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EDDY CURRENTS IN A STRAIGHT CYLINDRICAL CONDUCTOR, HAVING A CIRCULAR SECTION, IN PERIODICAL NON-HARMONIC STEADY-STATE

BY

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Abstract. The eddy currents in a straight cylindrical conductor, having a circular section, situated in a homogeneous, periodical, non-harmonic magnetic field, having the same direction as the conductor's axis, were studied. The performed study is based on a symbolical method which permits to represent the periodical non-harmonical signals through hypercomplex “images”. Using this proceeding the hypercomplex moduli of electromagnetic field's state vectors are determined in a point situated inside the cylindrical conductor. Firstly were determined these hypercomplex moduli of field's state vectors produced, separately, by the low, medium and high order harmonics, of the external magnetic field. The resultant expressions of these moduli are obtained applying the superposition theorem, considering that the cylindrical conductor has a linear behavior in the electromagnetic field.

Key words: eddy currents; straight cylindrical conductor; periodical non-harmonic external magnetic field; hypercomplex symbolic method.

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1. Introduction

In a previous paper (Rosman, 2011) the electromagnetic field produced by eddy currents in a conducting plate having a rectangular section, situated in a homogeneous magnetic field, periodically variable, but non-harmonic, in time, was studied.

The aim of this paper is to study the analogous problem referring to eddy currents induced in a straight cylindrical conductor, infinitely long, having a circular section. Such a conductor is considered having the radius of the straight section *a* (Fig. 1), being situated in a homogeneous magnetic field, $\mathbf{H}_0$, oriented parallel to the cylindrical conductor's axis. It is supposed that the considered conductor is situated sufficiently far with respect to other conductors through which currents are flowing, so that the proximity effect may be neglected.

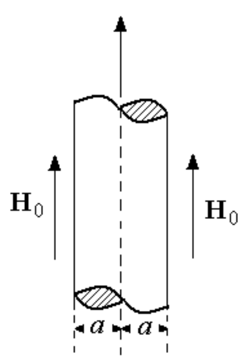

Fig. 1

If the external magnetic field's intensity is variable *vs*. the time according to a harmonic law, $\mathbf{H}_0(t) = \mathbf{k}\sqrt{2}H_0 \sin \omega t$, the electromagnetic field's state vectors complex moduli inside the conductor have the following complex RMS values (Moraru, 1984):

$$\underline{E}_{\text{int}}(r) = -\frac{\underline{\gamma}\underline{H}_0}{\sigma} \cdot \frac{\mathrm{I}_1(\underline{\gamma}r)}{\mathrm{I}_0(\underline{\gamma}a)}, \quad \underline{H}_{\text{int}}(r) = \underline{H}_0 \frac{\mathrm{I}_0(\underline{\gamma}r)}{\mathrm{I}_0(\underline{\gamma}a)},$$
$$\underline{J}(r) = -\underline{\gamma}\underline{H}_0 \frac{\mathrm{I}_1(\underline{\gamma}r)}{\mathrm{I}_0(\underline{\gamma}a)}, \tag{1}$$

with $r \in [0, a]$, and

$$\underline{\gamma} = \sqrt{\mathrm{j}\omega\sigma\mu} = (1 + \mathrm{j})\alpha, \quad \alpha = \sqrt{\frac{\omega\mu\sigma}{2}}, \tag{2}$$

$\underline{\gamma}$ being the complex propagation constant, α – the attenuation constant (equal with the phase constant, β), μ, σ – conductor's material constants considered invariable with respect to external magnetic field intensity, I_0 and I_1 – modified Bessel functions of first species and zero, respectively first order, $\underline{H}_0$ – the external magnetic field's intensity complex RMS value.

The complex Poynting vector modulus in a point situated on the conductor's surface, corresponding to the length unit of this one, is

$$\underline{S}_l(a) = -\frac{\underline{\gamma}H_0^2}{\sigma} \cdot \frac{\mathrm{I}_1(\underline{\gamma}a)}{\mathrm{I}_0(\underline{\gamma}a)}, \tag{3}$$

which permits to perform the active and reactive powers, P_l and Q_l , corresponding to the conductor's length unit too.

The functions $I_0(m)$ and $I_1(m)$ admit the series developments (Gray & Mathews, 1958)

$$I_0(m) = 1 + \frac{m^2}{2\cdot 2} + \frac{m^4}{2^2\cdot 4^2} + \ldots,\ I_1(m) = \frac{m}{2}\left[1 + \frac{m^2}{2(2+2)} + \frac{m^4}{2\cdot 4(2+2)(2+4)} + \ldots\right], \quad (4)$$

with $m = \underline{\gamma}r$, $\underline{\gamma}a$. If from these series only the first two terms are retained relations (1) become

$$\underline{E}_{\text{int}}(r) = -\frac{\underline{\gamma}^2 r \underline{H}_0}{4\sigma}\cdot\frac{8+\underline{\gamma}^2 r^2}{4+\underline{\gamma}^2 a^2},\ \underline{H}_{\text{int}}(r) = \underline{H}_0\frac{4+\underline{\gamma}^2 r^2}{4+\underline{\gamma}^2 a^2},$$
$$\underline{J}(r) = -\frac{\underline{\gamma}^2 r \underline{H}_0}{4}\cdot\frac{8+\underline{\gamma}^2 r^2}{4+\underline{\gamma}^2 a^2}, \quad (5)$$

and expression (3) leads to

$$\underline{S}_l(a) = -\frac{\underline{\gamma}^2 a H_0^2}{4\sigma}\cdot\frac{8+\underline{\gamma}^2 a^2}{4+\underline{\gamma}^2 a^2}. \quad (6)$$

Relations (5) and (6) represent approximations acceptable at *low* frequencies. In the case of *high* frequencies functions $I_0(m)$ and $I_1(m)$ may be approximated with theirs asymptotic values (Gray & Mathews, 1958)

$$I_0(m) = I_1(m) \approx \frac{e^m}{\sqrt{2\pi m}}, \quad (7)$$

so relations (1), (2) may be approximated at their turn with

$$\underline{E}_{\text{int}}(r) = -\frac{\underline{\gamma}\underline{H}_0}{\sigma}\sqrt{\frac{a}{r}}e^{\underline{\gamma}(r-a)},\ \underline{H}_{\text{int}}(r) = \underline{H}_0\sqrt{\frac{a}{r}}e^{\underline{\gamma}(r-a)},\ \underline{J}(r) = -\underline{\gamma}\underline{H}_0\sqrt{\frac{a}{r}}e^{\underline{\gamma}(r-a)}, \quad (8)$$

respectively

$$\underline{S}_l(a) = -\frac{\underline{\gamma}H_0^2}{\sigma}. \quad (9)$$

In what follows the material constants (σ, μ) dispersion with the frequency is neglected.

2. Study Method

As was indicated in a previous paper (Rosman, submitted) the study of eddy currents in periodical non-harmonic steady-state is advantageous to be performed using a symbolic method, proposed by B.A. Rozenfeld (1949), which is based on periodical non-harmonic signals representation through hypercomplex "images". So, to a periodical non-harmonic signal, $a(t)$, which admits the development in Fourier series

$$a(t) = \sum_{k=0}^{\infty} A_k^{'} \cos k\omega t + \sum_{k=0}^{\infty} A_k^{''} \sin k\omega t \,, \tag{10}$$

may be attached the hypercomplex "image"

$$\hat{A} = \sum_{k=0}^{\infty} 1_k \, A_k^{'} + \sum_{k=0}^{\infty} \mathrm{j}_k \, A_k^{''} \,, \tag{11}$$

where functions 1_k , j_k are orthonormalized. The so defined algebra is commutative, representing a direct sum of real numbers field (generated by 1_0) and the numberable set of complex numbers set (generated by the pair 1_k , j_k). The unit element of this algebra is

$$\sum_{k=0}^{\infty} 1_k = 1 \,. \tag{12}$$

The elements 1_k, j_k satisfy in the same time the relations

$$1_k^2 = 1_k,\ \mathrm{j}_k^2 = -1_k,\ \mathrm{i}_k \mathrm{j}_k = \mathrm{j}_k i_k = \mathrm{j}_k,\ 1_p 1_q = \mathrm{j}_p \mathrm{j}_q = 1_p \mathrm{j}_q = 1_q \mathrm{j}_p = 0,\ (p \neq q). \tag{13}$$

The identity

$$\frac{\mathrm{d}^m}{\mathrm{d}t^m} = \sum_{k=0}^{\infty} \left(\mathrm{j}_k k\omega \right)^m , \quad m \in \mathbb{N}, \tag{14}$$

is satisfied too by the signals hypercomplex "images", what renders evident the remarkable property of this symbolic method to "algebrize" the differential operations with regard to time. It results that to integro-differential eqs. satisfied by periodical non-harmonic signals correspond algebraic eqs. satisfied by these signals "images".

3. Hypercomplex Vectors $\hat{\mathbf{E}}_{\text{int}}(r)$, $\hat{\mathbf{H}}_{\text{int}}(r)$, $\hat{\mathbf{J}}(r)$ and $\hat{\mathbf{S}}(a)$

Denoting with $\hat{\mathbf{E}}_{\text{int}}(x)$, $\hat{\mathbf{H}}_{\text{int}}(x)$ and $\hat{\mathbf{J}}(x)$ the hypercomplex state vectors of an electromagnetic field in periodical non-harmonic steady state, in a point at a distance $x \in [0, a]$ from cylindrical conductor's axis, situated in a homogeneous external magnetic field having the intensity a periodical but non-harmonic function of time which admits the development in a Fourier series

$$\mathbf{H}_0(t) = \sqrt{2}\sum_{k=0}^{\infty}\mathbf{H}'_{0k}\cos k\omega t + \sqrt{2}\sum_{k=0}^{\infty}\mathbf{H}''_{0k}\sin k\omega t, \tag{15}$$

the hypercomplex expressions of field's state vectors moduli are similar to relations (1) with the specification that $\underline{\gamma}$ must be substituted with (Rosman, 1977)

$$\hat{\gamma} = \sqrt{\sum_{k=0}^{\infty}\mathrm{j}_k k\omega\sigma\mu} = \sum_{k=0}^{\infty}\left(1_k + \mathrm{j}_k\right)\alpha_k,\ \alpha_k = \sqrt{\frac{k\omega\sigma\mu}{2}}, \tag{16}$$

which represents the hypercomplex propagation constant, α_k being the attenuation constant (equal with the phase constant) of external's magnetic field k rank harmonic. In the same time H_0 must be substituted too with

$$\hat{H}_0 = \sum_{k=0}^{\infty}1_k H'_{0k} + \sum_{k=0}^{\infty}\mathrm{j}_k H''_{0k}, \tag{17}$$

representing the hypercomplex RMS value of the same magnetic field. In these conditions eqs. (1) may be substituted with

$$\hat{E}_{\text{int}}(r) = -\frac{\hat{\gamma}\hat{H}_0}{\sigma}\cdot\frac{\mathrm{I}_1(\hat{\gamma}r)}{\mathrm{I}_0(\hat{\gamma}a)},\ \hat{H}_{\text{int}}(r) = \hat{H}_0\frac{\mathrm{I}_0(\hat{\gamma}r)}{\mathrm{I}_0(\hat{\gamma}a)},\ \hat{J}(r) = -\hat{\gamma}\hat{H}_0\frac{\mathrm{I}_1(\hat{\gamma}r)}{\mathrm{I}_0(\hat{\gamma}a)}, \tag{18}$$

so that Poynting vector's hypercomplex modulus related to cylindrical conductor's length unit, in a point situated on his surface (s. rel. (2)), is

$$\hat{S}_l(a) = -\frac{\hat{\gamma}H_0^2}{\sigma}\cdot\frac{\mathrm{I}_1(\hat{\gamma}a)}{\mathrm{I}_0(\hat{\gamma}a)}. \tag{19}$$

As was established in a previous paper (Rosman, 1960) it is possible to define, in periodical non-harmonic steady-state, a hypercomplex apparent power

$$\hat{s} = \sum_{k=0}^{\infty} 1_k P_k + \sum_{k=0}^{\infty} \mathrm{j}_k Q_k \ , \tag{20}$$

where

$$P = \sum_{k=0}^{\infty} P_k \ , \ \ Q = \sum_{k=0}^{\infty} Q_k \tag{21}$$

represent the active, respectively the reactive power corresponding to all harmonics. It is easy to observe that

$$\hat{s}_l = 2\pi a \hat{S}_l(a), \tag{22}$$

and, consequently,

$$\hat{s}_l = -2\pi a \frac{\hat{\gamma} H_0^2}{\sigma} \cdot \frac{\mathrm{I}_1(\hat{\gamma} a)}{\mathrm{I}_0(\hat{\gamma} a)}, \tag{23}$$

where relation (19) was taken into account. In this stage exists the possibility to perform the calculus of the active power, P_l , and of the reactive power, Q_l , which correspond to cylindrical conductor's length unit.

If the external magnetic field's fundamental frequency is enough low, in her frequency spectrum may be marked three domains in which the eddy currents produced by the external magnetic field's harmonics are different from a qualitative point of view namely: a) the domain [0, *pf*], in which the harmonics frequencies are sufficiently low, being necessary to retain only two terms from the series development (4); b) the domain [*pf*, *qf*], in which the harmonics have medium values, in this case being necessary to take into account all the terms of series (4); c) the domain [*qf*, ∞) of harmonics having high frequencies, when functions $\mathrm{I}_0(m)$, $\mathrm{I}_1(m)$ can be approximated with theirs assymptotical values (s. rel. (7)).

3.1. Case of External Magnetic Field's Intensity Low Order Harmonics

The contribution of external magnetic field's intensity harmonics having low frequencies, at eddy currents engendering, induced in a cylindrical straight conductor may be estimated retaining for the functions $\mathrm{I}_0(m)$, $\mathrm{I}_1(m)$ series developments (s. rel. (4)) only the first two terms. The electromagnetic field's state vectors hypercomplex moduli, produced inside the conductor, may be determined using relations having the form (5) where $\underline{\gamma}$ is substituted with

$$\hat{\gamma}_p = \sqrt{\sum_{k=0}^{p} \mathrm{j}_k k \omega \sigma \mu} \tag{24}$$

and $\underline{H}_0$ with

$$\hat{H}_{0p} = \sum_{k=0}^{p} 1_k H'_{0k} - \sum_{k=0}^{p} \mathrm{j}_k H''_{0k}. \tag{25}$$

In these conditions

$$\begin{cases} \hat{E}_{\mathrm{int}\,p}(r) = -\dfrac{r\hat{H}_{0p}}{4} \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu \dfrac{8 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu r^2}{4 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu a^2}, \\ \hat{H}_{\mathrm{int}\,p}(r) = \hat{H}_{0p} \dfrac{4 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu r^2}{4 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu a^2}, \\ \hat{J}_p(r) = -\dfrac{r\hat{H}_{0p}}{4} \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu \dfrac{8 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu r^2}{4 + \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu a^2}. \end{cases} \tag{26}$$

Amplifying, in the right member of each relation, with the denominator's hypercomplex conjugate, $4 - \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu a^2$, and taking into account (12) expressions (26) become

$$\begin{cases} \hat{E}_{\mathrm{int}\,p}(r) = \dfrac{\hat{H}_{0p}}{16 + \sum\limits_{k=0}^{p} k^2\omega^2\sigma^2\mu^2 a^4} \Bigg[\sum\limits_{k=0}^{p} 1_k 4k^2\omega^2\sigma\mu^2 r \left(r^2 - 2a^2\right) - \\ \qquad - \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\mu r \left(32 + k^2\omega^2\sigma^2\mu^2 a^2\right) \Bigg], \\ \hat{H}_{\mathrm{int}\,p}(r) = \hat{H}_{0p} \dfrac{16 + \sum\limits_{k=0}^{p} k^2\omega^2\sigma^2\mu^2 a^2 r^2 + 4\sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu \left(r^2 - a^2\right)}{16 + \sum\limits_{k=0}^{p} k^2\omega^2\sigma^2\mu^2 a^4}, \\ \hat{J}_p(r) = \dfrac{\hat{H}_{0p}}{4\left(16 + \sum\limits_{k=0}^{p} k^2\omega^2\sigma^2\mu^2 a^4\right)} \Bigg[\sum\limits_{k=0}^{p} 1_k 4k^2\omega^2\sigma^2\mu^2 r \left(r^2 - 2a^2\right) - \\ \qquad - \sum\limits_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu r \left(32 + k^2\omega^2\sigma^2\mu^2 r^2 a^2\right). \end{cases} \tag{27}$$

In the same time Poynting vector's hypercomplex modulus, related to cylindrical conductor's length unit, is

$$\hat{S}_{lp}(a) = -\frac{aH_{0p}^2}{\sigma}\cdot\frac{\mathrm{I}_1\left(\hat{\gamma}_p a\right)}{\mathrm{I}_0\left(\hat{\gamma}_p a\right)} =$$

$$= -\frac{aH_{0p}^2}{4\sigma}\cdot\frac{\sum_{k=0}^{p} 1_k 4k^2\omega^2\sigma^2\mu^2a^2 + \sum_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu\left(32 + k^2\omega^2\sigma^2\mu^2a^4\right)}{16 + \sum_{k=0}^{p} k^2\omega^2\sigma^2\mu^2a^4}. \tag{28}$$

Having in view relations (19),…,(22), the expressions of active and reactive powers corresponding to the conductor's length unit, in a point situated on her surface, may be determined, namely

$$P_{lp} = -2\pi H_{0p}^2\frac{\sum_{k=0}^{p} k^2\omega^2\sigma^2\mu^2a^3}{16 + \sum_{k=0}^{p} k^2\omega^2\sigma^2\mu^2a^2},\ Q_{lp} = -2\pi H_{0p}^2\frac{\sum_{k=0}^{p} k\omega\mu a\left(32 + k^2\omega^2\sigma^2\mu^2a^4\right)}{16 + \sum_{k=0}^{p} k^2\omega^2\sigma^2\mu^2a^2}. \tag{29}$$

Taking into account that the material constants (σ, μ) dispersion with the frequency is neglected, relations (29) become

$$P_{lp} = -2\pi H_{0p}^2\frac{\omega^2\sigma\mu^2a^3\sum_{k=0}^{p} k^2}{16 + \omega^2\sigma^2\mu^2a^2\sum_{k=0}^{p} k^2}, \tag{30}$$

respectively

$$Q_{lp} = -2\pi H_{0p}^2\frac{\omega\mu a\left(32\sum_{k=0}^{p} k + \omega^2\sigma^2\mu^2a^2\sum_{k=0}^{p} k^3\right)}{16 + \omega^2\sigma^2\mu^2a^2\sum_{k=0}^{p} k^2}. \tag{31}$$

But (Ryžik & Gradshtein, 1951)

$$\sum_{k=0}^{p} k = \frac{p(p+1)}{2}, \sum_{k=0}^{p} k^2 = \frac{p(p+1)(p+2)}{6}, \sum_{k=0}^{p} k^3 = \frac{p^2(p+1)^2}{4}. \quad (32)$$

So expressions (30), (31) may be written

$$P_{lp} = -2\pi H_{0p}^2 \frac{\omega^2\sigma\mu^2 a^3 p(p+1)(p+2)}{96+\omega^2\sigma^2\mu^2 a^2 p(p+1)(p+2)}, \quad (33)$$

respectively

$$Q_{lp} = -3\pi H_{0p}^2 \frac{\omega\mu a\left[64p(p+1)+\omega^2\sigma^2\mu^2 a^2 p^2(p+1)^2\right]}{96+\omega^2\sigma^2\mu^2 a^2 p(p+1)(p+2)}. \quad (34)$$

3.2. Case of External Magnetic Field's Intensity Medium Order Harmonics

For the external magnetic field's harmonics order comprised in range [*pf*, *qf*], $q > p > 1$, f being the fundamental's frequency, the electromagnetic field's state vectors hypercomplex RMS values inside the cylindrical conductor may be determined with relations (20), where $\hat{H}_0$ is substituted with

$$\hat{H}_{0pq} = \sum_{k=p}^{q} 1_k H_k^{'} + \sum_{k=p}^{q} \mathrm{j}_k H_k^{''} \quad (35)$$

and $\hat{\gamma}$ with

$$\hat{\gamma}_{pq} = \sqrt{\sum_{k=p}^{q} \mathrm{j}_k k\omega\sigma\mu} = \sum_{k=p}^{q} \left(1_k + \mathrm{j}_k\right)\alpha_k , \quad (36)$$

so that

$$\hat{E}_{\mathrm{int}\,pq}(r) = -\frac{\hat{\gamma}_{pq}\hat{H}_{0pq}}{\sigma}\cdot\frac{\mathrm{I}_1\left(\hat{\gamma}_{pq}r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)}, \quad \hat{H}_{\mathrm{int}\,pq}(r) = \hat{H}_{0pq}\frac{\mathrm{I}_0\left(\hat{\gamma}_{pq}r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)},$$
$$\hat{J}_{pq}(x) = -\hat{\gamma}_{pq}\hat{H}_{0pq}\frac{\mathrm{I}_1\left(\hat{\gamma}_{pq}r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)}. \quad (37)$$

In the same time

$$\hat{S}_{pq}(a) = -\frac{\hat{\gamma}_{pq}\hat{H}_{0pq}^2}{\sigma} \cdot \frac{\mathrm{I}_1\left(\hat{\gamma}_{pq}a\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)}. \tag{38}$$

Having in view relations (19),…,(26) it is possible to determine the expressions of active and reactive powers on cylindrical conductor's surface and on his length unit

$$P_{lpq} = \sum_{k=p}^{q} P_{lpq}k, \quad Q_{lpq} = \sum_{k=p}^{q} Q_{lpq}k, \tag{39}$$

taking into account that

$$\sum_{k=p}^{q} 1_k P_{lpq}k + \sum_{k=p}^{q} \mathrm{j}_k Q_{lpq}k = 2\pi a \hat{S}_{lpq}(a). \tag{40}$$

3.3. Case of External Magnetic Field's Intensity High Order Harmonics

In such a case the electromagnetic field's state vectors hypercomplex moduli inside the cylindrical conductor, produced by the external magnetic field's intensity harmonics of order [q, ∞), $q > p$, may be determined, using relations of type (8), substituting $\underline{\hat{H}}_0$ with

$$\hat{H}_{0q} = \sum_{k=q}^{\infty} 1_k H_{0k}^{'} + \sum_{k=q}^{\infty} \mathrm{j}_k H_{0k}^{"} \tag{41}$$

and $\hat{\gamma}$ with

$$\hat{\gamma}_q = \sqrt{\sum_{k=q}^{\infty} \mathrm{j}_k k\omega\sigma\mu} = \sum_{k=q}^{\infty} \left(1_k + \mathrm{j}_k\right)\alpha_k. \tag{42}$$

It result the following expressions:

$$\hat{E}_{\mathrm{int}\,q}(r) = -\frac{\hat{\gamma}_q \hat{H}_{0q}}{\sigma}\sqrt{\frac{a}{r}}\mathrm{e}^{\hat{\gamma}_q(r-a)}, \quad \hat{H}_{\mathrm{int}\,q}(r) = \hat{H}_{0q}\sqrt{\frac{a}{r}}\mathrm{e}^{\hat{\gamma}_q(r-a)},$$
$$\hat{J}_q(r) = -\hat{\gamma}_q \hat{H}_{0q}\sqrt{\frac{a}{r}}\mathrm{e}^{\hat{\gamma}_q(r-a)}. \tag{43}$$

Also, in accordance with relation (9) it results that Poynting vector's hypercomplex modulus, related to cylindrical conductor's length unit, in a point situated on her surface, is

$$\hat{S}_{lq}(a) = -\frac{\hat{\gamma}_q H_{0q}^2}{\sigma}. \quad (44)$$

As regards the expressions of the corresponding active and reactive powers, having in view, in addition, relations (20),...,(22), these ones are

$$P_{lq} = -\sqrt{\frac{\omega\mu}{2\sigma}} H_{0q}^2 \sum_{k=q}^{n} \sqrt{k}, \quad Q_{lk} = -\sqrt{\frac{\omega\mu}{2\sigma}} H_{0q}^2 \sum_{k=q}^{n} \sqrt{k}, \quad (45)$$

when are considered only the harmonics till the $n > q$ order.

3.4. General Case

In view to determine the expressions of electromagnetic field's state vectors hypercomplex values inside the cylindrical conductor, as well as of the active and reactive powers related to conductor's length unit, in a point situated on the surface of this one, it is necessary to apply the superposition theorem. More precisely the contributions of low, medium and high order harmonics of magnetic external field are added, namely

$$\begin{cases} \hat{E}_{\text{int}}(x) = \hat{E}_{\text{int}\,p}(x) + \hat{E}_{\text{int}\,pq}(x) + \hat{E}_{\text{int}\,q}(x), \\ \hat{H}_{\text{int}}(x) = \hat{H}_{\text{int}\,p}(x) + \hat{H}_{\text{int}\,pq}(x) + \hat{H}_{\text{int}\,q}(x), \\ \hat{J}(x) = \hat{J}_p(x) + \hat{J}_{pq}(x) + \hat{J}_q(x). \end{cases} \quad (46)$$

Substituting in relation (46) the components $\hat{E}_{\text{int}\,p}(x)$, $\hat{E}_{\text{int}\,pq}(x)$,..., $\hat{J}_q(x)$ with expressions (26), (37) and (43) it results

$$\hat{E}_{\text{int}}(x) = -\frac{r\hat{H}_{0p}}{4} \sum_{k=0}^{p} \mathrm{j}_k k\omega\mu \frac{8 + \sum_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu r^2}{4 + \sum_{k=0}^{p} \mathrm{j}_k k\omega\sigma\mu a^2} - \\ -\frac{\hat{\gamma}_{pq}\hat{H}_{0pq}}{\sigma} \cdot \frac{\mathrm{I}_1\left(\hat{\gamma}_{pq} r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq} a\right)} - \frac{\hat{\gamma}_q \hat{H}_{0q}}{\sigma} \sqrt{\frac{a}{r}} \mathrm{e}^{\hat{\gamma}_q (r-a)}, \quad (47)$$

$$\hat{H}_{\text{int}}(x)=\hat{H}_{0p}\frac{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu r^2}{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu a^2}-\hat{H}_{0pq}\frac{\mathrm{I}_0\left(\hat{\gamma}_{pq}r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)}+ \tag{48}$$

$$+\hat{H}_{0q}\sqrt{\frac{a}{r}}\mathrm{e}^{\hat{\gamma}_q(r-a)},$$

$$\hat{J}(x)=-\frac{r\hat{H}_{0p}}{4}\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu\frac{8+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu r^2}{4+\sum_{k=0}^{p}\mathrm{j}_k k\omega\sigma\mu a^2}- \tag{49}$$

$$-\hat{\gamma}_{pq}\hat{H}_{0pq}\frac{\mathrm{I}_1\left(\hat{\gamma}_{pq}r\right)}{\mathrm{I}_0\left(\hat{\gamma}_{pq}a\right)}-\hat{\gamma}_q\hat{H}_{0q}\sqrt{\frac{a}{r}}\mathrm{e}^{\hat{\gamma}_q(r-a)}.$$

Using a similar proceeding regarding the active and reactive power, may be written the relations

$$P_l=P_{lp}+P_{lpq}+P_{lq},\quad Q_l=Q_{lp}+Q_{lpq}+Q_{lq}. \tag{50}$$

If, in these relations, are substituted $P_{lp},P_{lpq},\ldots,Q_{lq}$ with expressions (33), (39), (45_1), respectively (34), (39_2), (45_2) it results

$$P_l=-2\pi H_{0p}^2\frac{\omega^2\sigma\mu^2a^2p\left(p+1\right)\left(p+2\right)}{96+\omega^2\sigma^2\mu^2a^4p\left(p+1\right)\left(2p+1\right)}+\sum_{k=p}^{q}P_{lpqk}-\sqrt{\frac{\omega\mu}{2\sigma}}H_{0pq}^2\sum_{k=p}^{n}\sqrt{k}, \tag{51}$$

$$Q_l=-3\pi H_{0p}^2\frac{\omega\mu a\left[64p\left(p+1\right)+\omega^2\sigma^2\mu^2a^2p^2\left(p+1\right)^2\right]}{96+\omega^2\sigma^2\mu^2a^2p\left(p+1\right)\left(p+2\right)}+\sum_{k=p}^{q}Q_{lpqk}- \tag{52}$$

$$-\sqrt{\frac{\omega\mu}{2\sigma}}H_{0p}^2\sum_{k=q}^{n}\sqrt{k}.$$

In the above relations were considered only the first $n>q$ harmonics of the external magnetic field intensity. The explicit expressions of active power P_{lpqk} and reactive power, Q_{lpqk}, are, unfortunately, quite difficult to obtain.

3.5. Particular Case

If the external magnetic field's fundamental term frequency is sufficiently high, the electromagnetic field's state vectors hypercomplex moduli, in a point situated inside the cylindrical conductor, produced by the eddy currents, may be determined in the whole using relations (8) substituting γ with $\hat{\gamma}$ (rel. (16)) and H_0 with $\hat{H}_0$ (rel. (17)). This way expressions

$$\hat{E}_{\text{int}}(r) = -\frac{\hat{\gamma}\hat{H}_0}{\sigma}\sqrt{\frac{a}{r}}\text{e}^{\hat{\gamma}(r-a)},\ \hat{H}_{\text{int}}(r) = \hat{H}_0\sqrt{\frac{a}{r}}\text{e}^{\hat{\gamma}(r-a)},\ \hat{J}(r) = -\hat{\gamma}\hat{H}_0\sqrt{\frac{a}{r}}\text{e}^{\hat{\gamma}(r-a)} \quad (53)$$

are obtained. Also, using the same proceeding relation (9) becomes

$$\hat{S}_l(a) = -\frac{\hat{\gamma}\hat{H}_0^2}{\sigma}, \quad (54)$$

so that, having in view relations (20),...,(23), may be established the expressions of active and reactive powers related to cylindrical conductor's length unit namely

$$P_l(a) = -\sqrt{\frac{\omega\mu}{2\sigma}}H_0^2\sum_{k=0}^{n}\sqrt{k},\ Q_l(a) = -\sqrt{\frac{\omega\mu}{2\sigma}}H_0^2\sum_{k=0}^{n}\sqrt{k}, \quad (55)$$

considering this time only the first n harmonics of the external magnetic field.

It is possible to define, in this particular case, the *hypercomplex impedance waves impedance* (Rosman, 2010)

$$\hat{\zeta}_0 = \frac{\hat{E}_{\text{int}}(r)}{\hat{H}_{\text{int}}(r)} = -\frac{\hat{\gamma}}{\sigma} = \sum_{k=0}^{\infty}\left(1_k + \text{j}_k\right)\sqrt{\frac{k\omega\mu}{2\sigma}} = \sqrt{\frac{k\omega\mu}{\sigma}}\text{e}^{-\sum\limits_{k=o}^{\infty}\text{j}_k\pi/4}, \quad (56)$$

which is independent with respect to distance r, where relation (53) was taken into account.

4. Conclusions

The eddy currents induced in a straight cylindrical conductor having a circular section, situated in a homogeneous magnetic field, having a periodical, non-harmonic variation in time, oriented in the conductor's axis direction, are determined.

The performed study is based on a symbolic proceeding, which permits the representation of periodical, non-harmonic signals, through hypercomplex "images". The electromagnetc field's state vectors hypercomplex moduli, $\hat{E}_{int}(r)$, $\hat{H}_{int}(r)$, $\hat{J}(r)$, $r \in [0, a]$, are determined, a being the radius of the straight conductor's section. In the same time the Poynting's vector's hypercomplex modulus is determined too, permitting to obtain the expressions of the active and reactive powers related to cylindrical conductor's length unit.

The determination of these hypercomplex moduli are performed separately for external magnetic field's intensity harmonics of low, medium and high order. The final expressions of hypercomplex moduli, $\hat{E}_{int}(x)$, $\hat{H}_{int}(x)$, $\hat{J}(x)$ and $\hat{S}(a)$, are obtained applying the superposition theorem, supposing that the cylindrical conductor's material has a linear behavior in the electromagnetic field.

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CURENȚII TURBIONARI ÎNTR-UN CONDUCTOR CILINDRIC DREPT, DE SECȚIUNE CIRCULARĂ, ÎN REGIM PERMANENT PERIODIC NEARMONIC

(Rezumat)

Se studiază curenții turbionari induşi într-un conductor cilindric drept situat într-un câmp magnetic omogen coaxial cu conductorul. Studiul efectuat se bazează pe o metodă simbolică elaborată de B.A. Rozenfeld, care permite reprezentarea semnalelor

periodice nearmonice prin „imagini” hipercomplexe. Folosind acest procedeu se determină modulii hipercomplecşi ai vectorilor de stare $\hat{E}_{int}(x)$, $\hat{H}_{int}(x)$, $\hat{J}(x)$, ai câmpului electromagnetic, într-un punct situat în interiorul conductorului cilindric. De asemenea se determină şi modulul vectorului Poynting într-un punct situat pe suprafața conductorului corespunzător unității de lungime a acestuia. Aceşti moduli hipercomplecşi se determină separat, pentru armonicile joase, medii şi înalte ale intensității câmpului magnetic exterior. Expresiile rezultante ale modulilor hipercomplecşi ai vectorilor de stare se determină aplicând teorema suprapunerii considerând că materialul conductor din care este confecționat conductorul cilindric se comportă liniar în câmpul electromagnetic.