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THE CONSTANT TRANSFER COEFFICIENTS MODULI DIAGRAMS OF A LINEAR AND NON-AUTONOMOUS TWO-PORTS IN HARMONIC STEADY-STATE

BY

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Abstract. The equations of constant transfer coefficients moduli of a linear and non-autonomous two-port, in harmonic steady-state, are established. The restrictions which intervene as consequence of the receiver’s passive character are determined too.

Key words: transfer coefficients; linear and non-autonomous two-port; harmonic steady-state.

1. Introduction

In the two-ports study an important role is played by the transfer coefficients namely: voltage and current transfer coefficients and transfer impedance. In the case of linear and non-autonomous two-ports (LNT), working in harmonic steady-state, the expressions of these coefficients are (Şora, 1964)

$$k_U = \frac{\underline{U}_2}{\underline{U}_1},\ \ k_I = \frac{\underline{I}_2}{\underline{I}_1},\ \ Z_m = \frac{\underline{U}_2}{\underline{I}_1}, \tag{1}$$

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$\underline{U}_l$, $\underline{I}_l$, (l = 1, 2), being the signals complex RMS values at the two-port's gates (Fig. 1). In relations (1) $\underline{k}_U$ is the voltage's transfer coefficient, $\underline{k}_I$ – the current's transfer coefficient and $\underline{Z}_m$ – the transfer impedance.

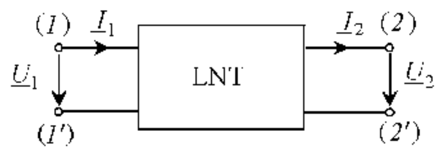

Fig. 1

The eqs. of such a two-port (LNT) are, in harmonic steady-state,

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}, \tag{2}$$

where $\underline{A}_{ij}$, (i, j = 1, 2), represent the LNT's fundamental parameters.

Having in view that the receiver's complex impedance is

$$\underline{Z}_2 = \frac{\underline{U}_2}{\underline{I}_2} = R_2 + \mathrm{j}X_2, \tag{3}$$

transfer coefficients expressions (1) become

$$\underline{k}_U = \frac{R_2 + \mathrm{j}X_2}{\underline{A}_{11}\left(R_2 + \mathrm{j}X_2\right) + \underline{A}_{12}}, \quad \underline{k}_I = \frac{1}{\underline{A}_{21}\left(R_2 + \mathrm{j}X_2\right) + \underline{A}_{22}},$$
$$\underline{Z}_m = \frac{R_2 + \mathrm{j}X_2}{\underline{A}_{21}\left(R_2 + \mathrm{j}X_2\right) + \underline{A}_{12}}, \tag{4}$$

where eqs. (2) were taken into account.

The moduli of these transfer coefficients are (Rosman, submitted)

$$k_U = \sqrt{\frac{R_2^2 + X_2^2}{A_{11}^2\left(R_2^2 + X_2^2\right) + 2\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)R_2 - 2\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)X_2 + A_{12}^2}}, \tag{5}$$

$$k_I = \frac{1}{\sqrt{A_{21}^2\left(R_2^2 + X_2^2\right) + 2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)R_2 - 2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)X_2 + A_{22}^2}}, \tag{6}$$

$$Z_m = \sqrt{\frac{R_2^2 + X_2^2}{A_{21}^2\left(R_2^2 + X_2^2\right) + 2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)R_2 - 2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)X_2 + A_{22}^2}}. \tag{7}$$

In what follows the constant moduli of transfer coefficients, k_U = const., k_I = const. and Z_m = const., in case of an LNT, working in harmonic steady-state, are studied.

2. k_U = const. Diagrams

Taking into account relation (5) the eq. of these diagrams are

$$R_2^2 + X_2^2 + \frac{2\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)k_U^2}{A_{11}^2 k_U^2 - 1} R_2 - \frac{2\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)k_U^2}{A_{11}^2 k_U^2 - 1} X_2 + \frac{\underline{A}_{12}^{*2} k_U^2}{A_{11}^2 k_U^2 - 1} = 0, \quad (8)$$

which represents, in the plane (R_2 , X_2), a circles family for different values of k_U , having the centers coordinates in

$$R_{2k_U} = -\frac{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)k_U^2}{A_{11}^2 k_U^2 - 1}, \quad X_{2k_U} = \frac{\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)k_U^2}{A_{11}^2 k_U^2 - 1} \quad (9)$$

and the radii

$$r_{k_U} = \frac{A_{12} k_U}{\left|A_{11}^2 k_U^2 - 1\right|}. \quad (10)$$

Evidently $k_U \neq 1/A_{11} = k_{U_0}$, where k_{U_0} is the voltage transfer coefficient at empty load working.

The centers of circles family (8) are situated on the straight line

$$\frac{X_2}{R_2} = \frac{\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)}{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)}. \quad (11)$$

Having in view that $\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right) = -P_{2\mathrm{sc}}/I_1^{'2} > 0$, $\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right) = -Q_{2\mathrm{sc}}/I_1^{'2} \lesseqgtr 0$, where $I_1^{'}$ represents the current's RMS value at the gate (1), ($1'$) when the LNT is inverted, relations (11) may be written

$$\frac{X_2}{R_2} = \frac{Q_{2\mathrm{sc}}}{P_{2\mathrm{sc}}}. \quad (12)$$

As a consequence of the receiver's passive character ($R_2 \geq 0$), intervene some restrictions regarding the voltage transfer modulus. It is necessary to have in view two different cases: a) when the centers of circles family (8) are situated in the half-plane $R_2 < 0$ and b) when these centers are situated in the half-plane

$R_2 > 0$. In the first case the voltage transfer coefficient must satisfy, firstly, the inequality

$$k_U < \frac{1}{A_{11}} = k_{U_0}. \tag{13}$$

Secondly, it is necessary to establish if the diagrams k_U = const. are situated entirely or partially in the half-plane $R_2 \geq 0$. With that end in view the eq.'s

$$X_2^2 - \frac{\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)}{A_{11}^2 k_U^2 - 1} X_2 + \frac{\underline{A}_{12}^* k_U^2}{A_{11}^2 k_U^2 - 1} = 0 \tag{14}$$

roots must be analysed. If

$$k_U > \frac{A_{12}}{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)} \tag{15}$$

the roots being imaginary, the diagrams are situated entirely in the half-plane $R_2 \geq 0$. On the contrary, if

$$k_U < \frac{A_{12}}{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)} \tag{16}$$

the roots are real and distinct and, consequently, the diagrams are situated only partially in the half-plane $R_2 \geq 0$.

If the centers of circles family are situated in the half-plane $R_2 < 0$ two situations are possible: either the diagrams are situated entirely in this half-plane, when

$$\frac{1}{A_{11}} < k_U < \frac{A_{12}}{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)}, \tag{17}$$

or the diagrams are situated partially in the half-plane $R_2 \geq 0$ if

$$k_U > \frac{1}{A_{11}} \text{ and } k_U > \frac{A_{12}}{\Re e\left(\underline{A}_{11}\underline{A}_{12}^*\right)}. \tag{18}$$

3. k_I = const. Diagrams

Relation (6) leads to these diagrams eq. namely

$$R_2^2+X_2^2+\frac{2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}R_2-\frac{2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}X_2+\frac{\underline{A}_{22}^*k_I^2-1}{A_{21}^2k_I^2}=0, \tag{19}$$

which represents a concentric circles family having the centers in

$$R_{2k_I}=-\frac{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2},\ X_{2k_I}=\frac{\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2} \tag{20}$$

and the radii

$$r_{k_I}=\frac{1}{k_I A_{21}}. \tag{21}$$

Having in view that $\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)=P_{20}/U_1^{'2}>0,\ \Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)=-Q_{20}/U_1^{'2}\lesseqgtr 0,$ where $U_1^{'}$ represents the voltage's RMS value at the gate (*1*), (*1'*) when the LNT is inverted, relations (20) become

$$R_{2k_I}=-\frac{P_{20}}{A_{21}^2U_1^{'2}},\ X_{2k_I}=\frac{Q_{20}}{A_{21}^2U_1^{'2}}. \tag{22}$$

Since $P_{20}>0$ it results that the centers of circles family (19) are situated in the half-plane $R_2\le 0$, Therefore the diagrams k_I = const. can be situated, in case of an LNT which supplies a passive receiver, only partially in the half-plane $R_2\ge 0$. This situation is realized when eq.'s

$$X_2^2-\frac{2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}X_2+\frac{\underline{A}_{22}^*k_I^2-1}{A_{21}^2k_I^2}=0 \tag{23}$$

roots are real and distinct, *i.e.* when inequality

$$k_I<\frac{A_{21}}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)} \tag{24}$$

is satisfied. Therefore, in case of an LNT supplying a passive receiver, the diagrams k_I = const., are a family of concentric circles partially situated in the half-plane $R_2 \geq 0$, namely for those values of transfer coefficient k_I which satisfy inequality (24).

4. Z_m = const. Diagrams

Using relation (7) the eq. of these diagrams may be established namely

$$R_2^2 + X_2^2 + \frac{2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2 Z_m^2 - 1} Z_m^2 R_2 - \frac{2\Im m\left(\underline{A}_{11}\underline{A}_{12}^*\right)}{A_{21}^2 Z_m^2 - 1} Z_m^2 X_2 + \frac{\underline{A}_{12}^* Z_m^2}{A_{11}^2 Z_m^2 - 1} = 0, \tag{25}$$

which represents, in the plane (R_2, X_2), a circles family having the centers in

$$R_{2Z_m} = -\frac{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2 Z_m^2 - 1} Z_m^2, \; X_{2Z_m} = -\frac{\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right) k_U^2}{A_{21}^2 Z_m^2 - 1} Z_m^2 \tag{26}$$

and the radii

$$r_{Z_m} = \frac{A_{22} Z_m^2}{\left|A_{21}^2 Z_m^2 - 1\right|}. \tag{27}$$

Evidently, for physical reasons $Z_m \neq 1/A_{22}$.

The circles family centers are situated on the strainght line which pass through the origin

$$\frac{X_{2Z_m}}{R_{2Z_m}} = -\frac{\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}. \tag{28}$$

Or $\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right) = -P_{20}/U_1^{'2} > 0$, $\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right) = -Q_{20}/U_1^{'2} \lesseqgtr 0$; consequently

$$\frac{X_{2Z_m}}{R_{2Z_m}} = \frac{Q_{20}}{P_{20}}. \tag{29}$$

If relations (8) and (25) are compared, it may be observed a strong similitude between these ones, to $\underline{A}_{11}$, $\underline{A}_{12}$ from first relation correspond $\underline{A}_{11}$, $\underline{A}_{22}$ in the second one. Having in view this similitude it is possible to express the conclusions regarding the Z_m = const. diagrams using the conclusions concerning the k_U = const. diagrams. In this case it is sufficiently to substitute in the final results concerning the diagrams k_U = const. fundamental parameters $\underline{A}_{11}$, $\underline{A}_{12}$ with $\underline{A}_{21}$, $\underline{A}_{22}$, to obtain the conclusions regarding the Z_m = const. diagrams. Briefly, if

$$\frac{1}{A_{21}} > Z_m > \frac{A_{22}}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)}, \tag{30}$$

the diagram Z_m = const. is situated entirely in the half-plane $R_2 \geq 0$. If

$$Z_m > \frac{1}{A_{21}} \text{ or } Z_m > \frac{A_{12}}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)} \tag{31}$$

the quoted diagram is situated only partially in the half-plane $R_2 \geq 0$. In the same time when

$$\frac{1}{A_{21}} < Z_m < \frac{A_{22}}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)} \tag{32}$$

the Z_m = const. diagram is situated entirely in the helf-plane $R_2 \leq 0$, situation which is without a physical meaning and when

$$Z_m > \frac{1}{A_{21}} \text{ and } Z_m > \frac{A_{22}}{\Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right)} \tag{33}$$

the Z_m = const. diagrams are situated partially in the half-plane $R_2 \geq 0$.

5. Conclusions

1. Considering a linear and non-autonomous two-port, supplying, in harmonic steady-state, a passive receiver, the k_U = const., k_I = const., Z_m = const. diagrams, were studied, where k_U represents the voltage transfer coefficient modulus, k_I – the current transfer coefficient modulus and Z_m – the transfer impedance modulus.

2. The equations of these diagrams are established.

3. The conditions satisfied by k_U , k_I and Z_m so that the diagrams be situated in the half-plane $R_2 \geq 0$, the unique case which have a physical meaning when the receiver is passive, are determined.

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DIAGRAMELE DE MODUL CONSTANT ALE COEFICIENŢILOR DE TRANSFER AI UNUI CUADRIPOL DIPORT LINIAR ŞI NEAUTONOM ÎN REGIM PERMANENTI ARMONIC

(Rezumat)

Se determină ecuaţiile diagramelor de modul constant ale coeficienţilor de transfer ai unui cuadripol diport liniar şi neautonom în regim permanent armonic. De asemenea se determină restricţiile necesar a fi satisfăcute în cazul în care atât cuadripolul cât şi receptorul sunt pasivi.