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COGNITIVE RADIO ENVIRONMENTS: GAME THEORETICAL MODELLING

BY

LIGIA CREMENE* **and AUREL VLAICU**

Technical University of Cluj-Napoca

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Abstract. Three oligopoly game models and new equilibrium concepts are considered in order to investigate spectrum access scenarios in cognitive radio environments: Nash, Pareto, and the joint Nash-Pareto, Pareto-Nash equilibria. The experimental observations may be especially relevant for designing new rules of behavior for emerging radio environments.

Key words: dynamic spectrum access; cognitive radio; oligopoly game modeling; rules of behaviour.

1. Introduction

New spectrum bands are being released around the world: 2.6 GHz in Europe, the 800 MHz digital dividend, 700 MHz and AWS – 1,700/2,100 MHz in the U.S., etc. Existing spectrum bands are being deregulated to allow coexistence of 2G, 3G, and 4G technologies. Technology neutral spectrum, in the context of infrastructure sharing by operators (to lower costs, improve capital efficiency), becomes more and more prevalent.

Current spectrum regimes are based on a highly prescriptive approach, centralized control and decisions. The administrative approach makes it easier for the regulators to ensure avoidance of excessive interference, to tailor appropriate license conditions based on guard bands and maximum power transmission levels (Doyle, 2009). But traditional spectrum planning was

*Corresponding author: *e-mail*: Ligia.Cremene@com.utcluj.ro

proved to be a slow process that cannot keep up with new innovations and technologies. Studies have shown up to 90% of the radio spectrum remains idle in any one geographical location. Also, the existing digital TV whitespaces can provide significantly higher data rates compared to the 2.4 GHz ISM band.

The problem of harmonized spectrum access in dynamic radio environments is addressed. Implementation of dynamic spectrum access tends to be synonyms with Cognitive Radio (Doyle, 2009). This assumption, together with reformulated game theoretical models, build up our approach.

Cognitive radios (CRs) are seen as the solution to the current low usage of the radio spectrum (Corderio *et al*., 2006; Niyato & Hossain, 2007; Doyle, 2009). CRs have the potential to utilize the large amount of unused spectrum in an intelligent way while not interfering with other incumbent devices in frequency bands already licensed for specific uses (Corderio *et al*., 2006). A cognitive radio has to manage a dynamic interaction profile. Such an interactive decision process may be analysed using Game Theory models and techniques.

Game Theory has been widely used as an analysis tool in economic systems and has recently emerged as an effective framework for the analysis and design of wireless networks. Radio resource allocation and dynamic spectrum access may be described as strategic interactions between cognitive radios (Neel *et al*., 2004; MacKenzie & Wicker, 2001;Huang & Krishnamurthy, 2009; Maskery *et al*., 2007) – each player payoff depends on the actions of all players. We consider three well known oligopoly game models: Cournot, Stackelberg, and Bertrand. Several equilibrium concepts are studied as game solutions: Nash, Pareto, and the joint Nash-Pareto, Pareto-Nash equilibria. The aim is to investigate the relevance of these equilibrium concepts for spectrum access and resource allocation. The observations may be especially relevant for designing new rules of behavior for dynamic radio environments.

The paper is structured as follows: Section **2** outlines the role of cognitive radios in efficient spectrum usage. Section **3** provides some basic insights to standard and joint game-equilibria. The game theoretical models modes for spectrum whitespace access are described in Section **4**. Section **5** presents and discusses the numerical results obtained from simulations. The conclusions are presented in Section **6**.

2. Cognitive Radios – a Solution for Low Usage of Radio Spectrum

WRAN IEEE 802.22 is the first wireless standard based on cognitive radios. The 802.22 groundbreaking wireless air interface is defined for use by license-exempt devices in the spectrum that is currently allocated to the Television service. In this range there is much unused or underused frequency spectrum. Since the new air interface is required to reuse the fallow TV spectrum without causing any harmful interference to incumbents (*i.e.*, the TV receivers), cognitive radio techniques are of primary importance in establishing/supporting adaptive strategies that facilitate coexistence. Such adaptive strategies regard dynamic spectrum access and radio environment characterization (Niyato & Hossian, 2007).

CRs are seen as the solution to the current low usage of the radio spectrum (Cordeiro *et al*., 2006; Niyato & Hossian, 2007; Doyle, 2009). CRs have the potential to utilize the large amount of unused spectrum in an intelligent way while not interfering with other incumbent devices in frequency bands already licensed for specific uses (Niyato & Hossian, 2007). Mechanisms based on frequency hopping have been widely used to enable wireless networks to use resources from the unlicensed spectrum without frequency planning (Cordeiro *et al*., 2006).

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A cognitive radio has to manage a dynamic interaction profile. In the radio environment this is reflected in the interference profile that may be prompted to change whenever another radio changes its profile. Such an interactive decision process can be analysed using Game Theory models and techniques (MacKenzie & Wicker, 2001; Huang & Krishnamurthy, 2009).

We may assume that the CRs know the form of the other CRs utility functions if all the interacting radios have the same objective (*e.g.*, maximizing SINR) or if the radios can poll the other radios in the environment. However, due to the variability of channel conditions it is unlikely that a CR will know the precise values of other radios' utility functions. Even without any ability to infer other players' utility functions, the equilibrium concept has a significant implication for cognitive radio interactions modeled as a repeated game (Neel *et al*., 2004).

A CR is defined as a radio that can sense its environment and then modify its behavior, based on a set of rules (policy), and without operator intervention (Mitola, 2000). We chose to model the interactions between CRs as interactions between players on an oligopoly market, which in this case is the frequency spectrum. For a more realistic formulation of the game, a modified rationality paradigm is considered. Within this paradigm radios may have several approaches and biases towards different equilibrium concepts (Dumitrescu *et al*., 2009). We investigate the Pareto equilibrium and a new equilibrium concept – the joint Nash-Pareto equilibrium.

3. Game Equilibria. Generative Relations and Detection

A game may be defined as a system $G = ((N, S_i, u_i), (i = 1,\ldots,n))$, where

(i) N represents the set of n players, $N = \{1,\ldots,n\}$;

(ii) for each player $i \in N$, S_i represents the set of actions $S_i = \{s_{i1}, s_{i2},\ldots, s_{im}\}$; $S = S_1 \times S_2 \times\ldots\times S_N$ is the set of all possible game situations;

(iii) for each player $i \in N$, $u_i : S \rightarrow R$ represents the payoff function.

A strategy profile (strategy or action vector) is a vector $s = (s_1,\ldots,s_n)$, where $s_i \in S_i$ is a strategy (or action) of player i. By (s_i, s^*_{-i}) we denote the strategy profile obtained from s^* by replacing the strategy of player I with s_i, *i.e.* $(s_i, s^*_{-i}) = (s^*_1, s^*_2, \ldots, s^*_{i-1}, s_i, s^*_{i+1}, \ldots, s^*_1)$.

Game Theory provides a number of tools for analysing games. The most frequently used steady-state concept is the Nash Equilibrium (NE), which has been extensively studied (Neel *et al*., 2004; Dumitrescu *et al*., 2009; Osborne, 2004). Informally, a strategy profile is an NE if no player can improve her payoff by unilateral deviation.

Considering two strategy profiles, x and y, from S, the strategy profile, x, is said to Pareto dominate the strategy profile, y (and we write $x < Py$) if the payoff of each player using strategy, x, is greater or equal to the payoff associated to strategy, y, and at least one payoff is strictly greater. The set of all non-dominated strategies (Pareto frontier) represents the set of Pareto equilibria of the game (Osborne, 2004).

In an n-player game consider that each player, i, acts based on a certain type of rationality, r_i, ($i = 1,\ldots, n$). We may consider a two-player game where r_1 = Nash and r_2 = Pareto. The first player is biased towards the Nash equilibrium and the other one is Pareto-biased. Thus, a new type of equilibrium, called the *joint Nash-Pareto equilibrium*, may be considered (Dumitrescu *et al*., 2009). The considered generalization involves heterogeneous players that are biased towards different equilibrium types or may act based on different types of rationality (Dumitrescu *et al*., 2009).

Let us consider an n player game where each player may be either Nash or Pareto-biased. We denote by I_N the set of Nash biased players (N-players) and by I_P the set of Pareto biased players (P-players). Therefore we have

$$I_N = \left\{ i \in \{1,\ldots,n\} : r_i = \text{Nash} \right\},$$
$$I_P = \left\{ j \in \{1,\ldots,n\} : r_j = \text{Pareto} \right\}.$$

An operator for measuring the relative efficiency of profile strategies has been introduced (Osborne, 2004)

$$E : S \times S \to N,$$

defined as

$$E(y,x) = \text{card}\left\{ i \in I_N : u_i(x_i, y_{-i}) \geq u_i(y), x_i \neq y_i \right\} +$$
$$+ \text{card}\left\{ j \in I_P : u_j(y) < u_j(x), x \neq y \right\}.$$

$E(x, y)$ measures the relative efficiency of the strategy profile, x, with respect to the strategy profile, y. The relative efficiency enables us to define a generative relation for the joint Nash-Pareto (NP) equilibrium.

Consider a relation < NP defined as y < NP x if and only if

$$E(y,x) < E(x,y).$$

The relation < NP is considered as the generative relation of the joint Nash Pareto equilibrium.

An evolutionary technique for equilibria detection, based on appropriate generative relations (Cremene *et al*., 2010) that allow the comparison of strategies, is considered. Multi-objective Optimization Algorithms (Deb *et al*., 2000) are efficient tools for evolving strategies based on a non-domination relation. Numerical experiments aim the detection of pure equilibria or a combination of equilibria in parallel with cognitive radios interaction. An adaptation of the popular NSGA2 (Deb *et al*., 2000) has been considered.

4. Oligopoly Game Modeling of Cognitive Radio Environments

In order to assess dynamic spectrum access scenarios of cognitive radios, three oligopoly game models are considered: Cournot, Stackelberg, and Bertrand. The commodity of this oligopoly market is the frequency spectrum. These models are computationally simple and therefore suitable for implementation in resource-limited software-defined radio transceivers. To illustrate the oligopoly environment models we consider a scene with two radios (duopoly) so that the radios' strategies and their equilibria can be represented two-dimensionally. However, the same approach can be applied to the case of more than two radios.

4.1. Cournot Game Modeling of a Cognitive Radio Access Scenario

We consider a cognitive radio access scenario that can be modeled as a simple reformulation of the Cournot oligopoly game. Suppose there are n cognitive radios attempting to access the radio environment. Each radio, i, is free to decide the number $c_i \in [0, \infty)$ of simultaneous frequency hopping channels the radio implements. How many simultaneous channels should each radio implement in order to maximize its operation efficiency?

Based on the above scenario, a Cournot game can be formulated as follows. The players are the cognitive radios attempting to access a certain frequency band. The strategy of each player, i, is the number, c_i, of simultaneous implemented (active) channels. A strategy profile is a vector $c = (c_1, ..., c_n)$. The payoff of each player is the difference between a function of goodput and power consumption (cost of implementing c_i simultaneous channels).

Let us denote by $P(c)$ the fraction of symbols that are not interfered with. The goodput for radio i is $P(c)c_i$. Radio i cost for supporting c_i simultaneous channels is $C_i(c_i)$.

The payoff of player (radio) i is thus

$$u_i(c) = P(c)c_i - C_i(c_i)\,.$$

In general, P decreases with the total number of implemented channels and C_i increases with c_i (more bandwidth implies more processing resources and more power consumption). If these effects are approximated as linear functions, the payoff function can be rewritten as

$$u_i(c) = \left(W - \sum_{k=1}^{n} c_k\right)c_i - Kc_i\,,$$

where: W is the total bandwidth available (set of available channels), K – the cost of implementing a channel.

The Nash equilibrium is considered as the solution of the game

$$c_i^* = \frac{W-K}{N+1},\ \forall i \in \mathrm{N}..$$

4.2. Stackelberg Modelling of a Cognitive Radio Access Scenario

The traditional spectrum access approach ensures co-existence of multiple systems by splitting the available spectrum into frequency bands and allocating them to licensed (primary) users. The dynamic spectrum access in cognitive environments improves the spectrum utilization by detecting unoccupied spectrum holes or whitespaces and assigning them to unlicensed (secondary) users. This situation, where we have incumbent monopoly and new entrants, may well be modeled using a Stackelberg game model.

The players are the CRs – licensed and unlicensed (primary and secondary) users. The strategy of player i is given by the number, c_i , of simultaneous implemented channels. The payoff of each player is the difference between the goodput and the power consumption (cost of implementing c_i simultaneous channels).

Using the same notations as for Cournot modelling, the payoff function of player i can be defined as

$$u_i(c_1, c_2) = c_i P_d(c_1 + c_2) - C_i(c_i),\ \text{ for } i = 1, 2.$$

considering $c_2 = b_2(c_1)$ as the output of the secondary user for primary user's output c_1 (Maskery *et al*., 2007). We consider a constant unit cost and a linear inverse demand function, $P_d(c)$, with the same definition as for the Cournot model.

The outcome of the equilibrium (Maskery *et al.*, 2007) is that radio 1 activates $c_1^* = (W - K)/2$ simultaneous channels and radio 2 activates $c_2^* = b_2(c_1^*) = (W - K)/4$ simultaneous channels.

4.3. Bertrand Spectrum Access Modelling

In the Bertrand competition, producers compete by varying the product price and thus adjusting the demand. A constant unit cost and linear demand function are assumed.

The Bertrand competition for spectrum access may be reformulated as follows: we consider *n* cognitive radios competing for access to c_i channels, in a given whitespace, *W*. The objective of each radio is to activate a subset of channels in order to satisfy its current demand level (*e.g.* target throughput). The strategy of each player, *i*, is a target number, $p_i(c)$, of non-interfered symbols. Each player payoff is the difference between a function of goodput and the cost of accessing, c_i, simultaneous channels. Using the same notations as for Cournot and Stackleberg models, the payoff function of radio *i* can be written

$$u_i(p_1, p_2) = (p_i - K)(W - p_i), p_i < p_j = (p_i - K)(W - p_i)/2, p_i = p_j = 0, \; p_i > p_j .$$

5. Numerical Experiments

The results represent a sub-set of more extensive simulations. For equilibria detection the evolutionary technique utilized by Dumitrescu *et al.* (2009) is considered. A population of 100 strategies is evolved using a rank based fitness assignment technique. In all experiments the process converges in less than 20 generations. Our tests show that the evolutionary method for equilibrium detection is scalable with respect to the number of available channels (Cremene *et al.*, 2010).

The simulation parameters for all the three models – Cournot, Stackelberg, and Bertrand – are $W = 10$ (available channels) and $K = 1$ (cost of accessing one channel).

5.1. Cournot Modelling – Numerical Experiments

Model evaluation results are presented for the Cournot competition with two radios trying to access the same whitespace at the same time. The payoff equilibria are captured in Fig. 1 (Nash, Pareto, Nash-Pareto, and Pareto-Nash). The four types of equilibria are obtained in separate runs.

The NE corresponds to the scenario where each of the two CRs activates 7 channels (from 24 available). The Pareto equilibrium describes a situation where the spectrum resource is completely used by one or both radios in different portions. The number of active channels lies in the range [0, 10.5] for each CR.

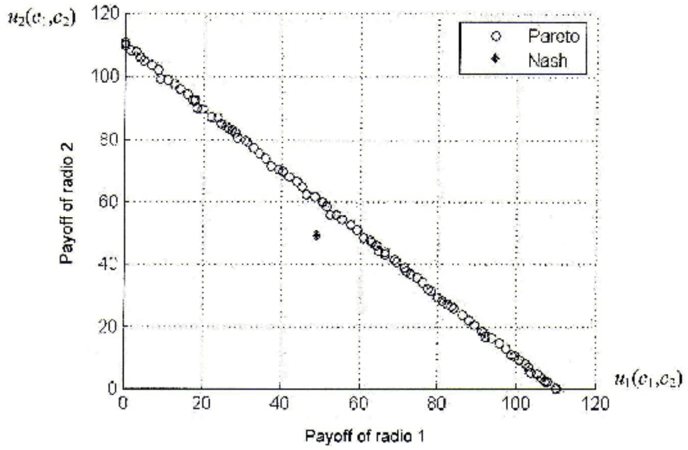

Fig. 1 – Cournot modelling – two radios (W = 24, K = 3); payoffs of the evolutionary detected equilibria: Nash (49, 49), Pareto, NP and PN.

Although each CR tries to maximize its utility, none of them can access more than half of the available channels. Moreover, the sum of active channels is less than that for the NE. In other words, for NE the number of accessed channels is maximum, and the available spectrum is efficiently used.

In some cases, the Nash-Pareto strategy enables the CR to access more channels than for the NE strategy. In the performed experiments – (W = 24, K = = 3), (W = 10, K = 1), and (W = 100, K = 1) – the PN equilibrium is symmetric to the NP equilibrium with respect to the first bisecting line. It is interesting to notice that none of the NP strategies actually reach NE.

The payoffs of P-players (payers from the Pareto front) are stuated in the range [0, 110] and their sum is always larger than the NE payoff (49, 49). For each strategy of the Nash-Pareto equilibrium the Pareto-player has a higher payoff. The Nash-player's payoff is smaller in a Nash-Pareto situation than in a case where all the players play Nash (are Nash-biased). Even if the NP strategies allow the CRs to access more channels, the payoffs are smaller than for the Pareto strategies. This is due to interference increasing with the number of accessed channels.

5.2. Stackelberg Modelling – Numerical Experiments

The payoffs of the evolutionary detected equilibria - Nash, Pareto, NP, and PN – are captured in Fig. 2. Any strategy from the Pareto front is also a Nash-Pareto strategy. If the primary user plays Nash then the secondary user may maximize its payoff by choosing any strategy. If the secondary user plays

Nash then the maximum payoff of the primary user is NE (55.12, 27.81) (Fig. 2).

The secondary user by access less channels than in the Cournot case (c_2 = 10.5 which is less than three channels, NE = (3,3)), its maximum payoff remains unaffected, 20 (Fig. 4). Instead, the primary user's maximum payoff is half (10) even if it accesses more channels (c_1 = 4.5). For the Stackelberg formulation of the game, the NE payoff of the secondary user (Fig. 4) is less then in the Cournot case (5 instead of 9). For the primary user the NE payoff is slightly increased (10.13 instead of 9).

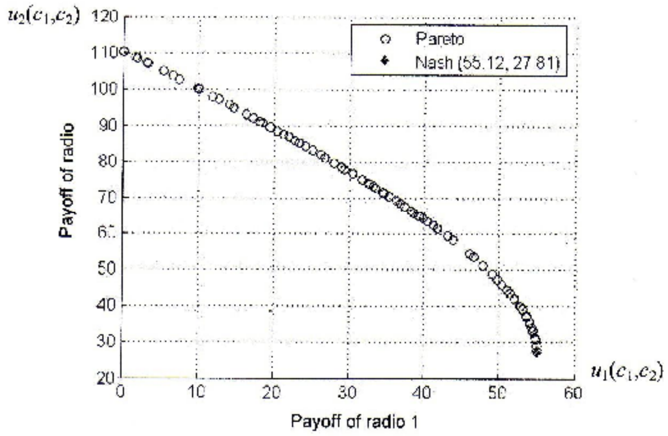

Fig. 2 – Stackelberg modelling – two radios (W = 24, K = 3); payoffs of the evolutionary detected equilibria: Nash (55.12, 27.81), Pareto, NP = Nash and PN = Pareto.

This situation is relevant for interference control in dynamic spectrum access scenarios between incumbents and new entrants. The analysis shows that payoffs are maximized for all users if the incumbents are Nash oriented and the new entrants are Pareto driven.

5.3. Bertrand Modelling – Numerical Experiments

We think the Bertrand oligopoly is suitable for modelling crowded spectrum access scenarios and the reformulation is as follows. The Bertrand strategy is the price. The equivalent of the price $P(c)$ in this game reformulation is the target number of non-interfered symbols of each radio. The lower this target is the higher the chances are for the radio to access one or several channels. On the other hand, as the number $P(c)$ of non-interfered symbols per channel decreases, the need for channels (the demand) increases. Thus, a radio willing to maximize its goodput will attempt to occupy as many low-rate channels as possible. Figs. 3 and 4 qualitatively illustrate the winning situations for two radios trying to access a limited bandwidth, W. The NE in this case,

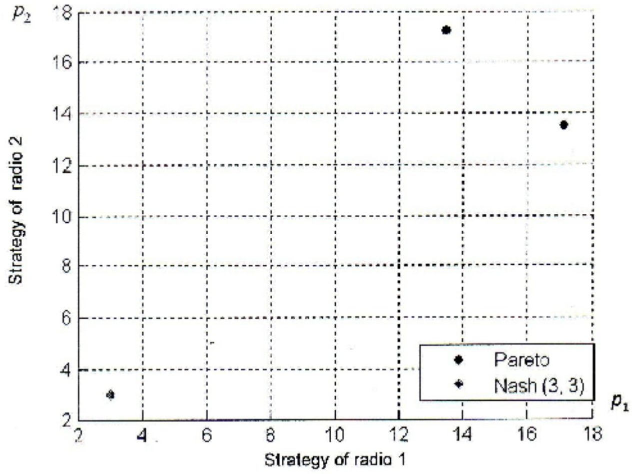

Fig. 3 – Bertrand modelling – two radios ($W = 24$, $K = 3$); evolutionary detected equilibria: Nash (3, 3), Pareto.

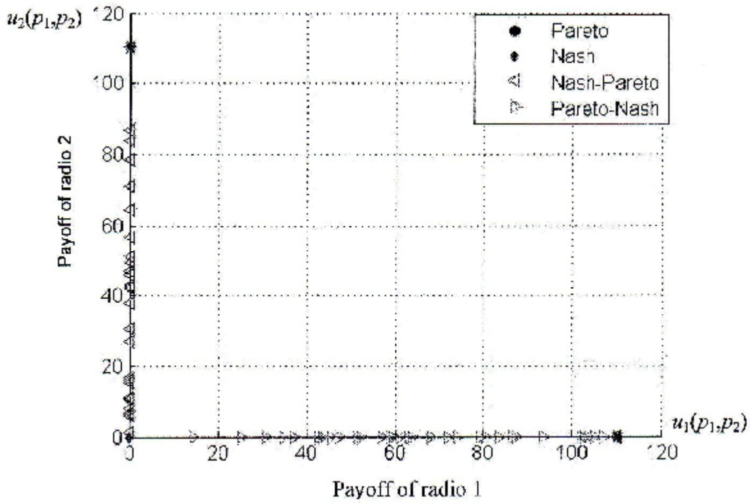

Fig. 4 – Bertrand modelling – two radios ($W = 24$, $K = 3$); payoffs of the evolutionary detected equilibria: Nash (0, 0), Pareto, NP, and PN.

means zero payoff for each radio while the Pareto strategy ensures the maximum possible payoff for one radio at a time. This indicates that, for a high interference scene, some sort of scheduling or sequential access scheme is required.

6. Conclusions and Future Work

Three oligopoly game models are considered in order to investigate the relevance of certain equilibrium concepts for the problem of dynamic spectrum access in cognitive radio environments. Besides standard Nash equilibrium and Pareto equilibrium a new approach based on heterogeneous players has been considered. In this last model players are biased towards several equilibrium types. This is a more realistic game formulation for interactions in cognitive radio environments. Numerical experiments indicate the effectiveness of the proposed approach. The observations may be especially relevant for designing new rules of behavior for heterogeneous radio environments. Future experiments include 3-player game modeling for simultaneous spectrum access and investigation of new equilibrium concepts (*e.g*. Lorentz equilibrium).

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REFERENCES

Cordeiro C., Challapali K., Birru D., *IEEE 802.22: An Introduction to the First Wireless Standard Based on Cognitive Radios*. J. of Comm., **1**, *1*, 38-47 (2006).

Cremene L.C., Dumitrescu D., Nagy R., *Oligopoly Game Modeling for Cognitive Radio Environments*. In Balandin S., Dunaytsev R., Koucheryavy Y. (Eds.), Proc. of the Smart Spaces a. Next Gen. Wired/Wireless Networking, Springer LNCS, *6294*, 2010, 219-230.

Deb K., Agrawal S., Pratab A., Meyarivan T., *A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimization: NSGA-II*. In Schoenauer M., Deb K., Rudolph G., Yao X., Lutton E., Merelo J. J., Schwefel H.P. (Eds.), Proc. of the PPSN2000, Springer LNCS, *1917*, 2000, 849-858.

Doyle L.E., *Essentials of Cognitive Radio*. Cambridge Univ. Press, UK, 2009.

Dumitrescu D., Lung R.I., Mihoc T.D., *Generative Relations for Evolutionary Equilibria Detection*. GECCO '09, 2009, 1507-1512.

Huang J.W., Krishnamurthy V., *Game Theoretic Issues in Cognitive Radio Systems*. J. of Comm., **4**, *10*, 790-802 (2009).

Lung R.I., Dumitrescu D., *Computing Nash Equilibria by Means of Evolutionary Computation*. IJCCCl, **3**, 364-368 (2008).

MacKenzie A., Wicker S., *Game Theory in Communications: Motivation, Explanation, and Application to Power Control*. Globecom2001, 821-825 (2001).

Maskery M., Krishnamurthy V., Zhao Q., *GT Learning and Pricing for Dynamic Spectrum Access in Cognitive Radio*. Cogn. Wireless Commun. Networks, Hossain E., Bhargava V.K. (Eds.), Springer Science +Business Media., NY, 2007.

Mitola J., *Cognitive Radio: An Integrated Agent Architecture for Software Defined*

Radio. Ph. D. Diss., Royal Inst. Technol. (KTH), Stockholm, Sweden, 2000.
Neel J.O., Reed J.H., Gilles R.P., *Game Models for Cognitive Radio Algorithm Analysis*. http://www.mprg.org/people/gametheory/presentations.shtml, 2004.
Niyato D., Hossain E., *Microeconomic Models for Dynamic Spectrum Management in Cognitive Radio Networks*. Cogn. Wireless Commun. Networks, Hossain E., Bhargava V.K. (Eds.), Springer Science+Business Media, NY, 2007.
Osborne M.J., *An Introduction to Game Theory*. Oxford Univ. Press, UK, 2004.
Popovski P., Yomo H., Prasad R., *Strategies for Adaptive Frequency Hopping in the Unlicensed Bands*. IEEE W. Comm., 60-67 (2006).
* * * http://www.wun-cogcom.org accessed 2010.

INTERACŢIUNI ÎN MEDII CU ECHIPAMENTE RADIO COGNITIVE – O ABORDARE BAZATĂ PE TEORIA COMPUTAŢIONALĂ A JOCURILOR

(Rezumat)

Se prezintă trei modele din teoria computaţională a jocurilor, reformulate pentru analiza unor scenarii de acces la spectrul radio. Se investighează relevanţa unor noi tipuri de echilibre pentru această problemă: echilibrele combinate Nash-Pareto şi Pareto-Nash, alături de echilibrele clasice Nash şi Pareto. Rezultatele experimentelor numerice pot fi relevante pentru reglementarea interacţiunilor între echipamente radio cognitive.