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SPATIO-TEMPORAL DYNAMICS IN CELLULE NEURAL NETWORK'S USING OTA'S WITH NONIDEAL FREQUENCY RESPONSE

BY

PAUL-MIHAI PUŞCAŞU[1,*] and LIVIU GORAŞ[1,2]

[1]"Gheorghe Asachi" Technical University of Iaşi
[2]Institute of Computer Science, Romanian Academy, Iaşi Branch

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Abstract. In this paper the dynamics of an analog parallel structure is discussed and simulation results at system and transistor level are given. The main idea is that of using the decoupling technique to analyse the parameter influence on the spatial modes evolution in time, in the hypothesis of piecewise linear cells working in the central linear part of the characteristics.

Key words: analog parallel architectures; Cellule Neural Networks; spatial modes.

1. Introduction

Analog parallel computing has, at least in principle, the highest potential regarding speed. A remarkable class of parallel architectures are the Cellular Neural Networks (CNN's) introduced by Chua and Yang (1988). They consist of identical and identically coupled cells within a certain neighborhood. Indeed, more general structures can be imagined if one renounces to the homogeneity of the array in what concern the cells, the interconnections or both. Let us observe

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[*]Corresponding suthor: *e-mail*: ppuscasu@etti.tuiasi.ro

that the interconnection nonhomogeneity is specific to classical Neural Networks.

CNN's are by definition nonlinear systems. However, for architectures made of piecewise linear cells working in the central linear part of the cell characteristic the dynamic is linear as far as the signals on any cell does not leave the linear domain. A particular architecture of the above kind consisting of second order two-port cells sandwiched between two resistive grids which have been shown to exhibit Turing patterns has been studied by Goraş *et al*., (1995). The architecture proved to be useful for building banks of spatial filters for texture classification (Goraş & Puşcaşu, 2011). However, because of the complexity and the large number of configuration parameters, the CMOS implementation is less attractive from the area, power consumption and accuracy points of view. A simpler topology with the principle schematic of the basic cell is suggested in Fig. 1.

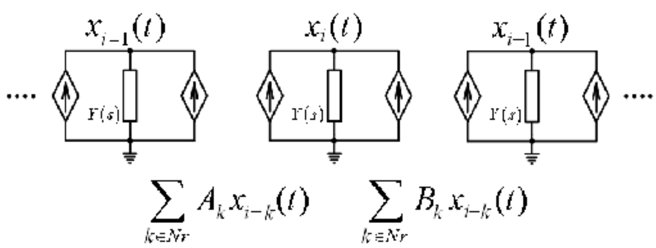


Fig. 1 – 1D CNN architecture.

In the most general case, the voltages of nodes of the parallel array are influenced both by the neighboring nodes and by external sources. In what follows we will consider only autonomous arrays, the dynamics being determined by initial conditions. The aim of this paper is to report further results for the case when the coupling between the cells is no more algebraic but dynamic.

2. The Model

In the following we will consider only the 1D case with the model shown in Fig.2 consisting as before of linear cells represented by admittances denoted by *Y*(*s*) coupled using nonideal voltage controlled current sources over a neighborhood number of radius *r*.

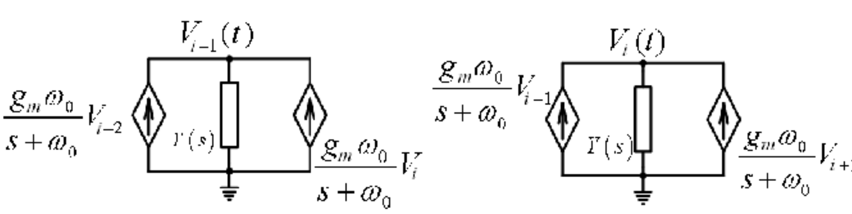


Fig. 2 – The CNN model.

In order to take into consideration the frequency response of the OTA's the model shown in Fig. 2 has been used (Goraş & Puşcaşu, 2011). The nonidealities of the OTA in what concerns the finite input and output impedances can be immediately absorbed by $Y(s)$. The frequency response of the OTA has been modeled as shown in Fig. 3.

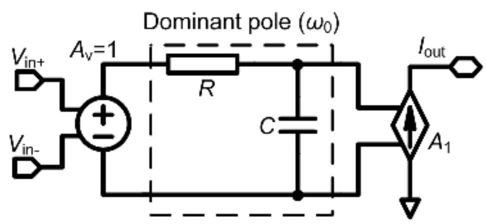


Fig. 3 – Nonideal OTA model.

In the above conditions the differential eqs. that describe the dynamics of the array in symbolic form for $Y(s)$ consisting of a capacitor in parallel with a conductor with conductance A_0 are

$$CsV_i = -A_0V_i + \left(\frac{A_{i-1}\omega_0}{s+\omega_0}\right)V_{i-1} + \left(\frac{A_{i+1}\omega_0}{s+\omega_0}\right)V_{i+1},\ (i=1,...,M-1). \tag{1}$$

For a homogeneous network, with symmetric templates $A_{i-1}= A_{i+1}=A_1$ with ring boundaries conditions, eq. (1) becomes

$$\left(Cs + A_0\right)\left(\frac{s+\omega_0}{\omega_0}\right)V_i = A_1(V_{i-1}+V_{i+1}). \tag{2}$$

Using the decoupling mode technique (Goraş *et al.*, 1995; Goraş & Puşcaşu, 2011) the following set of decoupled eqs. is obtained:

$$s^2\left(\frac{C}{\omega_0}\right)\hat{x}_m(t) + s\left(C+\frac{A_0}{\omega_0}\right)\hat{x}_m(t) = -A_0\hat{x}_m(t) + K_A\left(m\right)\hat{x}_m(t), \tag{3}$$

where $x_m(t)$ is the amplitude of the m-th spatial mode, and $K_A(m)$ represents the spatial eigenvalues, which are complex, in general. For ring boundaries conditions, and for first and second order connections, $K_A(m)$ can be written in the following form:

$$K_A(m) = \begin{cases} 2A_1\cos\frac{2\pi}{M}m;\ \text{for I order,} \\ 2A_1\cos\frac{2\pi}{M}m + 2A_2\cos\frac{4\pi}{M}m;\ \text{for II order,} \end{cases} \tag{4}$$

where $K_A(m)$ represents the dispersion curve.

The characteristic polynomial governing the dynamics of the m-th spatial mode will be

$$s^2\left(\frac{C}{\omega_0}\right)+s\left(C+\frac{A_0}{\omega_0}\right)+A_0-K_A\left(m\right)=0\,, \tag{5}$$

or

$$s^2C+sC\omega_0+sA_0+A_0\omega_0-\omega_0K_A\left(m\right)=0\,, \tag{6}$$

Eq. (6) can be rearranged in the form

$$1+\omega_0\frac{s+\dfrac{A_0-K_A\left(m\right)}{C}}{s\left(s+\dfrac{A_0}{C}\right)}=0\,, \tag{7}$$

which suggests the possibility of using the root locus method considering ω_0 as a generalized amplification. As it can be observed from (7), there are two poles and one zero, all real, namely

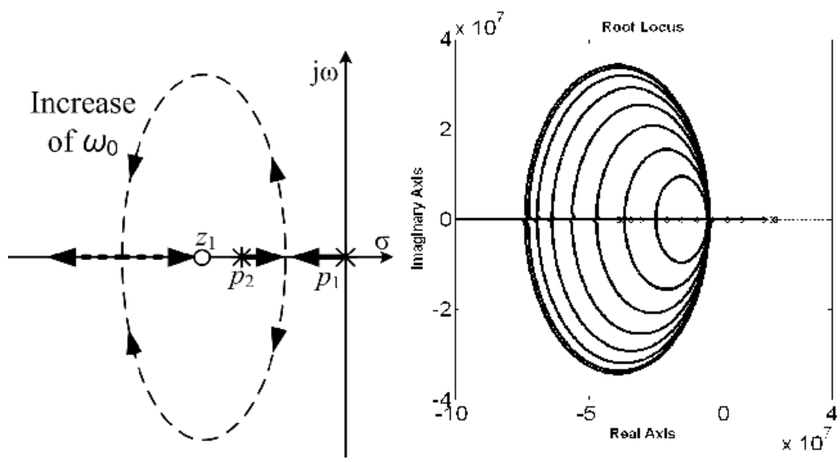
$$\begin{cases} p_{1,2}=0,\ -\dfrac{A_0}{C}; \\ z_1=-\dfrac{A_0-K_A\left(m\right)}{C}. \end{cases} \tag{8}$$

An interesting situation is when one pole is in the origin, and the other one is situated on the real axis in the left half plane, and the zero lies to the left of the two poles. This happens when the following inequality is valid:

$$\frac{A_0-\ 2\ A_1\cos\left(\dfrac{2m\pi}{M}\right)}{C}>\frac{A_0}{C},\ \text{or}\ K_A\left(m\right)<0. \tag{9}$$

If $K_A(m) < 0$, where m is a spatial stable mode, the shape of the root locus for one stable spatial mode with variation of ω_0 , is plotted in Fig. 4.

It can be observed that for some values of the ω_0, the CNN can have an oscillatory spatio-temporal behavior, because of the pair of complex conjugate roots.



a *b*

Fig. 4 – *a* – Root locus for one stable spatial mode;
b – root locus for all spatial modes.

3. Simulation Results

At transistor level simulations, the interconnections between the cells were implemented using a variable gain transconductors similar to the proceeding used by Tanaka *et al*. (2008), shown in Fig. 5.

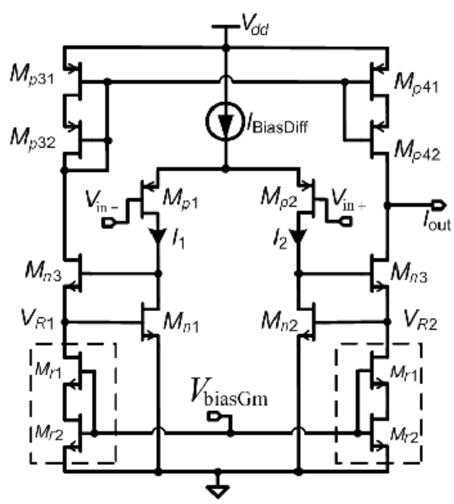


Fig. 5 – OTA's schematic, used in the implementation of the CNN.

The admittance $Y(s)$ consists of a capacitor ($C = 1$ pF), and a resistor (with conductance $A_0 = 10$ μS). The simulations at transistor level and system level have been performed.

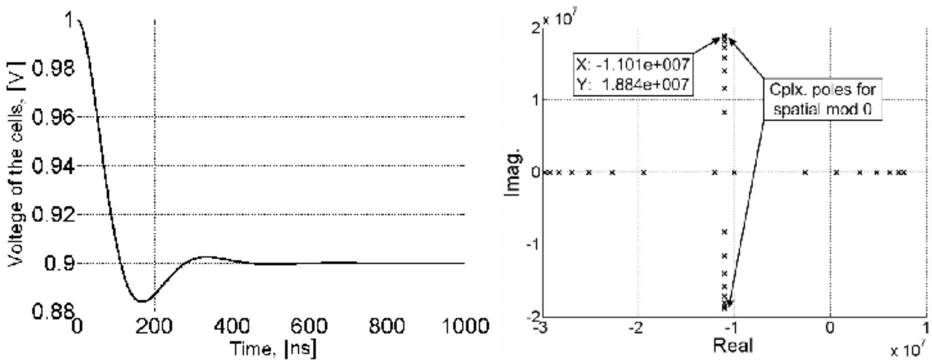
The dominant pole of the OTA is $\omega_0 = 1.202756 \times 10^7$ rad/s; and gm = $= A_1 = A_{-1} = \pm 14.79$ μS.

For simulations, an array of 32 cells, connected in ring boundary conditions, was used. Simulations were done using ideal, ideal with real frequency response, and transistor level implementations.

A first example described below is the evolution in time of the CNN, configured as a high pass filter ($A_1 = A_{-1} = -14.8$ μS), seeded with a dc offset component with amplitude of 100 mV (stable spatial mode 0, which will be attenuated). The solution in time for a CNN implemented with transconductors with $\omega_0 = 1.202756 \times 10^7$ rad/s is

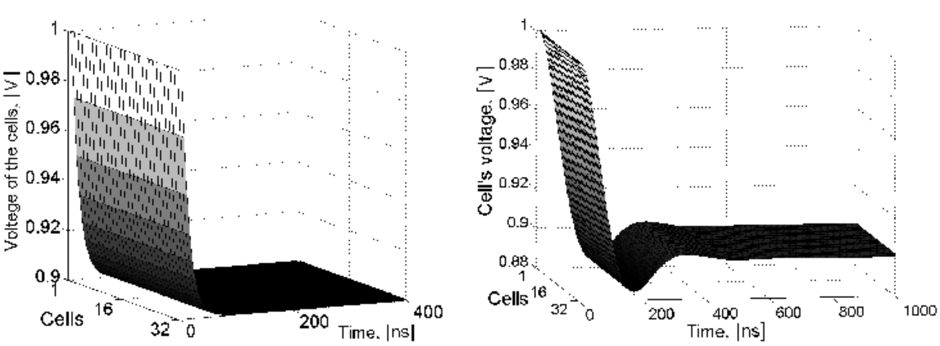
$$y(t) = 0.1\mathrm{e}^{-1.1013\times 10^7 t}\left[\cos\left(1.883\times 10^7 t\right) + 0.5845\sin\left(1.883\times 10^7 t\right)\right]. \qquad (10)$$

The response and the roots constellation of the characteristic polynomial are plotted in Fig. 6.



a *b*

Fig. 6 – Calculated response for mode 0, of the CNN: *a* – transient evolution; *b* – the pole constellation.



a *b*

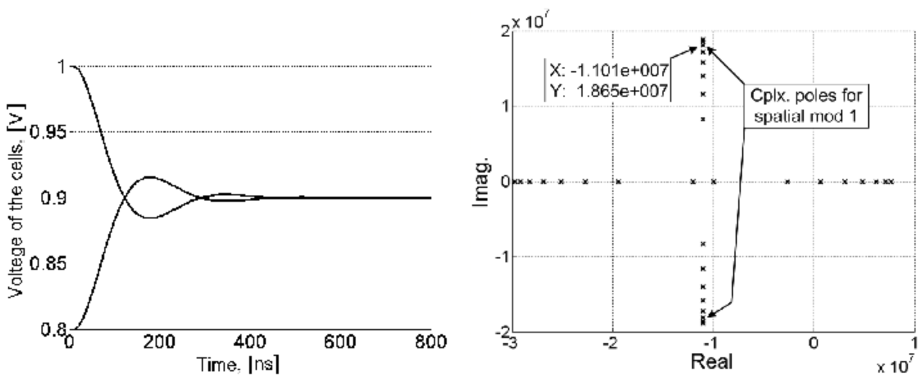
Fig. 7 – Response for mode 0, of the CNN: *a* – with ideal OTAs; *b* – with transistor level OTAs with ω_0 dominant pole.

In Fig. 7, the evolution in time of the above CNN, seeded with spatial mode 0, is presented. The simulations were done using ideal models and transistor level implementations.

Next, we will present the evolution in time of the same CNN (configured as a high pass filter), seeded with the spatial mode 1, with amplitude of 100 mV. The solution is

$$y(t)=\pm 0.1\mathrm{e}^{-1.1013\times 10^{7}t}\left[\cos\left(1.865\times 10^{7}t\right)+0.59\sin\left(1.865\times 10^{7}t\right)\right]. \qquad (11)$$

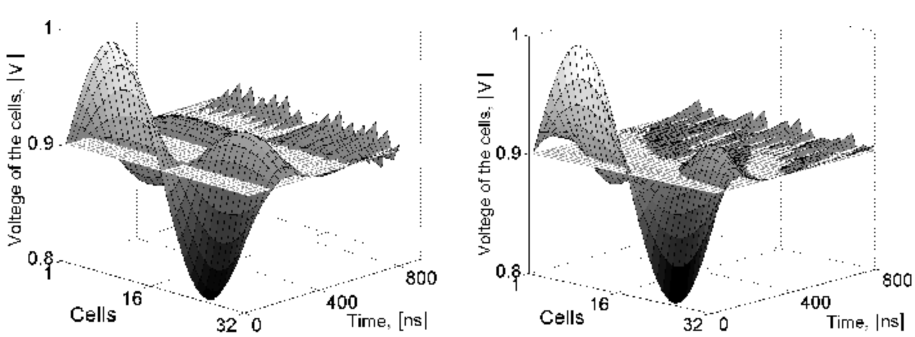
The response and the roots constellation of the characteristic polynomial are plotted in Fig. 8.



a *b*

Fig. 8 – Calculated response for mode 0, of the CNN: *a* – transient evolution; *b* – root constellation.

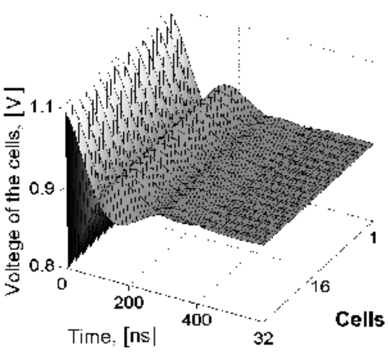
In Fig. 9, the evolution in time of the above CNN, seeded with spatial mode 1, is presented.



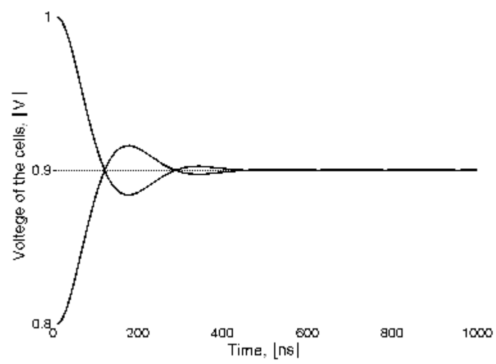
a *b*

Fig. 9 – Response for mode 1, of the CNN: *a* – with ideal OTAs with ω_0 dominant pole; *b* – at transistor level OTA with ω_0 dominant pole.

In Fig. 10 we presented the evolution in time of the CNN configured as a low pass filter ($A_1 = A_{-1} = +14.8$ μS), seeded with the highest spatial mode.



a



b

Fig. 10 – Transient response for highest spatial frequency mode of the CNN: *a* – simulated; *b* – calculated response in time.

The analytical solution is

$$y(t) = 0.1\mathrm{e}^{-1.1013\times10^7 t}\left[\cos\left(1.883\times10^7 t\right) + 0.584\sin\left(1.883\times10^7 t\right)\right]. \tag{12}$$

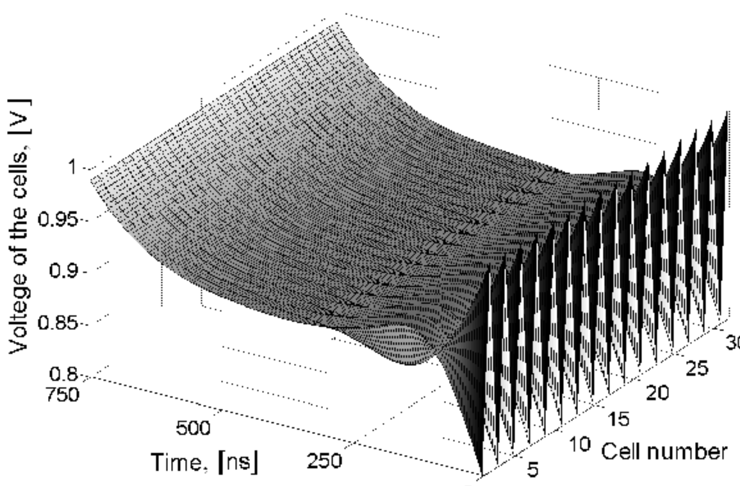


Fig. 11 – Transient response (transistor level) for a CNN seeded with the highest spatial mode.

When the CNN is configured as a low pass filter, it will amplify the offsets of the OTAs. This behavior is present only in low pass filters, with unstable spatial modes. In Fig. 11 it is plotted the transient response at transistor level.

4. Conclusions

The dynamics of a CNN with linear frequency dependent interconnections has been investigated. It has been shown theoretically, at system level and at transistor level, that the dynamics can exhibit oscillatory behavior. The simulation results confirmed with very good accuracy the theoretical approach.

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DINAMICA SPAŢIO-TEMPORALĂ ÎN REŢELE CELULARE NEURALE FOLOSIND TRANSCONDUCTANŢE CU RĂSPUNS NEIDEAL ÎN FRECVENŢĂ

(Rezumat)

Se studiază dinamica unei structuri paralele analogice şi sunt prezentate rezultatele simulărilor la nivel de sistem şi tranzistor. Ideea principală este de a folosi tehnica de decuplare şi de a analiza influenţa unui parametru asupra evoluţiei modurilor spaţiale în timp, în ipoteza că regimul de funcţionare a celulelor este situat în regiunea centrală şi liniară a caracteristicii de lucru.