

SOME GENERALIZATIONS OF MANLEY-ROWE RELATIONS IN THE ELECTROMAGNETIC FIELD

BY

HUGO ROSMAN*

“Gheorghe Asachi” Technical University of Iași,
Faculty of Electrical Engineering, Energetics
and Applied Informatics

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Abstract. The local and global forms of generating (primitive), generalized generating (primitive) and generalized Manley-Rowe (M.-R.) relations as well as of generalized Manley-Rowe-Kontorovich relations in the electromagnetic field were established. These generalizations are referred to the radiated power as well as to the in field stored power..

Key words: electromagnetic field; Manley-Rowe relations; Manley-Rowe-Kontorovich relations; their generalization.

1. Introduction

In a previous paper (Rosman, 1971), was proved that the radiated active power and the in field stored active power, either in volume unit or in the whole volume limited by a closed surface, situated in an electromagnetic field evolving in a motionless, non-linear, homogeneous, isotropic, non-polarized and non-magnetized permanently, non-dissipatif, without hereditary properties, in which non-periodic phenomena not occur, satisfy, in certain conditions, Manley-Rove (M.-R.) type relations.

Namely, if an electromagnetic field excited by a double periodical

* *e-mail:* adi_rotaru2005@yahoo.com

source with respect to time is considered, evolving in a medium having the above mentioned properties, let be $\mathbf{E} = \mathbf{E}(\mathbf{r}, t)$, $\mathbf{H} = \mathbf{H}(\mathbf{r}, t)$ the electric, respectively, magnetic field intensities. If these vectorial functions of point and time are double-periodical with respect to time, using the complex form of double Fourier series (Tolstov, 1953) the expressions

$$\begin{cases} \mathbf{E}(\mathbf{r}, t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \underline{\mathbf{E}}_{p,q}(\mathbf{r}) e^{j(p\omega_1 + q\omega_2)t}, \\ \mathbf{H}(\mathbf{r}, t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \underline{\mathbf{H}}_{p,q}(\mathbf{r}) e^{j(p\omega_1 + q\omega_2)t}, \end{cases} \quad (1)$$

may be written, where

$$\begin{cases} \underline{\mathbf{E}}_{p,q}(\mathbf{r}, t) = \frac{1}{4\pi^2} \int_0^{2\pi} d(\omega_2 t) \int_0^{2\pi} \mathbf{E}(\mathbf{r}) e^{-j(p\omega_1 + q\omega_2)t} d(\omega_1 t), \\ \underline{\mathbf{H}}_{p,q}(\mathbf{r}, t) = \frac{1}{4\pi^2} \int_0^{2\pi} d(\omega_2 t) \int_0^{2\pi} \mathbf{H}(\mathbf{r}) e^{-j(p\omega_1 + q\omega_2)t} d(\omega_1 t). \end{cases} \quad (2)$$

In the same time was supposed that the function $\mathbf{E}(\mathbf{H})$ is singled valued

$$\int \mathbf{E}(\mathbf{H}) d\mathbf{H} = 0. \quad (3)$$

In the above mentioned paper (Rosman, 1971) was proved that in such situation relations

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p \Re(\nabla \underline{\mathbf{S}}_{p,q})}{p\omega_1 + q\omega_2} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} \frac{q \Re(\nabla \underline{\mathbf{S}}_{p,q})}{p\omega_1 + q\omega_2} = 0 \quad (4)$$

and

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p P_{r_{p,q}}}{p\omega_1 + q\omega_2} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} \frac{q P_{r_{p,q}}}{p\omega_1 + q\omega_2} = 0 \quad (5)$$

are satisfied, where

$$\underline{\mathbf{S}}_{p,q} = \underline{\mathbf{E}}_{p,q} \times \underline{\mathbf{H}}_{p,q}^* \quad (6)$$

represents the Poynting complex vector while

$$P_{r_{p,q}} = \Re \oint_{\Sigma} \underline{\mathbf{S}}_{p,q} \cdot d\mathbf{A} = \Re \iiint_{V_{\Sigma}} \nabla \cdot \underline{\mathbf{S}}_{p,q} dv \quad (7)$$

constitutes the active radiated power through the closed surface which limits the volume v_Σ situated in the studied electromagnetic field. In the same paper (Rosman, 1971), relations (4) and (5) were named the local form, respectively the global form of M.-R. relations in the electromagnetic field.

2. Generating (Primitive) Manley-Rowe Relations in the Electromagnetic Field

Let be $\mathbf{F}(\mathbf{r}, t)$ and $\mathbf{G}(\mathbf{r}, t)$ two vectorial functions of point and time, double-periodical with respect to time, having the periods $T_1 = 2\pi/\omega_1$, $T_2 = 2\pi/\omega_2$. Such functions admit developments in complex double Fourier series (Tolstov, 1953) namely

$$\begin{cases} \mathbf{F}(\mathbf{r}, t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \underline{\mathbf{F}}_{p,q} e^{j(p\omega_1 + q\omega_2)t}, \\ \mathbf{G}(\mathbf{r}, t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \underline{\mathbf{G}}_{p,q} e^{j(p\omega_1 + q\omega_2)t}, \end{cases} \quad (8)$$

with

$$\begin{cases} \underline{\mathbf{F}}_{p,q} = \frac{1}{4\pi^2} \int_0^{2\pi} d(\omega_2 t) \int_0^{2\pi} \mathbf{F}(\mathbf{r}, t) e^{-j(p\omega_1 + q\omega_2)t} d(\omega_1 t), \\ \underline{\mathbf{G}}_{p,q} = \frac{1}{4\pi^2} \int_0^{2\pi} d(\omega_2 t) \int_0^{2\pi} \mathbf{G}(\mathbf{r}, t) e^{-j(p\omega_1 + q\omega_2)t} d(\omega_1 t). \end{cases} \quad (9)$$

If function $\mathbf{F}(\mathbf{G})$ is single valued namely

$$\int \mathbf{F}(\mathbf{G}) d\mathbf{G} = 0 \quad (10)$$

relations

$$\begin{cases} \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re(\underline{\mathbf{F}}_{p,q} \underline{\mathbf{G}}_{p,q}^*) = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re(\underline{\mathbf{F}}_{p,q} \underline{\mathbf{G}}_{p,q}^*) = 0, \end{cases} \quad (11)$$

are satisfied, as it was proved in a previous work (Rosman, 1973). Similarly relations

$$\begin{aligned}
\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re(\underline{\mathbf{F}}_{p,q} \times \underline{\mathbf{G}}_{p,q}^*) &= 0, \\
\sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re(\underline{\mathbf{F}}_{p,q} \times \underline{\mathbf{G}}_{p,q}^*) &= 0,
\end{aligned} \tag{12}$$

were established too. Relations (11) and (12) were named *generating (primitive) M.-R. relations*.

Considering now that

$$\mathbf{F}(\mathbf{r}, t) \equiv \mathbf{E}(\mathbf{r}, t) \text{ and } \mathbf{G}(\mathbf{r}, t) \equiv \mathbf{H}(\mathbf{r}, t) \tag{13}$$

if, besides, condition (3) is fulfilled, relations (11) and (12) lead to

$$\begin{cases} \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re(\underline{\mathbf{E}}_{p,q} \underline{\mathbf{H}}_{p,q}^*) = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re(\underline{\mathbf{E}}_{p,q} \underline{\mathbf{H}}_{p,q}^*) = 0, \end{cases} \tag{14}$$

respectively

$$\begin{cases} \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re(\underline{\mathbf{E}}_{p,q} \times \underline{\mathbf{H}}_{p,q}^*) = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re(\underline{\mathbf{E}}_{p,q} \times \underline{\mathbf{H}}_{p,q}^*) = 0. \end{cases} \tag{15}$$

Having in view relation (6), expressions (15) may be written also

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re \underline{\mathbf{S}}_{p,q} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re \underline{\mathbf{S}}_{p,q} = 0. \tag{16}$$

Applying to relations (16) the operator ∇ and having in view (7), expressions (16) become

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p P_{r_{p,q}} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q P_{r_{p,q}} = 0. \tag{17}$$

Relations (17) can be considered as representing the *generating*

(primitive) *M.-R. relations concerning the radiated active power* in an electromagnetic field.

In another work (Rosman, 1976) relations

$$\left\{ \begin{array}{l} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) \Re(\underline{\mathbf{F}}_{p, q} \underline{\mathbf{G}}_{p, q}^*) = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) \Re(\underline{\mathbf{F}}_{p, q} \times \underline{\mathbf{G}}_{p, q}^*) = 0, \end{array} \right. \quad (18)$$

were established, named *generalized generating (primitive) M.-R. relations*. Here

$$\psi(p, q) = -\psi(-p, -q) \quad (19)$$

is an arbitrary odd function.

Expression (18₂) becomes

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) \Re \underline{\mathbf{S}}_{p, q} = 0, \quad (20)$$

where relation (6) was taken into account. Finally if to this last relation is applied the operator ∇ and having in view (7) one obtain

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) P_{r_{p, q}} = 0, \quad (21)$$

which may be considered as representing the *generalized generating M.-R. relation concerning the radiated active power* in an electromagnetic field.

Having in view that the medium in which evolves the electromagnetic field has the properties indicated in § 1 (motionless, non-linear, homogeneous, isotropic, non-polarized and non-magnetized permanently, non-dissipatif, without hereditary properties), the differential form of electromagnetic energy theorem can be written as (Mocanu, 1981)

$$\frac{\partial w}{\partial t} + \nabla \mathbf{S} = 0, \quad (22)$$

where w represents the volumetric density of electromagnetic field energy stored in the field and \mathbf{S} – Poynting vector. The integral form of same theorem is

$$\frac{\partial W}{\partial t} + \oint_{\Sigma} \mathbf{S} d\mathbf{A} = 0, \quad (23)$$

with W – the stored energy in the finite electromagnetic field domain bounded by closed surface Σ .

Considering the harmonics pair of rang p, q and using the complex symbolic representation of harmonic signals, relations (22) and (23) become

$$\frac{\partial \underline{s}_{p,q}}{\partial v} + \nabla \underline{\mathbf{S}}_{p,q} = 0, \quad (24)$$

respectively

$$\underline{s}_{p,q} + \oint_{\Sigma} \underline{\mathbf{S}}_{p,q} \cdot d\mathbf{A} = 0, \quad (25)$$

where $\underline{s}_{p,q}$ is the complex apparent power stored in the electromagnetic field of the considered harmonics.

Having in view relations (16) it is obvious that relations

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p d(\Re \underline{s}_{p,q}) / dv = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q d(\Re \underline{s}_{p,q}) / dv = 0 \quad (26)$$

are satisfied or, by integrating in a finite volume bounded by closed surface, Σ , one obtains

$$\sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} p \Re \underline{s}_{p,q} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=0}^{\infty} q \Re \underline{s}_{p,q} = 0. \quad (27)$$

Relations (26) and (27) may be considered as representing the differential, respectively the integral form of *generating (primitive) M.-R. relations concerning the electromagnetic powers stored in the studied electromagnetic field*.

Using an analogous proceeding may be established the differential, respectively the integral form of *generalized generating M.-R. relations concerning the electromagnetic power stored in the electromagnetic field*

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) d(\Re \underline{s}_{p,q}) / dv = 0, \quad (28)$$

respectively

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \psi(p, q) \Re(\underline{s}_{p,q}) = 0. \quad (29)$$

3. Generalized Manley-Rowe Relations in the Electromagnetic Field

This time are considered the relations (Rosman, 1980)

$$\begin{cases} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p \Re(\underline{\mathbf{F}}_{p+\mu, q+\nu} \underline{\mathbf{G}}_{p+\sigma, q+\tau}^*)}{p\omega_1 + q\omega_2} = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p \Re(\underline{\mathbf{F}}_{p+\mu, q+\nu} \times \underline{\mathbf{G}}_{p+\sigma, q+\tau})}{p\omega_1 + q\omega_2} = 0, \end{cases} \quad (30)$$

with $\mu, \nu, \sigma, \tau \in \mathbb{N}$, named *Manley-Rowe-Kontorovich (M.-R.-K.) relations*.

Using a similar proceeding as those utilized in § 2, namely taking into account relations (12), the second relation (30) becomes

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p \Re \underline{\mathbf{S}}_{p+\mu, q+\nu; p+\sigma, q+\tau}}{p\omega_1 + q\omega_2} = 0, \quad (31)$$

where

$$\underline{\mathbf{S}}_{p+\mu, q+\nu; p+\sigma, q+\tau} = \underline{\mathbf{E}}_{p+\mu, q+\nu} \times \underline{\mathbf{H}}_{p+\sigma, q+\tau}^* \quad (32)$$

represents the *pseudo Poynting complex vector*, in the sense utilized by A. Țugulea (1981).

Applying, this time, too, the operator ∇ and having in view relation (7) it results

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p P_{r_{p+\mu, q+\nu; p+\sigma, q+\tau}}}{p\omega_1 + q\omega_2} = 0, \quad (33)$$

where $P_{r_{p+\mu, q+\nu; p+\sigma, q+\tau}} = \Re(\underline{\mathbf{E}}_{p+\mu, q+\nu} \times \underline{\mathbf{H}}_{p+\sigma, q+\tau}^*)$ represents the *radiated active pseudo-power* in the studied electromagnetic field. It is necessary to mention that the notion of pseudo-power was introduced for the first time by Penfield, Spence and Dunker (1970).

Relation (33) may be considered as representing the *M.-R.-K. relation in an electromagnetic field*.

Finally are considered the relations (Rosman, 2004)

$$\begin{cases} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q) \Re(\mathbf{F}_{p+\mu,q+\nu} \mathbf{G}_{p+\sigma,q+\tau}^*)}{p\omega_1 + q\omega_2} = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q) \Re(\mathbf{F}_{p+\mu,q+\nu} \times \mathbf{G}_{p+\sigma,q+\tau}^*)}{p\omega_1 + q\omega_2} = 0, \end{cases} \quad (34)$$

with $\mu, \nu, \sigma, \tau \in \mathbb{N}$ and $\psi(p,q)$ representing an arbitrary odd function (s. rel. (19)). These relations were called *generalized M.-R.-K. relations*. Taking into account, in this case too, relations (12) and (32), expression (34) becomes

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q) P_{r_{p+\mu,q+\nu;p+\sigma,q+\tau}}}{p\omega_1 + q\omega_2} = 0, \quad (35)$$

which represents the *generalized M.-R.-K. relations in an electromagnetic field*, $P_{r_{p+\mu,q+\nu;p+\sigma,q+\tau}}$ being the radiated active pseudo-power as above defined.

Analogously is possible to establish the differential, respectively the integral form of either the *M.-R.-K. relations concerning the electromagnetic power stored in the volume unit of the studied field*,

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p d(\Re s_{-p+\mu,q+\nu;p+\sigma,q+\tau})/dv}{p\omega_1 + q\omega_2} = 0, \quad (36)$$

respectively

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{q d(\Re s_{-p+\mu,q+\nu;p+\sigma,q+\tau})/dv}{p\omega_1 + q\omega_2} = 0, \quad (37)$$

or the *generalized M.-R.-K. relations concerning the electromagnetic power stored in the volume unit of the field*, namely

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q) d(\Re s_{-p+\mu,q+\nu;p+\sigma,q+\tau})/dv}{p\omega_1 + q\omega_2} = 0 \quad (38)$$

for the differential form or

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q) \Re s_{-p+\mu,q+\nu;p+\sigma,q+\tau}}{p\omega_1 + q\omega_2} = 0, \quad (39)$$

for the integral form, where $\psi(p, q)$ is an arbitrary odd function (s. rel. (19)) and

$$\underline{S}_{p+\mu, q+\nu; p+\sigma, q+\tau} = \Re \left(\underline{U}_{p+\mu, q+\nu} \underline{I}_{p+\sigma, q+\tau}^* \right) + j \Im \left(\underline{U}_{p+\mu, q+\nu} \underline{I}_{p+\sigma, q+\tau}^* \right) \quad (40)$$

is the *complex apparent pseudo-power stored in the electromagnetic field*.

4. Conclusions

Choosing as model the relations valid in the case of non-linear networks were established the following expressions, valid in the case of an electromagnetic field evolving in a motionless, non-linear, homogeneous, isotropic, non-polarized and non-magnetized permanently, nondissipatif, without hereditary properties, in which non-periodic phenomena not occur:

- a) Generating (primitive) Manley-Rowe relations concerning the radiated active power.
- b) Generalized generating (primitive) Manley-Rowe relations concerning the radiated active power.
- c) Manley-Rowe-Kontorovich relations concerning the radiated active power.
- d) Generalized Manley-Rowe-Kontorovich relations concerning the radiated active power.

Also, were established, in the above mentioned electromagnetic field, similar relations concerning either the electromagnetic active power stored in unit volume or the electromagnetic active power stored in the entire field.

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CÂTEVA GENERALIZĂRI ALE RELAȚIILOR MANLEY-ROWE ÎN CÂMPUL ELECTROMAGNETIC

(Rezumat)

Se stabilesc formele locale și globale ale relațiilor Manley-Rowe (M.-R.) generatoare (primitive), relațiilor M.-R. generatoare (primitive) generalizate, relațiilor M.-R. generalizate, relațiilor Manley-Rowe-Kontorovich (M.-R.-K.) și relațiilor M.-R.-K. generalizate într-un câmp electromagnetic care evoluează într-un mediu imobil, neliniar, omogen și izotrop, nedisipativ, nepolarizat și nemagnetizat permanent, lipsit de proprietăți ereditare, în care nu au loc fenomene neperiodice.

Generalizările relațiilor M.-R. și M.-R.-K. se referă la puterea activă radiată cât și, parțial, la puterea activă înmagazinată în câmp.