recognize that differential eq. (11) is a nonlinear one, of first order, belonging to the type

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{M(x,y)}{N(x,y)} = 0, \tag{12}$$

where

$$\begin{cases} M(x,y) = A(x^2 - y^2) + 2Bxy + Cx + Dy + E, \\ N(x,y) = -B(x^2 - y^2) + 2Axy - Dy + Cy + F. \end{cases} \tag{13}$$

2.2. Solution of Differential Equation (11)

Performing the derivatives

$$\frac{\partial M}{\partial y} = -2Ay + 2Bx + D,\ \ \frac{\partial N}{\partial x} = -2Bx + 2Ay - D, \tag{14}$$

it results that

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}; \tag{15}$$

consequently is obviously that expression $M(x,y)\mathrm{d}x + N(x,y)\mathrm{d}y$ isn't an exact total differential. In this case the differential eq. (11) may be integrated only using numerical proceedings.

In the particular case when the GLNPT's fundamental parameter satisfy either the relation

$$\underline{A}_{13} = 0 \tag{16}$$

or the relation

$$\underline{A}_{11}^{*}\underline{A}_{32} + \underline{A}_{12}^{*}\underline{A}_{31} = 0 \tag{17}$$

the differential eq. (11) admits, eventually, an analytical solution. As a matter of fact in the first case (s. rel. (16))

$$\begin{cases} A = 0,\ B = 0,\ C = 0,\ D = 0, \\ E = 0,\ F = -2A_{23}^{*}\Im m(\underline{A}_{33})\Re e(\underline{A}_{31}\underline{A}_{32}^{*}), \end{cases} \tag{18}$$

eq. (12) becomes

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 0, \tag{19}$$

which is inacceptable from a physical point of view. In the second case (s. rel. (17))

$$\begin{cases} A = 0,\ B = 0,\ C = 2A_{13}^{2}\Re e(\underline{A}_{31}\underline{A}_{32}^{*}),\ D = 0, \\ E = -2A_{13}^{2}\Re e(\underline{A}_{33})\Re e(\underline{A}_{31}\underline{A}_{32}^{*}),\ F = -2A_{13}^{2}\Im m(\underline{A}_{33})\Re e(\underline{A}_{31}\underline{A}_{32}^{*}) \end{cases} \tag{20}$$

and consequently eq. (12) becomes

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{Cx + E}{Cy + F} = 0, \tag{21}$$

which is a nonlinear differential eq. with separate variables. The solution of this eq. leads to the expression

$$x^2 + y^2 + \frac{2E}{C}x + \frac{2F}{C}y + \frac{2K}{C} = 0, \tag{22}$$

where K is an integration constant. Having in view relations (18) and notations (10) expression (22) becomes finally

$$R_3^2(I_m)+X_3^2(I_m)-2\Re e(\underline{A}_{33})R_3(I_m)-2\Im m(\underline{A}_{33})X_3(I_m)+\frac{K}{A_{13}^2\Re e(\underline{A}_{31}\underline{A}_{32}^*)}=0, \quad (23)$$

which represents the eq. of a circle having the center in $\left(\Re e(\underline{A}_{33}),\Im m(\underline{A}_{33})\right)$ and the radius $\sqrt{A_{33}^2-K/A_{13}^2\Re e(\underline{A}_{31}\underline{A}_{33}^*)}$. Since the circle's radius must be a positive quantity it results that the integration constant must satisfy the inequality

$$K<A_{13}^2A_{33}^2\Re e(\underline{A}_{31}\underline{A}_{33}^*). \quad (24)$$

It is possible to consider that the circle's (23) arc situated in the half-plane $R_3 \geq 0$ represents the geometric-locus diagram of the complex impedance $\underline{Z}_3(I_m)$ in the harmonic steady-state working regimes when the maximum active power (5), considered as a function of one independent variable, $P_{2\,\max}(I_m)$, has an extreme value.

The solution with respect to $X_3(I_m)$ of eq. (23) is

$$X_3(I_m)=\Im m(\underline{A}_{33})\pm\sqrt{\left[\Im m(\underline{A}_{33})\right]^2-\left[R_3^2(I_m)-2\Re e(\underline{A}_{33})R_3(I_m)+\frac{K}{A_{13}^2\Re e(\underline{A}_{31}\underline{A}_{33}^*)}\right]}. \quad (25)$$

Having in view that $X_3(I_m)$ must be a real (positive or negative) quantity, this condition is fulfilled only if

$$\Re e(\underline{A}_{33})-\sqrt{A_{33}^2-\frac{K}{A_{13}^2\Re e(\underline{A}_{31}\underline{A}_{32}^*)}}<R_3(I_m)<\Re e(\underline{A}_{33})+\sqrt{A_{33}^2-\frac{K}{A_{13}^2\Re e(\underline{A}_{31}\underline{A}_{32}^*)}}. \quad (26)$$

The inferior limit of these double inequality is a positive one only if

$$K>A_{13}^2\left[\Im m(\underline{A}_{33})\right]^2\Re e(\underline{A}_{31}\underline{A}_{32}^*), \quad (27)$$

which is in agreement with (24).

3. Maximum Active Power's Transfer Efficiency to a Linear and Passive Receiver

If the same proceeding as in § **2** is utilized it is possible to study, in what follows, the expression (Rosman, 2003)

$$\eta_{P_{2\max}} = \frac{R_3^2 + X_3^2 + aR_3 + bX_3 + c}{2\left\{m\left[R_3^2 + X_3^2\right] + nR_3 + oX_3 + p\right\}}. \tag{28}$$

of the maximum active power's transfer efficiency through a GLNPT (Fig. 1) in harmonic steady-state, to a linear and passive receiver. The expressions of coefficients m, n, o, p, which depend only on the GLNPT's fundamental parameters, are given in Appendix 1.

Having in view relations (6), expression (28) becomes

$$\eta_{P_{2\max}}(I_m) = \frac{R_3^2(I_m) + X_3^2(I_m) + aR_3(I_m) + bX_3(I_m) + c}{2\left\{m\left[R_3^2(I_m) + X_3^2(I_m)\right] + nR_3(I_m) + oX_3(I_m) + p\right\}}. \tag{29}$$

3.1. Differential Equation Satisfied by the Function $X_3(R_3)$ in the Studied Case

Proceeding in an analogous manner as in § **2** it results that the derivative $\mathrm{d}\eta_{P_{2\max}}(I_m)/\mathrm{d}I_m$ is null when the differential eq. (s. (11))

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{A'(x^2 - y^2) + 2B'xy + C'x + D'y + E'}{-B'(x^2 - y^2) + 2A'xy - D'x + C'y + F'} \tag{30}$$

is satisfied, where

$$\begin{cases} A' = 2(n - am),\ B' = d(o - bm),\ C' = 4(p - cm), \\ D' = 2(ao - bn),\ E' = d(ap - cn),\ F' = 2(bp - co). \end{cases} \tag{31}$$

The expressions of coefficients A', B',…, F' are given in Appendix 2, depending only on GLNPT's fundamental parameters, $\underline{A}_{ij}$, $(i, j = 1, 2, 3)$.

Differential eq. (30) is a nonlinear one, of first order, belonging to the type

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{M'(x,y)}{N'(x,y)} = 0, \tag{32}$$

analogous to (12), where

$$\begin{cases} M'(x,y) = A'(x^2 - y^2) + 2B'xy + C'x + D'y + E', \\ N'(x,y) = -B'(x^2 - y^2) + 2A'xy - D'y + C'y + F'. \end{cases} \tag{33}$$

3.2. Solution of Differential Equation (32)

Taking into account relations (33) theirs derivatives are

$$\frac{\partial M'}{\partial y} = -2A'y + 2B'x + D', \quad \frac{\partial N'}{\partial x} = -2B'x + 2A'y - D', \tag{34}$$

and consequently

$$\frac{\partial M'}{\partial y} \neq \frac{\partial N'}{\partial x}. \tag{35}$$

It results that the expression $M'(x,y)\mathrm{d}x + N'(x,y)$ isn't an exact total differential; in this case the integration of differential eq. (32) must be performed utilizing only numerical methods. It is possible that in certain particular cases, like in § **2**, differential eq. (32) admit an analytical solution.

The curve's $X_3(R_3)$ are (as solution of differential eq. (32)), situated in the half-plane $R_3 \geq 0$ represents, in this case, the geometric-locus diagram of complex impedance $\underline{Z}_3(I_m)$ in the harmonic steady-state working regimes when the transferred maximum active power's efficiency, through an GLNPT to a passive nonlinear inertial receiver considered as a function of one independent variable $\eta_{P_{2\max}}(I_m)$, has an extreme value.

4. Conclusions

1. The differential equation satisfied by the function $X_3(R_3)$ when the maximum active power transferred through a general, linear, non-autonomous and passive two-port to a nonlinear inertial and passive receiver, in harmonic steady-state, has an extreme value, is determined.

2. The analytical solution of this differential equations in a particular case, is obtained.

3. The differential equation satisfied by the same function, $X_3(R_3)$, when the efficiency of transferred maximum active power through the above considered two-port to a nonlinear and passive receiver, in harmonic steady-state, has an extreme value, is determined too.

Appendix 1

The coefficients a, b,…, g and m, n, o, p, established in a previous paper (Rosman, 2003), are the following

$$\begin{aligned}
&a = -2\Re e(\underline{A}_{33}),\ b = -2\Im m(\underline{A}_{33}),\ c = A_{33}^2,\ d = \Re e(\underline{A}_{11}\underline{A}_{12}^*),\\
&e = \Re e\left[\underline{A}_{13}\left(\underline{A}_{11}^*\underline{A}_{32} + \underline{A}_{12}^*\underline{A}_{31}\right)\right] - 2\Re e(\underline{A}_{33})\Re e(\underline{A}_{11}\underline{A}_{12}^*),\\
&f = \Im m\left[\underline{A}_{13}\left(\underline{A}_{11}^*\underline{A}_{32} + \underline{A}_{12}^*\underline{A}_{31}\right)\right] - 2\Im m(\underline{A}_{33})\Re e(\underline{A}_{11}\underline{A}_{12}^*),\\
&g = \Re e\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{33}^*\right),
\end{aligned} \tag{A.1}$$

$$
\begin{aligned}
&m=\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right),\\
&n=\Re e\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}\left(\underline{A}_{22}^{*}\underline{A}_{31}+\underline{A}_{21}^{*}\underline{A}_{32}\right)\right]-2\Re e(\underline{A}_{33})\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right),\\
&o=-\Im m\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)-\underline{A}_{13}\left(\underline{A}_{22}^{*}\underline{A}_{31}+\underline{A}_{21}^{*}\underline{A}_{32}\right)\right]-2\Im m(\underline{A}_{33})\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right),\\
&p=\Re e\left[\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{32}^{*}-\underline{A}_{22}^{*}\underline{A}_{33}^{*}\right)-\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*}-\underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right].
\end{aligned}
\tag{A.1}
$$

Appendix 2

Taking into account relations (8) and (A.1) one obtains

$$
\begin{aligned}
&A=\Re e\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)\right],\ B=\Im m\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)\right],\\
&C=2\Re e\left[\underline{A}_{13}\left(\underline{A}_{13}^{*}\underline{A}_{31}\underline{A}_{32}^{*}-\underline{A}_{31}^{*}\underline{A}_{32}\underline{A}_{33}^{*}-\underline{A}_{12}^{*}\underline{A}_{31}\underline{A}_{33}^{*}\right)\right],\\
&D=2\Im m\left[\underline{A}_{12}\underline{A}_{33}^{*}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)\right],\\
&E=\underline{A}_{33}^{2}\left\{2\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{11}\underline{A}_{12}^{*}\right)-\Re e\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)\right]\right\}-\\
&\quad-2\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{13}^{*}\underline{A}_{32}^{*}-\underline{A}_{12}^{*}\underline{A}_{33}^{*}\right),\\
&F=\underline{A}_{33}^{2}\left\{2\Im m\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{11}\underline{A}_{12}^{*}\right)-\Im m\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)\right]\right\}-\\
&\quad-2\Im m\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{13}^{*}\underline{A}_{32}^{*}-\underline{A}_{12}^{*}\underline{A}_{33}^{*}\right).
\end{aligned}
\tag{A.2}
$$

Similarly, having in view relations (31) and (A.1) it results

$$
\begin{aligned}
&A'=2\Re e\left[\underline{A}_{23}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)+\underline{A}_{13}\left(\underline{A}_{22}^{*}\underline{A}_{31}+\underline{A}_{21}^{*}\underline{A}_{32}\right)\right],\\
&B'=2\Im m\left[\underline{A}_{23}\left(\underline{A}_{11}^{*}\underline{A}_{32}+\underline{A}_{12}^{*}\underline{A}_{31}\right)+\underline{A}_{13}\left(\underline{A}_{22}^{*}\underline{A}_{31}+\underline{A}_{21}^{*}\underline{A}_{32}\right)\right],\\
&C'=8\Re e\left(\underline{A}_{13}\underline{A}_{23}^{*}\right)\Re e\left(\underline{A}_{31}\underline{A}_{32}^{*}\right)-4\Re e\left\{\underline{A}_{33}\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}^{*}\left(\underline{A}_{22}\underline{A}_{31}^{*}+\underline{A}_{21}\underline{A}_{32}^{*}\right)\right]\right\},\\
&D'=-8\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{13}\underline{A}_{23}^{*}\right)\Re e\left(\underline{A}_{31}\underline{A}_{32}^{*}\right)+2\left\{\left[\Re e\left(\underline{A}_{33}\right)\right]^{2}-\left[\Im m\left(\underline{A}_{33}\right)\right]^{2}\right\}\times\\
&\quad\times\Re e\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}^{*}\left(\underline{A}_{12}\underline{A}_{31}^{*}+\underline{A}_{21}\underline{A}_{32}^{*}\right)\right]-\\
&\quad-4\Re e\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{33}\right)\Im m\left[\underline{A}_{23}^{*}\left(\underline{A}_{21}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}^{*}\left(\underline{A}_{12}\underline{A}_{31}^{*}+\underline{A}_{21}\underline{A}_{32}^{*}\right)\right],\\
&E'=-8\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{12}\underline{A}_{23}^{*}\right)\Re e\left(\underline{A}_{31}\underline{A}_{32}^{*}\right)+2\left\{\left[\Re e\left(\underline{A}_{33}\right)\right]^{2}-\left[\Im m\left(\underline{A}_{33}\right)\right]^{2}\right\}\times\\
&\quad\times\Re e\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}^{*}\left(\underline{A}_{12}\underline{A}_{31}^{*}+\underline{A}_{21}\underline{A}_{32}^{*}\right)\right]-\\
&\quad-4\Re e\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{33}\right)\Im m\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*}+\underline{A}_{12}\underline{A}_{31}^{*}\right)+\underline{A}_{13}^{*}\left(\underline{A}_{23}\underline{A}_{31}^{*}+\underline{A}_{21}\underline{A}_{32}^{*}\right)\right],
\end{aligned}
\tag{A.2}
$$

$$F' = -8\Im m\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{13}\underline{A}_{23}^{*}\right)\Re e\left(\underline{A}_{31}\underline{A}_{32}^{*}\right) + 2\left[\Re e\left(\underline{A}_{33}\right)\right]^{2} \times$$
$$\times \Im m\left[\underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*} + \underline{A}_{12}\underline{A}_{31}^{*}\right)\right] - 2\underline{A}_{33}^{2}\Im m\left[\underline{A}_{13}\left(\underline{A}_{22}^{*}\underline{A}_{31} + \underline{A}_{21}^{*}\underline{A}_{32}\right)\right] - \tag{A.3}$$
$$-4\Re e\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{33}\right)\Re e\left[\underline{A}_{13}^{*}\left(\underline{A}_{12}\underline{A}_{31}^{*} - \underline{A}_{21}\underline{A}_{32}^{*}\right) - \underline{A}_{23}^{*}\left(\underline{A}_{11}\underline{A}_{32}^{*} + \underline{A}_{12}\underline{A}_{31}^{*}\right)\right].$$

REFERENCES

Philippow E., *Nichtlineare Elektrotechnik.* Akad. Ierlagsges. Geest u. Portig, K.G., Leipzig, 1963.

Rosman H., *Sur le transfert de la puissance active maximum par un quadripôle général linéaire, non-autonome, en régime permanent harmonique*. Bul. Inst. Politehnic, Iaşi, **XLIX (LIII)**, *1-2*, s. Électrot., Énerg., Électron., 25-31 (2003).

Rosman H., *La transfert de la puissance active maximum, en régime permanent harmonique, par un quadripôle général linéaire non-autonome, dont l'impédance de couplage entre les portes (1), (1') et (2), (2') est nonlinéaire inertielle* (I). Bul. Inst. Politehnic, Iaşi, **LIII (LVII)**, *1-2*, s. Électrot., Énerg., Électron., 23-29 (2007).

Sigorsky V.P., *General Theory of Two-Ports* (in Russian). Ed. Acad. of Sci., Ukraine, Kiev, 1956.

TRANSFERUL PUTERII ACTIVE MAXIME, ÎN REGIM PERMANENT ARMONIC, PRINTR-UN CUADRIPOL GENERAL LINIAR, NEAUTONOM ŞI PASIV, AVÂND LATURA DE CUPLAJ DINTRE PORȚILE (*1*), (*1'*) ŞI (*2*), (*2'*) NELINIARĂ, INERȚIALĂ ŞI PASIVĂ (II)

(Rezumat)

În cazul unui cuadripol general liniar, neautonom, având latura de cuplaj dintre porțile (*1*), (*1'*) şi (*2*), (*2'*), de asemenea liniară, funcționând în regim permanent armonic, puterea activă maximă transferată unui receptor liniar este o funcție de două variabile independente, R_3 şi X_3 , reprezentând parametrii independenți ai laturii de cuplaj dintre porțile (*1*), (*1'*) şi (*2*), (*2'*) ale cuadripolului. Dacă latura de cuplaj este neliniară, inerțială, aceşti parametri pot fi considerați ca depinzând de amplitudinea, I_m , a unui curent arbitrar cu variația armonică în timp, încât puterea activă maximă transferată receptorului devine o funcție de o singură variabilă independentă, I_m. În acest caz se stabilesc ecuațiile diferențiale (neliniare şi de primul ordin) satisfăcută de funcția $X_3(R_3)$ astfel încât: a) valoarea maximă a puterii active transferate receptorului să aibă valori extreme; b) randamentul cu care este transferată puterea activă maximă să aibă valori extreme. În general soluțiile acestor ecuații diferențiale pot fi obținute numai prin metode numerice. Soluția analitică a primei ecuații a fost obținută într-un caz particuler.