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MOSFET THERMAL NOISE MODULATING FUNCTIONS OF AN *LC* OSCILLATOR

BY

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Abstract. Analytical expressions for MOSFET thermal noise modulating functions of an *LC* oscillator are presented. These functions are used to analyse the phase noise within the time-varying impulse sensitivity function (ISF) theory. The phase noise due to thermal noise is investigated. The theoretical results are in good agreement with the simulation results.

Key words: *LC* oscillator; MOSFET thermal noise; phase noise; impulse sensitivity function; noise modulating functions; $1/f^2$ phase-noise region.

1. Introduction

The time-varying impulse sensitivity function theory (Hajimiri & Lee, 1999) gives a model for the phase noise generation process. The need for development of noise modulating functions and their analytical expressions are presented by Hajimiri & Lee (1998) and Andreani *et al*. (2005), respectively. The noise spectrum of an electrical oscillator consists of three regions, according to Hajimiri & Lee (1999). At large offset frequencies, Δf, from the

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fundamental, there is a constant noise floor. Closer to the carrier the thermal device noise is shaped as $1/f^2$. Close to the carrier the phase noise is dominated by the up-converted $1/f$ noise and the noise power is shaped as $1/f^3$. In this paper an analysis of the phase-noise generated by the thermal noise will be made and the phase noise behaviour of an oscillator will be investigated. Section **2** reviews the basics of the Impulse Sensitivity Function (ISF) theory. In section **3** MOSFET thermal noise modulating functions for a *LC* oscillator are determined. Simulation results are presented in section **4** and section **5** is devoted to conclusions.

2. Impulse Sensitivity Function Theory

The Impulse Sensitivity Function (ISF), $\Gamma(\omega_0 t)$, (Hajimiri & Lee, 1999) determines the sensitivity of an oscillator to an impulsive input. For a small area of the current impulse into an oscillator node, the resultant phase shift is proportional to the voltage change, ΔV, and hence to the injected electrical charge, Δq,

$$\Delta\Phi = \Gamma(\omega_0 t)\frac{\Delta q}{q_{\max}}, \tag{1}$$

where $q_{\max}$ is the maximum charge swing across the capacitor in parallel with the impulsive current source.

From (1) it is seen that the impulse sensitivity function is a dimensionless, frequency and amplitude independent function periodic in 2π.

The phase angle change, $\Delta\Phi$, becomes

$$\Delta\Phi(t) = \int_0^t \Gamma(\omega_0 t)\frac{i_n(t)}{q_{\max}}\mathrm{d}t. \tag{2}$$

Considering a stationary thermal noise current source, $i_{\text{noise, thermal}}$, with a power spectral density, S_{thermal}, [A^2/Hz], the phase noise at a specific frequency, Δf, is given by (Hajimiri & Lee, 1999)

$$L(\Delta f) = 10\log\left[\frac{\Gamma_{\text{rms}}^2 S_{\text{thermal}}}{q_{\max}^2\, 2(2\pi\Delta f)^2}\right], \tag{3}$$

where Γ_{rms} represents the root mean square of Γ. From (3) it is seen that in order to reduce the phase noise it is necessary to have a low value for the root mean square of Γ.

In the case of cyclo-stationary noise current sources the noise modulating function, $\alpha(\omega_0 t)$, describes the modulation of the device noise current during one period:

$$i_n(t) = i_{n0}(t)\alpha(\omega_0 t)\,, \tag{4}$$

where $i_{n0}(t)$ is the stationary noise current source that would occur in a DC-biased device and $i_n(t)$ – a cyclo-stationary noise source that would occur under the time-varying operating points of the oscillator. Usually $i_{n0}(t)$ is defined at the bias point with maximum noise so that the following inequality holds:

$$0 < \alpha(\omega_0 t) < 1\,. \tag{5}$$

As it can be seen, cyclo-stationary noise can be treated as a stationary noise applied to an oscillator with a new impulse sensitivity function given by

$$\Gamma_{\text{eff}}(\omega_0 t) = \Gamma(\omega_0 t)\alpha(\omega_0 t)\,. \tag{6}$$

Next, in order to derive an analytical expression for the noise modulating function (NMF) the bias dependence of the noise source has to be taken into consideration.

The noise current power spectral density in the case of a MOSFET's thermal channel noise is given by

$$S_{\text{thermal}} = 4KT\gamma g_m\,, \tag{7}$$

where: γ = 2/3 for long channel devices (Jones & Martin, 1997) and g_m represents the transistor's trans-conductance.

Grozing *et al.*, (2004) have demonstrated that the thermal noise modulating function is given by

$$\alpha_{\text{thermal}}(\omega_0 t) = \sqrt{\frac{S_{\text{thermal}}(\omega_0 t)}{S_{\text{thermal,max}}}} = \sqrt{\frac{g_m(\omega_0 t)}{g_{m,\text{max}}}}\,. \tag{8}$$

In order to accommodate (8) for transient simulations, Hajimiri & Lee, (1999), have considered that the trans-conductance may be expressed as

$$g_m = \beta(V_{gs} - V_T)\,. \tag{9}$$

The thermal noise modulating function becomes

$$\alpha_{\text{thermal}}(\omega_0 t) = \sqrt{\frac{V_{gs}(\omega_0 t) - V_T}{V_{gs\max} - V_T}}\,. \tag{10}$$

The noise modulating function given by (10) shows that the channel thermal noise only occurs when the MOSFET's channel is conductive.

3. Phase Noise in the $1/f^2$ Region

In the following, analytical expressions for the phase noise of the *LC* oscillator, shown in Fig. 1, will be derived.

The principle of operation is similar to the one in the case of classic CMOS Colpitts oscillator (Clarke & Hess, 1971). The proposed structure is realized with two transistors in the signal path, M_1 and M_2.

The voltage sources, V_B, are needed in order to bias M_1-M_2 into saturation. For these sources not to shunt at ground C_2 capacitor at large frequencies of operation, a large resistor R_B of value 100 kΩ is inserted in series with them.

The signal at the drain of M_2 transistor is coupled *via* C_1 and C_2 capacitors to the gate of M_1 transistor.

According to Hajimiri & Lee (1999), in the case of a general oscillator, the presence of a resistance, *R*, between one of the oscillator's node and ground causes a phase noise, L, at the offset frequency, $\Delta\omega$, given by

$$L(\Delta\omega) = 10\log\frac{\Gamma_{R,\text{rms}}^2}{q_{\max}^2}\cdot\frac{\overline{i_R^2/\Delta f}}{2\Delta\omega^2}, \tag{11}$$

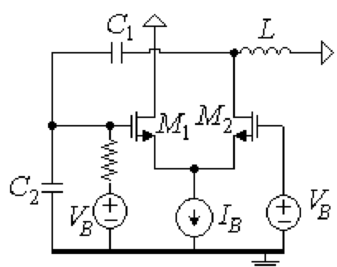

Fig. 1 – *LC* oscillator.

where $q_{\max}$ is the maximum dynamic charge injected into the capacitance in parallel to resistance *R* across one oscillation period, $\overline{i_R^2}$ – the power density of the stationary white noise current generated by the resistance *R*, whose expression is

$$\overline{i_R^2} = 4k_BT\frac{1}{R}\Delta f, \tag{12}$$

where Γ_R is the impulse sensitivity function of such a noise source. Hajimiri & Lee (1999) have proved that

$$\Gamma_R = \sin\Phi, \tag{13}$$

From (13) the root mean square value of Γ_R results

$$\Gamma^2_{R,\mathrm{rms}} = \int_{-\pi}^{\pi} \Gamma^2_R(\Phi)\ \mathrm{d}\Phi = \frac{1}{2}\,. \tag{14}$$

If a cyclo-stationary noise source is taken into account, then the ISF is replaced by an effective impulse sensitivity function as given by relation (6). If a cyclo-stationary noise source generating thermal noise is taken into consideration, then the effective impulse sensitivity function becomes

$$\Gamma_{\mathrm{eff}}(\omega_0 t) = \Gamma(\omega_0 t)\alpha_{\mathrm{thermal}}(\omega_0 t)\,. \tag{15}$$

From (14) the root means square value of the effective impulse sensitivity function shows the conversion of the thermal noise power to phase noise according to relation (3).

Eq. (15) will be used to analyse the effect of the MOS transistor noise current, $\overline{i^2_{ds}}$, which is a cyclo-stationary noise current source.

In order to find the impulse sensitivity function associated to $\overline{i^2_{ds}}$ we use the definition of this special function as the excess phase generated by a current impulse injected into the same oscillator node where the noise current source flows.

Before determining the ISF associated to the MOS transistor's noise current source, we make a simplifying assumption namely, we consider in Fig. 1 that the moment when the oscillator is most susceptible to a phase perturbation caused by noise injection into its output node is the moment when its output voltage at the drain of M_2 crosses zero. The oscillator's output voltage crosses zero when the total voltage at the gate of M_1 transistor is equal to the voltage at the gate of M_2 transistor. The situation when the oscillator's output voltage reaches a maximum (or a minimum) is not taken into consideration due to the fact that, according to the theory of ISF (Hajimiri & Lee, 1999), the phase perturbation caused by a noise injection at that moment is minimum. To prove this fact we consider that the oscillator's output voltage has reached a minimum. In this case M_1 transistor is off and all the bias current flows through M_2. We will relate the impulse sensitivity function associated to the M_2 transistor's noise to the impulse sensitivity function associated to the loss resistance of the oscillator's *LC* tank (Andreani *et al.*, 2005). To find $\Gamma_{ids,2}$ we send a current impulse between the drain and source of M_2 transistor as in Fig.2.

In Fig. 2 C_p represents the parasitic capacitance associated with the bias current source, I_B , and C is the equivalent capacitance formed by C_1 in series with C_2. The thermal noise generated by R resistance is filtered by large C_2 capacitance and has no influence (Jones & Martin, 1997). If the approximate time constant given by C_p/g_{m2} is much lower than the period of oscillation, the charge $-\Delta Q$ from C_p parasitic capacitance is suddenly transferred to the equivalent capacitance formed by C_1 and C_2, at the same moment when C is charged by ΔQ. As a result, the voltage variation on C is zero and thus $\Gamma_{ids,2}$ is zero as well. This behaviour is similar to the one found by Razavi (1998) regarding the noise rejection mechanism in a cascode transistor with large source degeneration.

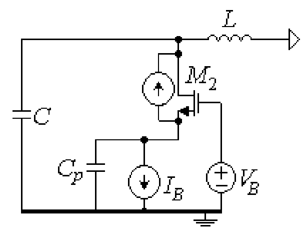

Fig. 2 – Circuit used for the determination of the ISF when only M_2 is on.

The situation when the oscillator's output voltage crosses zero corresponds to the moment when both M_1 and M_2 transistors are on. Thus M_1 and M_2 have maximum contribution to phase noise when they are in the differential mode. The total current noise injected into the oscillator's output in this case is given by

$$\overline{i^2_{\text{noise\_total}}} = \frac{1}{2}\overline{i^2_{\text{noise\_bias}}} + \overline{i^2_{\text{noise\_}ds,M_1}} + \overline{i^2_{\text{noise\_}ds,M_2}},$$

where $\overline{i^2_{\text{noise\_bias}}}$ is the noise current source associated to the bias current source I_B, $\overline{i^2_{\text{noise\_}ds,M_1}}$ – the noise current source associated to the M_1 transistor and $\overline{i^2_{\text{noise\_}ds,M_2}}$ – the noise current source associated to the M_2 transistor. If we neglect the noise introduced by the bias current source and we consider that the noise current in the drain is given by $\overline{i^2_{ds}} = 4k_B T\gamma g_m \Delta f$, where $\gamma = 2/3$ for long channel transistors, relation (8) becomes

$$\overline{i^2_{\text{noise\_total}}} = 2\left(4k_B t\frac{2}{3}g_m\right).$$

By writing Kirchhoff's second law across the loop formed by the gate sources of transistors M_1 and M_2 the following relation is obtained

$$V_i - V_{gs1} + V_{gs2} - V_B = 0, \tag{16}$$

where V_i is the total voltage applied at the gate of M_1 transistor, *i.e.* $V_i = v_i + V_B$, and v_i is equal to a fraction of the output voltage fundamental

$$v_i = \frac{C_1}{C_1 + C_2} V_{\text{out}} \cos \omega_0 t = V_A \cos \omega_0 t \text{ , where } V_A = \frac{C_1}{C_1 + C_2} V_{\text{out}} .$$

After some rearrangements the drain current of M_2 transistor is equal to (Gray *et al*., 2000)

$$I_{d2} = \begin{cases} I_B, \; v_i \le -v_{\max}, \\ \dfrac{I_B}{2} - \dfrac{v_i}{2}\left[k' \dfrac{W}{L}\left(I_B - v_i^2 \dfrac{k' W}{4L}\right)\right]^{1/2}, \; -v_{\max} \le v_i \le v_{\max}, \\ 0, \; v_i \ge v_{\max}, \end{cases} \tag{17}$$

where $v_{\max} = \left(\dfrac{2I_B}{k' W/L}\right)^{1/2}$.

The normalized drain current with respect to the bias current source is

$$\frac{I_{d2}}{I_B} = \begin{cases} 1, \; v_{\text{in}} \le -1, \\ \dfrac{1}{2} - \dfrac{v_{\text{in}}}{\sqrt{2}}\left(1 - \dfrac{v_{\text{in}}^2}{2}\right)^{1/2}, \; -1 \le v_{\text{in}} \le 1, \\ 0, \; v_{\text{in}} \ge 1. \end{cases} \tag{18}$$

where $v_{\text{in}} = v_i / v_{\max}$.

Next the dc and fundamental components of the drain current are obtained. To simplify the analysis we note that $\theta = \omega_0 t$, $a = V_A/v_{\max}$. With these notations the drain current can be written

$$I_{d2} = \begin{cases} 1, \; a\cos\theta \le -1, \\ \dfrac{1}{2} - \dfrac{a\cos\theta}{2}\left[1 - \dfrac{(a\cos\theta)^2}{2}\right]^{1/2}, \; -1 \le a\cos\theta \le 1, \\ 0, \; a\cos\theta \ge 1. \end{cases} \tag{19}$$

The condition $a \cos \theta \ge 1$ can be described by a cut-off angle defined as $\cos \theta_{\text{off}} = 1/a$. Similarly, the condition $a \cos \theta \le 1$ can be described by a conduction angle defined as $\cos \theta_{\text{off}} = -1/a$. The drain current through M_2 transistor normalized to the bias current source, I_B, value now becomes

$$\frac{I_{d2}}{I_B}=\begin{cases}0,\ 0\le\theta\le\theta_{\text{off}}, \\ \dfrac{1}{2}-\dfrac{a\cos\theta}{2}\left[1-\dfrac{(a\cos\theta)^2}{2}\right]^{1/2},\ \theta_{\text{off}}\le\theta\le\theta_{\text{on}}, \\ 1,\ \theta_{\text{on}}\le\theta\le 2\pi-\theta_{\text{on}}, \\ \dfrac{1}{2}-\dfrac{a\cos\theta}{2}\left[1-\dfrac{(a\cos\theta)^2}{2}\right]^{1/2},\ 2\pi-\theta_{\text{on}}\le\theta\le 2\pi-\theta_{\text{off}}, \\ 0,\ 2\pi-\theta_{\text{off}}\le\theta\le 2\pi.\end{cases} \tag{20}$$

In order to determine the effective impulse sensitivity function associated to this total noise current source we use relation (15), where the small signal gate-source voltage when both M_1 and M_2 are on is given by

$$v_{gs}=\frac{C_1}{2(C_1+C_2)}V_o\sin\omega_o t \tag{21}$$

and V_0 represents the amplitude of the oscillation at the drain of M_2 transistor. Relation (18) is valid for $\theta_{\text{off}}\le\theta\le\theta_{\text{on}}$ or $2\pi-\theta_{\text{on}}\le\theta\le 2\pi-\theta_{\text{off}}$.

According to (21), (10) becomes

$$\alpha_{\text{thermal}}(\omega_0 t)=\sqrt{\frac{V_{gs}(\omega_0 t)-V_T}{V_{gs,\max}-V_T}}=\sqrt{\frac{\dfrac{C_1}{2(C_1+C_2)}V_0\sin\omega_0 t-V_T}{\left(\dfrac{2I_B}{\mu_n C_{0x}W/L}\right)^{1/2}}}\,. \tag{22}$$

Using (22) the effective impulse sensitivity function due to the two drain current noise sources is calculated as

$$\Gamma_{\text{eff}}(\omega_0 t)=\Gamma(\omega_0 t)\alpha_{\text{thermal}}(\omega_0 t)=\sin\omega_0 t\sqrt{\frac{\dfrac{C_1}{2(C_1+C_2)}V_0\sin\omega_0 t-V_T}{\left(\dfrac{2I_B}{\mu_n C_{0x}W/L}\right)^{1/2}}}\,. \tag{23}$$

4. Simulation Results

As an application example the *LC* oscillator shown in Fig. 1 is analysed. The impulse sensitivity functions are determined by transient simulations. A current source placed between the drain of M_2 transistor and ground injects a short current pulse of area ΔQ at specific phase angles during one oscillation cycle. From top to down we have represented the noise modulating function (NMF) for the thermal noise, $\alpha_{\text{thermal}, M_2}$, and the impulse sensitivity function ISF Γ_{M_2}. The shape of the NMF shows the time when the thermal noise current is injected into the output node (Fig. 3). Thermal noise current is injected only when the transistors are in conducting state, meaning that their gate source voltage is larger than the threshold voltage, V_T.

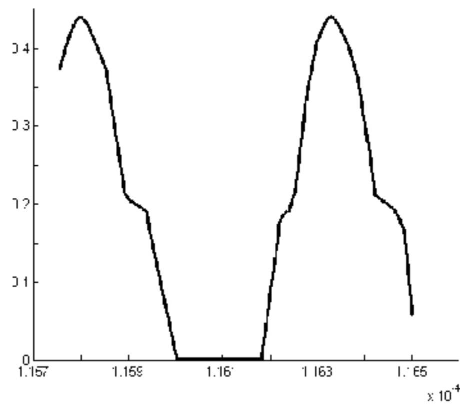

Fig. 4 – The noise modulating function (NMF) for M_2 transistor.

The result of simulation to determine the impulse sensitivity function (ISF) is shown in Fig. 4. This is done by applying small impulse between the drain-source of M_2. It is shown that when the output voltage reaches a maximum, the impulse sensitivity function (ISF) reaches a minimum.

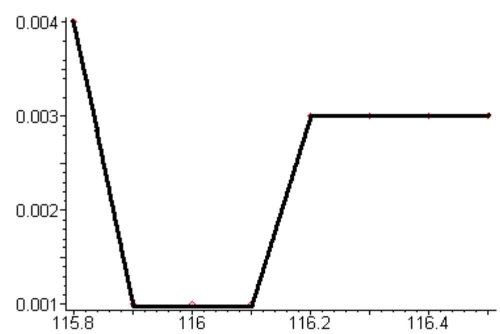

Fig. 5 – The noise impulse sensitivity function (ISF).

5. Conclusions

A simple noise modulating function for MOSFET thermal noise is given and a new *LC* oscillator with its functionality similar to that of a simple CMOS Colpitts oscillator were proposed. The noise modulating function is used to analyse the phase noise of the proposed structure within the time-varying impulse sensitivity function (ISF) theory.

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FUNCŢII DE MODULAŢIE A ZGOMOTULUI TERMIC CARACTERISTIC TRANZISTORULUI MOS ÎNTR-UN OSCILATOR *LC*

(Rezumat)

Se prezintă o serie de expresii analitice pentru funcţiile de modulaţie a zgomotului termic într-un oscilator *LC*. Aceste funcţii sunt utilizate pentru a analiza zgomotul de fază în cadrul teoriei privind funcţiile de senzitivitate la impuls. Este analizat zgomotul de fază datorat zgomotului termic. Rezultatele simulărilor confirmă teoria elaborată.