BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVIII (LXII), Fasc. 1, 2012
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

ON DISTRIBUTION NETWORK OPERATION ANALYSIS

BY

VIRGIL ALEXANDRESCU, GHEORGHE CÂRŢINĂ and GHEORGHE GRIGORAŞ[*]

“Gheorghe Asachi” Technical University of Iaşi,
Faculty of Electrical Engineering, Energetics
and Applied Informatics

Received, January 10, 2012
Accepted for publication: March 6, 2012

Abstract. An algorithm to analyse the power flow problem for urban and rural distribution networks of 6…20 kV is proposed. Such analysis is absolutely necessary in the network monitoring, operation and planning studies. The algorithm, based on previous author’s works, has been tested on realistic networks. It is also shown that the algorithm can be extended to analyse simultaneously more networks of different nominal voltages, having more sub-networks, constituting an aggregated network.

Key words: sub-network; operation; aggregated networks.

1. Introduction

The authors’ previous works (Alexandrescu *et al.*, 2009, 2010) propose an algorithm to solve the power flow problem of distribution networks of 6…20 kV. This work refers to the networks with tree structure, supplied from the buses of medium voltage of the electric substations.

The classic method of power flow calculation supposes that the load powers of the transformers are known.

---

[*] Corresponding author: *e-mail*: ggrigor@ee.tuiasi.ro

At present, in usual operation of a distribution network, the active and reactive loads connected at low voltage side of the transformers are unknown.

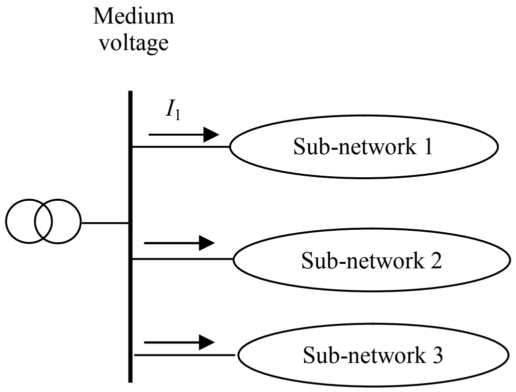

Fig.1– Substation supplying three sub-networks.

This work proposes an adequate algorithm to solve the power flow problem for urban and rural distribution networks of 6…20 kV. Such analysis is very necessary in monitoring of the network operation and planning studies.

Nowadays, in usual operation of a distribution network there is an important degree of load uncertainty because of small number of real time measurements. Actually, only the currents injected into the source node of the sub-network by supply substation, is hourly measured and recorded. The medium voltage of the substation are known and also the medium value of the power factor can be evaluated by practical experience of the network operators.

In the following, a convenient algorithm to solve the power flow problem of the distribution networks is proposed. That algorithm has been tested by a computer program, performed in MATLAB.

2. Particularities of the Proposed Algorithm

In the Fig. 2, a general draft of a distribution network is presented, in which the following quantities are denoted: $\underline{I}_1$ – the current injected from sub-station; $\underline{I}_{ci}, (i=\overline{1,n})$ – the nodal capacitive currents of the equivalent circuit; $\underline{I}_{li}, (i=\overline{1,n})$ – the load currents considered on the medium voltage windings of the transformers.

The proposed algorithm is based on the equivalent monophase circuit in which the lines and transformers are modeled by Π- and respectively Γ- circuits.

To analyse the network operation, the following hypotheses are taken into account:

a) The currents of primary windings of the transformers are considered as load currents, that is $\underline{I}_{li}, (i=\overline{1,n})$ from Fig. 2.

b) The lines of medium voltage and the load currents without transformers represent a separated network that will be calculated by an iterative way.

c) It is also assumed that the loads of the transformers are proportionally with their nominal apparent power.

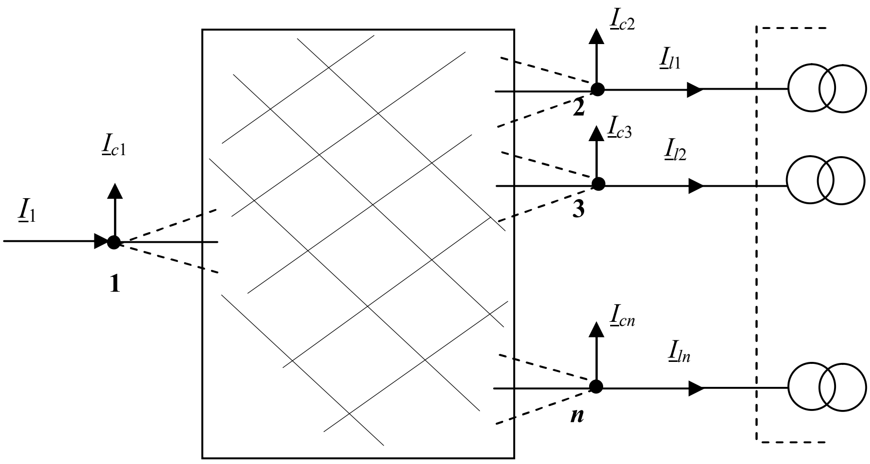

Fig. 2 – Single line diagram of the sub-network.

In the algorithm, there are procedures to determine network parameters, using databases of the lines and transformers.

3. Mathematical Model of Network Operation

To analyse a sub-network with *n* nodes, it is necessary to define the main quantities. Thus, the nodal complex voltages can be written $\underline{U}_i = U_i \mathrm{e}^{\mathrm{j}\theta_i}$, $(i = \overline{1,n})$. The source node voltage, $\underline{U}_1$, is fixed and chosen as phase origin, so that $\underline{U}_1 = U_1$.

The sum of all load currents is

$$\underline{I}_l = \sum_{i=2}^{n} \underline{I}_{li} \tag{1}$$

and from Fig. 2 it results the balance of currents into sub-network

$$\underline{I}_1 = \sum_{i=1}^{n} \underline{I}_{ci} + \sum_{i=2}^{n} \underline{I}_{li} \,. \tag{2}$$

Consequently

$$\underline{I}_l = \underline{I}_1 - \sum_{i=1}^{n} \underline{I}_{ci} \,. \tag{3}$$

By neglecting the transversal conductance, b_{i0} being the capacitive susceptance connected at node i, then, the nodal capacitive currents in the equivalent circuit are

$$\underline{I}_{ci} = \mathrm{j} b_{i0} \underline{U}_i, \ (i = \overline{1,n}) \,. \tag{4}$$

To evaluate the load currents it is assumed that the transformers are loaded proportionally with their apparent power, so that the loading coefficients have the expressions

$$k_{li} = \frac{S_i}{\sum\limits_{k \in L} S_k}, \ i \in L \,, \tag{5}$$

where S_i is the nominal apparent power of the transformers connected at node i and L is the set of the load nodes.

Under these conditions, the load current at nodes is

$$\underline{I}_{li} = \underline{I}_l k_{li}, \ i \in L \,. \tag{6}$$

To solve the power flow problem, the following quantities are known: I_1, U_1 and medium value of power factor, cosφ. In consequence, the active and reactive powers injected into the network can be calculated.

4. The Algorithm Description

The algorithm consists of two parts:

Part 1 is an iterative calculus extent to medium voltage lines and load currents connected at nodes, without considering transformers. The purpose of this part is to determine iteratively the voltages and the load currents into sub-network, corresponding to entry fixed quantities: I_1, U_1 and cosφ. The global results are: P_1, Q_1 – the active and reactive powers injected into the network by the substation; P_l, Q_l – the total powers at load nodes; Q_c – the total capacitive reactive power of the lines; ΔP_L, ΔQ_L – the active and reactive power losses into the lines; P_{ij}, Q_{ij} – line power flow.

Part 2 contains the calculus of the power flow of the transformers, which is performed by direct calculation, without iterations. The global results can be found in § **5**, in which the following quantities are introduced:

a) ΔP_r, ΔP_{Fe} – the active losses in resistance and iron;

b) ΔQ_x, ΔQ_m – the reactive loss in reactance and magnetizing loss;

c) P_{lv}, Q_{lv} – the powers of consumers at low voltage.

The iterative algorithm, corresponding to Part 1, consists in the following steps:

1. *Approximating the nodal voltages by real value equal to* U_1.
2. *Compuiing the capacitive currents and the total load current*.
3. *Computing the loading coefficients and the load currents corresponding to current iteration*.
4. *As usually, calculate: the currents flows by backward sweep and then, by forward sweep, calculate the voltage drops and the new values of complex voltages*.
5. *With load currents and nodal voltages in the current iteration, calculate active and reactive load powers*.
6. *Verify the convergence test applied to the load powers from two consecutive iterations imposing the values of errors.*

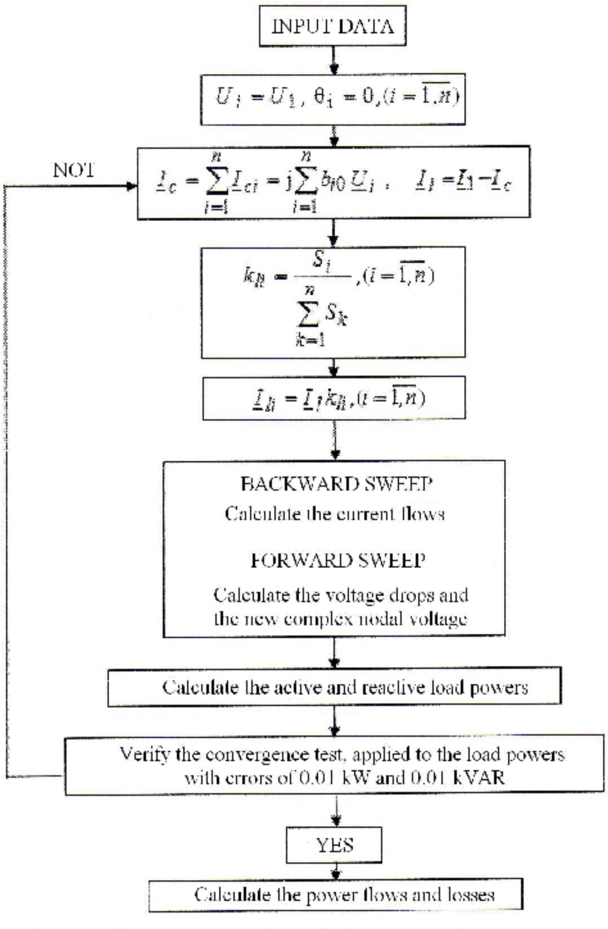

Fig. 3 – The graphic representation of the successive steps to calculate one sub-network operation.

The algorithm is briefly represented in Fig. 3. It is convenient to analyse the sub-networks. Such analysis for a sub-network with 23 nodes has been performed by the authors in a previous work (Alexandrescu *et al*., 2010).

The main interest regarding distribution network is, finally, the evaluation, as well as possible, of active power losses for optimizing operation and planning studies.

It is important to underline that the proposed algorithm has been tested on more real urban and rural distribution network.

5. Simultaneous Analysis of Distribution Networks

Below it is shown that the algorithm described above to analyse, one by one, the sub-networks, can be extended to analyse simultaneously more sub-networks, even of different nominal voltages. These sub-networks form, by aggregation, an unique network. The aggregate network will be treated as one sub-network. For that purpose a MATLAB software application was performed. which represents an extension of the algorithm presented above. .

The network structure includes more substations, each of them supplying more sub-networks.

The medium voltage bus of each substation represents a source node, which will be considered as node 1. The presence of more nodes, denoted as node 1, in the aggregated network, is not dangerous even the substations have different nominal voltages. That is because the source nodes supply sub-networks are completely different.

The lines of whole network are ordered pointing out the hierarchical ranks, described in previous works together with structure vectors.

The algorithm consists in the following steps:

1. *Approximating the nodal voltages from each sub-network by real values equal to* U_1*, corresponding to voltage of the supply substation.*

2. *Computing the nodal capacitive currents .with relation (4) and total load current with (1). The obtained values must be stored separately for each sub-network.*

3. *Computing the loading coefficients and the nodal load currents with relations (5) and (6) for each subnetwork.*

4. *With the aid of the structure vectors, the backward sweep and the forward sweep will be performed following simultaneously all the lines from each rank.*

6. Case Study

The algorithm was tested on a real complex network, representing an aggregated urban and rural network, composed of 5 sub-networks, supplied from two substations of 20 kV, respectively 6 kV.

The whole network is shown in Fig. 4 (Appendix). The medium voltage buses of the two substations are both considered as node 1. The other nodes are numbered from 2 to 77. In the same figure it can see also the lines numbers. The sum of the lines is 76 and their total length is of 51 km.

In the network there are 54 load nodes, marked by arrows.

The transformers connected to the load nodes have the nominal apparent powers from 100 to 1,600 kVA.

For lack of space the nodal and line data can not be specified here.

The currents injected into sub-networks, expressed in amperes, have the values: 60 A, 25 A, 30 A, 140 A and 215 A, with the same power factor equal to 0.83. The real and imaginary parts of these currents are stored in matrices.

The power flow of the aggregated network will be performed under condition of maximum loading, in which the voltages of substations are fixed at values of 21 kV and, respectively, 6.3 kV. In the results, the quantities of the form: $U, \theta, P, Q, \Delta P$ and ΔQ are expressed, respectively, in: kV, degrees, kW, and kVAr.

The power flow of lines has been computed in three iterations with the power errors of 0.01 kW and 0.01 kVAr.

The results regarding the aggregated network , which includes the load currents, are presented in the Tables 1...5. The global results contain also the active and reactive losses in transformers.

Optionally, the operator can chose among different variants of printing the results: voltage and nodal power, power flow in lines and transformers, active and reactive losses or other combinations of them. For example, referring to sub-network 4, the Tables 1 and 2 contain the resulted nodal variables and, respectively, the power flow and losses of the lines.

Table 1

Nodal Quantities of Sub-Network 4

Load node	U	θ	P_l	Q_l
49	6.19	–0.048	537.03	360.88
53	6.13	–0.051	335.07	225.17
54	6.17	–0.053	214.38	144.06
55	6.15	–0.049	133.41	89.65

Table 2

Power Flow and Losses

i	j	P_{ij}	Q_{ij}	ΔP_{ij}	ΔQ_{ij}
1	48	1,245	815	0.34	0.40
48	49	1,245	814	20.5	14.8
49	50	473	316	0.63	0.20
50	51	472	316	2.07	1.69
51	52	336	225	0.93	0.76
52	53	335	224	0.39	0.09
49	54	215	136	0.42	0.30
51	55	133	90	0.08	0.03

The Table 3 comprises the injected powers into sub-networks P_1 and Q_1, known before, and load powers, P_l and Q_l.

To test the accuracy of calculation it is sufficient to observe that the following balance relations are verified:

$$P_s = P_l + \Delta P_L,\ \ Q_s = Q_l + \Delta Q_L - Q_c,$$

where Q_c is the total capacitive power from the Table 5.

Table 3

Injected and Load Powers

Sub-network	P_1	Q_1	P_l	Q_l
1	1,486	609	1,483	997
2	602	240	600.5	404
3	878	553	863.5	580
4	1,245	815	1,220	820
5	1,910	1,221	1,796	1,207
Total	6,122	3,439	5,963	4,008

Table 4

Active Power Losses

Sub-network	ΔP_L	ΔP_r	ΔP_{Fe}
1	3.09	4.81	19.23
2	0.49	2.15	7.94
3	14.50	5.89	6.97
4	25.04	7.49	8.52
5	114.12	6.28	21.55
Total	157.57	26.62	64.20

Table 5

Reactive Power Losses and Capacitive Power

Sub-network	ΔQ_L	ΔQ_x	ΔQ_m	Q_c
1	2.11	21.5	212	390
2	0.29	8.7	89.5	164
3	7.00	20.1	66.3	34.5
4	18.3	32.1	93.3	23.3
5	62.3	26.3	247.0	48.5
Total	90.0	108.7	708.1	660.3

7. Conclusions

A general algorithm concerning the power flow problem of medium voltage distribution networks, under condition of lack of information about the nodal loads, is proposed. To solve the problem it is absolutely necessary to approximate the loads by suitable methods. The authors considered that it is convenient to evaluate the loading of transformers proportionally with their apparent powers. It is shown that the power flow analysis of a sub-network can be solved by separating the sub-network in two parts, one of them using an iterative method and the other, containing transformers, by direct calculation. The test results, obtained for more urban and rural real distribution networks, confirmed the validity of the proposed algorithm.

In order to increase the efficiency of the distribution network power flow analysis an appropriate MATLAB software application was performed, extending the algorithm used for a sub-network to analyse simultaneously more sub-networks, even of different nominal voltages. The case study has confirmed the algorithm validity.

Appendix

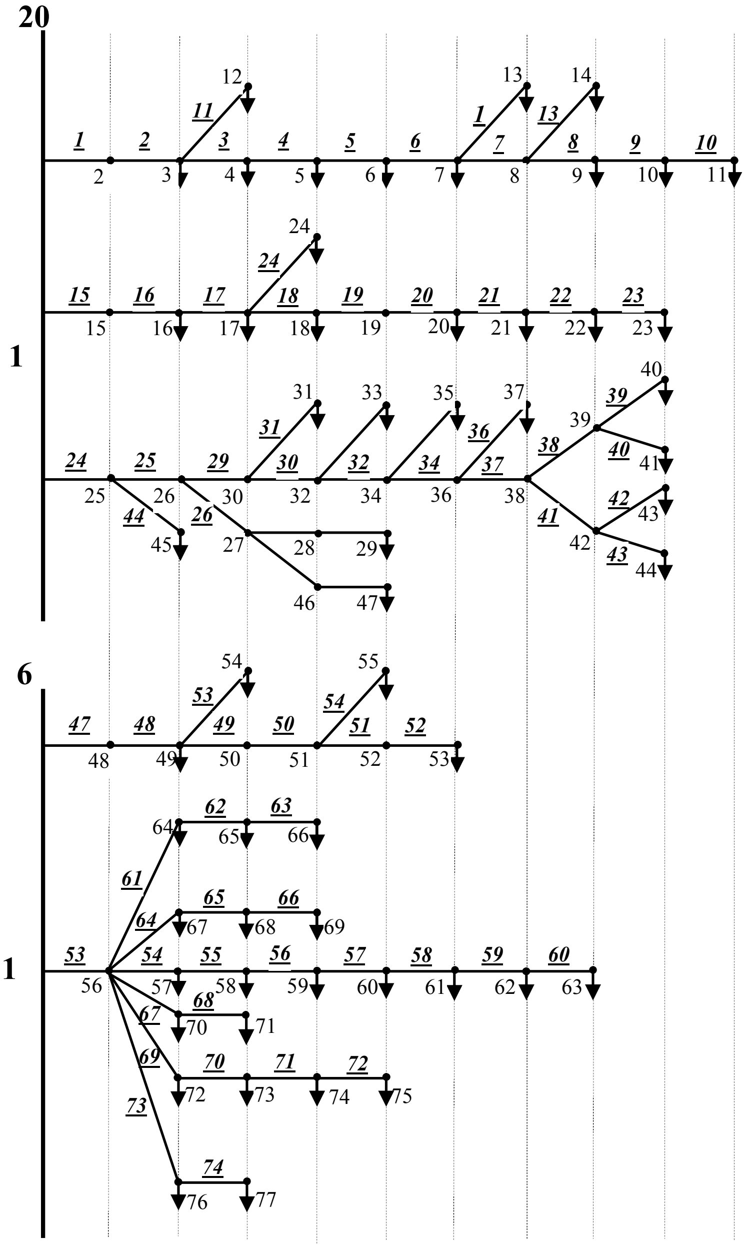

Fig. 4 – Single line diagram of the aggregated network.

REFERENCES

Alexandrescu V., Cârţină Gh., Grigoraş Gh., *An Algorithm to Solve the Power Flow Problem of Medium Voltage Distribution Networks*. Proc. of the 7th Internat. Conf. on Electromech. a. Power Syst., **1** . Iaşi, October 6-10, 2009, 107-110.

Alexandrescu V., Cârţină Gh., Grigoraş Gh., *On Power Flow Problem of Distribution Networks. Under Condition of Lack of Information*. Bul. Inst. Polit. Iaşi, **LVI** (**LX**), *2*, s. Electrot., Energ., Electron., 77-87 (2010).

ANALIZA FUNCŢIONĂRII REŢELELOR ELECTRICE DE DISTRIBUŢIE

(Rezumat)

Se propune un algoritm pentru analiza regimurilor de funcţionare ale reţelelor de distribuţie urbane şi rurale de 6...20 kV. Algoritmul propus are la bază lucrări anterioare ale autorilor. Analiza este utilă în monitorizarea şi planificarea reţelelor de distribuţie. Algoritmul a fost testat pe un număr mare de reţele reale. De asemenea, se arată într-un studiu de caz posibilitatea extinderii algoritmului pentru analiza simultană a unui număr mare de reţele. Analiza simultană are avantajul unui calcul efficient în care se obţin, pe zone mari, circulaţiile de puteri şi pierderile.