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RESONANCE AT THE ACCESS GATES OF A LINEAR NON-AUTONOMOUS AND RECIPROCAL TWO-PORT, SUPPLIED SIMULTANEOUSLY, AT THE TWO GATES, WITH HARMONICAL VOLTAGES HAVING THE SAME FREQUENCY

BY

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Abstract. The conditions which assure the resonance realization at each gate of a linear and autonomous two-port are established when this one is supplied at both gates with harmonical voltages having the same frequency. The conditions which assure the realization of the resonance simultaneously at both gates are established too.

Key words: linear and non-autonomous two-ports; simultaneous supply at the two-port’s both gates; resonance at the access gates.

1. Introduction

The systematic stady of in restricted sense two-ports, supplied, simultaneously at the two gates with harmonic voltages having the same frequency was performed for the first time by C. Şora, in 1961, in his Ph. D. dissertation. The main contribution of the cited author consists in the definition, thanks to this working regime, of a new set of two-ports (in restricted sense)

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parameters, $(Y_1)_1$, $(Y_2)_1$, $(Y_1)_{-1}$, $(Y_2)_{-1}$, which have the dimensions of admittances. C. Şora points out, in his dissertation, a series of advantages which result from utilization of these new introduced parameters. The results obtained by C. Şora concern the case when the two-port is supplied, simultaneously, at his gates with harmonic *voltages* having the same frequency. Interesting conclusions may be obtained when the studied two-port it supplied, simultaneously, at his gates by harmonic *currents* having the same frequency (Rosman, 2009).

The aim of this paper is to study the possibility to realize the resonance at each of the two gates of a linear, non-autonomous and reciprocal two-port (in restricted sense), supplied simultaneously at both gates with harmonic voltages having the same frequency (Fig. 1).

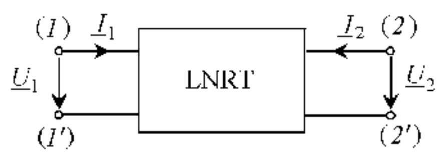

Fig. 1

The eqs. of such a two-port are (Şora, 1961)

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ -\underline{I}_2 \end{bmatrix}, \quad (1)$$

where $\underline{A}_{ij}$, $(i, j = 1, 2)$, represent the fundamental parameters of two-port (LNRT). As well

$$\underline{Z}_{e1} = \frac{\underline{U}_1}{\underline{I}_1} = R_{e1} + \mathrm{j}X_{e1}, \quad \underline{Z}_{e2} = \frac{\underline{U}_2}{\underline{I}_2} = R_{e2} + \mathrm{j}X_{e2} \quad (2)$$

represent the equivalent complex impedances at the two LNRT's gates. Evidently, if the LNRT is passive, $R_{e1} \geq 0$, $R_{e2} \geq 0$.

Having in view the reciprocity relation

$$\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21} = 1 \quad (3)$$

eqs. (1) may be written

$$\begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{A}_{22} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{11} \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ -\underline{I}_1 \end{bmatrix}. \quad (4)$$

Expressions (1), (2) and (4) permit to establish the relations between equivalent complex impedances $\underline{Z}_{e1}$, $\underline{Z}_{e2}$, at the LNRT's gates

$$\underline{Z}_{e1} = \frac{\underline{A}_{11}\underline{Z}_{e2} - \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_{e2} - \underline{A}_{22}} \quad \text{or} \quad \underline{Z}_{e2} = \frac{\underline{A}_{22}\underline{Z}_{e1} - \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_{e1} - \underline{A}_{11}}. \quad (5)$$

These relations represent *conformal transformations* (Stoilov, 1964) from complex plan $\underline{Z}_{e2}$ into complex plane $\underline{Z}_{e1}$ and reciprocally.

The equivalent complex impedance at the gate (*1*), (*1*') can be determined with relations (1) and (2) namely

$$\underline{Z}_{e1}=\frac{\underline{A}_{11}\underline{A}_{21}^{*}\left(R_{e2}^{2}+X_{e2}^{2}\right)-\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right)R_{e2}-\mathrm{j}\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)X_{e2}+\underline{A}_{12}\underline{A}_{22}^{*}}{A_{21}^{2}\left(R_{e2}^{2}+X_{e2}^{2}\right)-2\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)R_{e2}+2\Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)X_{e2}+A_{22}^{2}},\quad(6)$$

while the equivalent complex impedance at the gate (*2*), (*2*') may be obtained utilizing relations (2) and (4)

$$\underline{Z}_{e2}=\frac{\underline{A}_{21}^{*}\underline{A}_{22}\left(R_{e1}^{2}+X_{e1}^{2}\right)-\left(\underline{A}_{11}^{*}\underline{A}_{22}+\underline{A}_{12}\underline{A}_{21}^{*}\right)R_{e1}-\mathrm{j}\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}\underline{A}_{21}^{*}\right)X_{e1}+\underline{A}_{11}^{*}\underline{A}_{12}}{A_{21}^{2}\left(R_{e1}^{2}+X_{e1}^{2}\right)-2\Re e\left(\underline{A}_{11}^{*}\underline{A}_{21}\right)R_{e1}+2\Im m\left(\underline{A}_{11}^{*}\underline{A}_{21}\right)X_{e1}+A_{11}^{2}}.\quad(7)$$

2. The Resonance at the Gate (*1*), (*1*')

Such a regime is realized if

$$X_{e1}=\Im m\left(\underline{Z}_{e1}\right)=0.\quad(8)$$

Having in view relation (6), condition (8) is realized if

$$\begin{aligned}&\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\left(R_{e2}^{2}+X_{e2}^{2}\right)-\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right)R_{e2}-\\&-\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right)X_{e2}+\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)=0.\end{aligned}\quad(9)$$

Expression (9) represents, in plane (R_{e2} , X_{e2}), the equation of a circle with the center in

$$R_{e2}^{*}=\frac{\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}\underline{A}_{21}^{*}\right)}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)},\quad X_{e2}^{*}=\frac{\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}\quad(10)$$

and the radius

$$r_{e2}=\frac{1}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}\times$$
$$\times\sqrt{\left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{11}\underline{A}_{22}^{*}\right)\right]^{2}+\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)\right]^{2}-4\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)},\quad(11)$$

where was considered that $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) \neq 0$.

In a previous paper (Rosman *et al.*, 1964) was proved that the fundamental parameters, $\underline{A}_{ij}$, $(i, j = 1, 2)$, of an LNRT satisfy the identity

$$\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2} + \left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2} - 4\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) = 1 \quad (12)$$

so the radius (11) becomes

$$r_{e2} = \frac{1}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}. \quad (13)$$

Therefore eq. (8) may be written

$$\left[R_{e2} - \frac{\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right)}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}\right]^{2} + \left[X_{e2} - \frac{\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right)}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}\right]^{2} = \frac{1}{4\left[\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\right]^{2}}. \quad (14)$$

In view to determine the possibilities to realize the resonance at the LNRT’s gate (*1*), (*1'*) is important to study the position of circle (9) (or (14)) in the plane (R_{e2} , X_{e2}). Thus if

$$\operatorname{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right)\right] = \operatorname{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\right] \quad (15)$$

the circle’s center abscissa is situated in half-plane $R_{e1} > 0$. The circle (9) (or (14)) is situated entirely in this half-plane if

$$\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) \in \left(-\infty, -1\right] \cup \left[1, \infty\right), \quad (16)$$

or partially if

$$\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) \in \left[-1,\ 1\right]. \quad (17)$$

If

$$\operatorname{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right)\right] = -\operatorname{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\right] \quad (18)$$

the circle’s (9) center abscissa is situated in the half-plane $R_{e2} < 0$. This circle is situated entirely in this half-plane if relation (16) is satisfied and only an arc is contained in the half-plane R_{e2} > o, when the relation (17) is satisfied.

The specification of these details permits to establish the conditions which assure the resonance realization at the considered LNRT's gate $(1), (1')$. For instance it is impossible to realize the resonance if relations (16) and (18) are satisfied.

Relations (15) and (17) were established using identity (12).

The circle arc situated in the half-plane $R_{e2} \geq 0$ may be considered as representing the geometrical-locus diagram of the equivalent complex impedance at the gate $(2), (2')$, $\underline{Z}_{e2}$, which corresponds to those regimes which realize the resonance at the gate $(1), (1')$ of an LNRT supplied simultaneously at both gates with harmonic voltages having the same frequency.

3. Resonance at the Gate $(2), (2')$

This regime is possible when

$$X_{e2} = \Im m\left(\underline{Z}_{e2}\right) = 0. \tag{19}$$

Taking into account relation (7) condition (19) leads to

$$\begin{gathered}\Im m\left(\underline{A}_{11}^{*}\underline{A}_{21}\right)\left(R_{e1}^{2}+X_{e1}^{2}\right)-\Im m\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)R_{e1}-\\-\Re e\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)X_{e1}+\Im m\left(\underline{A}_{11}^{*}\underline{A}_{12}\right)=0\end{gathered} \tag{20}$$

which represents, in (R_{e1} , X_{e1}) plane, the equation of a circle having the center in

$$R_{e1}^{*}=\frac{\Im m\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)}{2\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)},\quad X_{e1}^{*}=\frac{\Re e\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)}{2\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)} \tag{21}$$

and the radius

$$\begin{gathered}r_{e1}=\frac{1}{2\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)}\times\\ \times\sqrt{\left[\Im m\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2}+\left[\Re e\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2}-4\Im m\left(\underline{A}_{11}^{*}\underline{A}_{12}\right)\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)}\end{gathered}, \tag{22}$$

where was considered that $\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right) \neq 0$.

Having in view the identity (Rosman *et al*., 1964)

$$\left[\Re e\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2}+\left[\Im m\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)\right]^{2}-4\Im m\left(\underline{A}_{11}^{*}\underline{A}_{12}\right)\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)=1 \quad (23)$$

the radius (22) may be written

$$r_{e1}=\frac{1}{2\Im m\left(\underline{A}_{11}^{*}\underline{A}_{21}\right)} \quad (24)$$

and eq. (20) becomes

$$\left[R_{e1}-\frac{\Im m\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)}{2\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)}\right]^{2}+\left[X_{e1}-\frac{\Re e\left(\underline{A}_{11}^{*}\underline{A}_{22}-\underline{A}_{12}^{*}\underline{A}_{21}\right)}{2\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)}\right]^{2}=\frac{1}{4\left[\Im m\left(\underline{A}_{21}^{*}\underline{A}_{22}\right)\right]^{2}}. \quad (25)$$

In this case too it is useful to study the circle's position (20) (or (25)) in the plane (R_{e2} , X_{e2}). With that and in view it is necessary to precise that if

$$\text{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)\right]=-\text{sign}\left[\Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)\right] \quad (26)$$

the circle's center abscissa is situated in the half-plane $R_{e1} > 0$. The circle (20) (or (25)) is contained entirely in this plane if

$$\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)\in\left(-\infty,-1\right]\cup\left[1,\infty\right), \quad (27)$$

and only an arc of this one is situated in this half-plane when

$$\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)\in\left[-1,\ 1\right]. \quad (28)$$

When

$$\text{sign}\left[\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}\underline{A}_{21}^{*}\right)\right]=\text{sign}\left[\Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)\right] \quad (29)$$

the circle's (20) center abscissa is situated in half-plane $R_{e1} < 0$ and the circle is contained entirely in this half-plane if relation (27) is satisfied. Only an arc of this circle is situated in the half-plane $R_{e1} > 0$ if relation (27) is satisfied.

These specifications facilitate the establishing of conditions which assure the realization of a resonance regime at the LNRT's gate $(2), (2')$.

The circle arc situated in the half-plane $R_{e1} > 0$ represents, strictly speaking, the geometrical-locus diagram of the equivalent complex impedance, $\underline{Z}_{e1}$, at the gate $(1), (1')$, when the resonance at the gate $(2), (2')$ of an LNRT supplied simultaneously at both gates with harmonic voltages having the same frequency, is realized.

Relations (27) and (28) were obtained using the identity (23).

4. Double Resonance at the Two Gates

When the resonance is realized at the LNRT's gate $(1), (1')$, the regime at the gate $(2), (2')$ is also a resonance one if the geometric-locus diagram of complex impedance $\underline{Z}_{e2}$, that is circle (20) (or (25)), intersects the $X_{e2} = 0$ axis. This situation is realized when

$$\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right) \in [-1,\ 1]. \tag{30}$$

Inversely, if the resonance is realized at the LNRT's gate $(2), (2')$ the condition that at the gate $(1), (1')$ be realized a resonance regime too is assured by the same condition (30). This result is obtained if the condition in which the geometrical-locus diagram of the equivalent complex impedance $\underline{Z}_{e1}$ (the circle (20) or (25), situated in the half-plane $R_{e1} \geq 0$) intersects the $X_{e1} = 0$ axis is satisfied, when the resonance at the LNRT's gate $(2), (2')$ is realized.

Condition (30) is obtained taking into account identities (12) and (23).

Evidently, it is necessary to have in view that to realize the resonance at any gate of an LNRT supplied simultaneously with harmonic voltages having the same frequencies, the conditions established in §§**2** and **3** must be satisfied too.

Having in view the above observations the following situations may be summarized:

1° When $\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) < 0$ are possible: a) two double resonance regimes if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) < 0,\ \Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) < 0$; b) one double resonance regime if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) < 0$; c) no double resonance regime if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) > 0,\ \Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) > 0$.

2° When $\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right) > 0$ are possible: a) two double resonance regimes if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) < 0,\ \Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) > 0$; b) one double resonance regime if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)<0$; c) no double resonance regime if $\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)<0$, $\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)<0$.

Consequently it results that at the most two different double resonance regimes may be obtained in the case of an LNRT supplied simultaneously at the two gates with harmonic voltages having the same frequency.

5. Conclusions

The performed study leads to following conclusions:

1° It is possible to realize the resonance either at the gate (*1*), (*1'*) or at the gate (*2*), (*2'*) of a linear, non-autonomous and reciprocal two-port, if conditions (15),…,(18), respectively (26),…,(29) are satisfied.

2° The resonance may be obtained simultaneously at the two gates of the above considered two-port if, in addition, relation (30) is satisfied. At the most two such double resonances may be obtained.

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REZONANȚA LA PORȚILE DE ACCES ALE UNUI CUADRIPOL DIPORT LINIAR, NEAUTONOM ŞI RECIPROC, ALIMENTAT SIMULTAN PE LA AMBELE PORȚI CU TENSIUNI ARMONICE DE ACEEAŞI FRECVENȚĂ

(Rezumat)

Se stabilesc condițiile în care poate fi realizată rezonanța la frecare dintre cele două porți ale unui cuadripol diport liniar, neautonom şi reciproc, alimentat simultan pe la aceste porți cu tensiuni armonice de aceeaşi frecvență. De asemenea se determină condițiile în care, la un astfel de cuadripol se poate realiza rezonanța dublă, la ambele sale porți. Se arată că pot exista cel mult două astfel de regimuri.