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IMPROVED NYQUIST PULSES PERFORMANCE INVESTIGATION

BY

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Abstract. A new family of improved Nyquist pulses using concave–convex frequency responses is introduced in this paper. The use of these pulses in intersymbol interference (ISI) free data communications allows the diminution of error probability in the presence of symbol timing error comparing to the reference raised-cosine pulse. These pulses are suitable for root and truncation operations as well.

Key words: intersymbol interference; Nyquist pulse; symbol error rate (SER).

1. Introduction

Considering a baseband digital transmission system composed of transmitter, channel, and receiver, where the transmitter input signal is given by

$$s_{\text{in}}(t) = \sum_n a_n h\left(t - nT_s\right) = \left[\sum_n a_n \delta\left(t - nT_s\right)\right] * h\left(t\right), \tag{1}$$

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where: $h(t)$ denotes the symbol signaling pulse, a_n – constants, $\delta(t)$ – the Dirac delta function, and * – the convolution operator, then the receiver output signal is given by

$$s_{\text{out}}(t) = \left[\sum_n a_n \delta\left(t - nT_s\right) \right] * h_e(t) . \tag{2}$$

The system equivalent impulse response given by

$$h_e(t) = h(t) * h_T(t) * h_C(t) * h_R(t). \tag{3}$$

is the convolution between the symbol signaling pulse, the transmitter impulse response, the channel impulse response, and the receiver impulse response, respectively.

Nyquist's method (Nyquist, 1928) for elimination of ISI consists is using a system equivalent transfer function such that the impulse response satisfies the condition given by

$$h_e\left(kT_s + t\tau\right) = \begin{cases} C, & k = 0, \\ 0, & k \neq 0, \end{cases} \tag{4}$$

where: k is an integer, T_s – the symbol (sample) clocking period, τ – the offset in the receiver sampling clock times when compared with the clocking times of the input symbols, and C – a nonzero constant.

The most commonly used Nyquist (ISI free) pulse is the raised-cosine (RC) pulse, whose frequency response (for positive frequencies) is given by

$$S_{\text{RC}}(f) = \begin{cases} 1, & 0 \le f \le B\left(1-\alpha\right), \\ \frac{1}{2}\left(1 + \cos\left\{\frac{\pi}{2B\alpha}\left[f - B\left(1-\alpha\right)\right]\right\}\right), & B\left(1-\alpha\right) < f \le B\left(1+\alpha\right), \\ 0, & B\left(1+\alpha\right) < f, \end{cases} \tag{5}$$

and impulse response is given by

$$p_{\text{RC}}(t) = \left(1/T\right)\sin c\left(t/T\right)\frac{\cos(2\pi\alpha\, t/T)}{1 - 4\alpha^2 t^2/T^2}, \tag{6}$$

where: α is the roll-off factor (excess bandwidth), B=1/(2T) – the Nyquist frequency, and T – the transmission symbol period.

The frequency response of all Nyquist pulses has a global even symmetry with respect to 0 Hz (yielding a real impulse response), and local odd symmetry with respect to Nyquist frequency B, [Hz], in [B(1 – α), B(1 + α)] band (and –B, [Hz], in [–B(1 – α), –B(1 + α)] band, respectively).

Using a pulse for data signaling which allows no ISI and low sensitivity to sampling errors is a very important issue in data communications (Proakis, 2000).

The new family of improved Nyquist pulses is denoted as concave–convex-i (CCi), where i is a positive integer. The frequency responses are given (for positive frequencies) by

$$\underset{i\text{-even}}{S_{\mathrm{CC}i}(f)} = \begin{cases} 1, & 0 \le f \le B(1-\alpha), \\ \dfrac{(f-B)^i}{2(B\alpha)^i} + \dfrac{1}{2}, & B(1-\alpha) < f \le B, \\ \dfrac{-(f-B)^i}{2(B\alpha)^i} + \dfrac{1}{2}, & B < f \le B(1+\alpha), \\ 0, & B(1+\alpha) < f; \end{cases} \tag{7}$$

$$\underset{i\text{-odd}}{S_{\mathrm{CC}i}(f)} = \begin{cases} 1, & 0 \le f \le B(1-\alpha), \\ \dfrac{(f-B)^i}{2(-B\alpha)^i} + \dfrac{1}{2}, & B(1-\alpha) < f \le B(1+\alpha), \\ 0, & B(1+\alpha) < f. \end{cases} \tag{8}$$

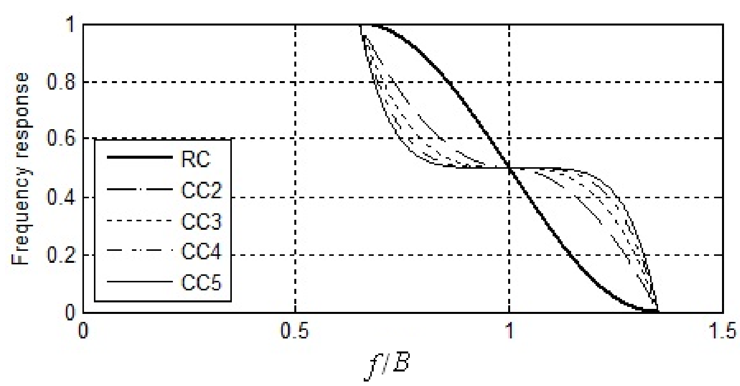

Fig. 1 – Frequency responses of RC and CCi, ($i \in \{2, 3, 4, 5\}$), pulses for α = 0.35.

The plot of these frequency responses is generated with MATLAB and it is shown in Fig. 1.

The scaled (T = 1) impulse responses of the new CCi pulses are numerically evaluated using MATLAB and the plots of these are shown in Fig. 2.

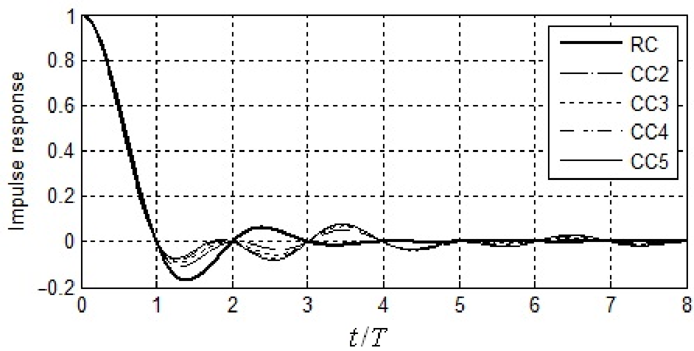

Fig. 2 – Scaled impulse responses of RC and CCi, ($i \in \{2, 3, 4, 5\}$), pulses for α=0.35.

2. Transmission Error Probability

SER is numerically evaluated (Beaulieu, 1991) using MATLAB for various roll-off factors and sampling errors (t/T). The results are presented in Tables 1,…, 3 and the smallest SERs are highlighted.

Table 1

SER for N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR = 15 dB, and α = 0.25

α	Pulse	$t/T=\pm0.05$	$t/T=\pm0.1$	$t/T=\pm0.2$	$t/T=\pm0.3$
0.25	RC	8.2186e-8	2.8182e-6	9.7454e-4	0.0259
	CC2	5.2825e-8	1.0773e-6	2.8253e-4	0.0122
	CC3	5.0130e-8	9.9554e-7	2.6223e-4	0.0113
	CC4	4.9055e-8	9.7675e-7	**2.6216e-4**	0.0111
	CC5	**4.8506e-8**	**9.7333e-7**	2.6575e-4	**0.0110**

Table 2

SER for N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR = 15 dB, and α = 0.35

α	Pulse	$t/T=\pm0.05$	$t/T=\pm0.1$	$t/T=\pm0.2$	$t/T=\pm0.3$
0.35	RC	5.9996e-8	1.3896e-6	3.9083e-4	0.0155
	CC2	3.5509e-8	4.5827e-7	**8.6036e-5**	**0.0050**
	CC3	3.3987e-8	**4.4712e-7**	9.1263e-5	0.0052
	CC4	3.3632e-8	4.6013e-7	1.0142e-4	0.0056
	CC5	**3.3599e-8**	4.7623e-7	1.1127e-4	0.0060

Table 3
SER for N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR = 15 dB, and α = 0.5

α	Pulse	$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$	$t/T = \pm 0.3$
0.5	RC	3.9723e-8	5.4890e-7	1.0217e-4	0.0060
	CC2	2.2050e-8	**1.6608e-7**	**2.2149e-5**	**0.0018**
	CC3	**2.1685e-8**	1.7156e-7	2.9173e-5	0.0026
	CC4	2.2031e-8	1.8417e-7	3.7671e-5	0.0035
	CC5	2.2517e-8	1.9755e-7	4.6290e-5	0.0043

According to Tables 1,…, 3, the new CC*i* pulses allow for better SER comparing to standard RC pulse.

A common practice is to deploy a root (of frequency response) pulse at the transmitter side, and the same root pulse at the receiver side. The filters are digitally implemented with a truncated oversampled causal version.

SER is numerically evaluated using MATLAB for the root-pulses impulse responses truncated in [–5.5*T*, 5.5*T*] time interval, various roll-off factors and sampling errors. The results are presented in Tables 4,…, 6 and the smallest SERs are highlighted.

Table 4
SER for the Truncated Root-Pulses, N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR = 15 dB, and α = 0.25

α	Pulse	$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$	$t/T = \pm 0.3$
0.25	RC	8.5445e-8	2.9226e-6	9.9919e-4	0.0262
	CC2	7.5141e-8	1.3784e-6	3.0852e-4	0.0125
	CC3	**5.8862e-8**	1.0535e-6	2.4527e-4	0.0108
	CC4	7.8123e-8	1.3251e-6	2.8587e-4	0.0113
	CC5	5.9881e-8	**1.0423e-6**	**2.4300e-4**	**0.0104**

Table 5
SER for the Truncated Root-Pulses, N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR=15 dB, and α=0.35

α	Pulse	$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$	$t/T = \pm 0.3$
0.35	RC	6.0729e-8	1.4082e-6	3.9555e-4	0.0156
	CC2	3.9228e-8	5.1529e-7	**9.7343e-5**	**0.00547**
	CC3	**3.7626e-8**	**4.9928e-7**	1.0090e-4	0.00552
	CC4	4.4460e-8	5.6658e-7	1.1050e-4	0.0058
	CC5	3.9786e-8	5.2658e-7	1.1116e-4	0.0059

Table 6

SER for the Truncated Root-Pulses, N = 23 Interfering Symbols, Signal-to-Noise Ratio SNR=15 dB, and α=0.5

α	Pulse	$t/T = \pm 0.05$	$t/T = \pm 0.1$	$t/T = \pm 0.2$	$t/T = \pm 0.3$
0.5	RC	3.9943e-8	5.5179e-7	1.0257e-4	0.0060
	CC2	2.2905e-8	**1.7790e-7**	**2.4512e-5**	**0.0019**
	CC3	**2.2666e-8**	1.8620e-7	3.2531e-5	0.0028
	CC4	2.4773e-8	2.1387e-7	4.3977e-5	0.0038
	CC5	2.4369e-8	2.2135e-7	5.1982e-5	0.0046

According to Tables 4,…, 6, the truncated root CC*i* pulses allow for better SER comparing to standard RC pulse.

3. Conclusions

In this paper has been introduced and evaluated a new family of improved Nyquist filters which allow a smaller SER in the presence of symbol timing error comparing to the RC pulse with the same excess bandwidth. This family of pulses is also suitable for root and truncation operations, allowing better SER than the RC pulse.

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INVESTIGAREA PERFORMANŢELOR UNOR FILTRE NYQUIST ÎMBUNĂTĂȚITE

(Rezumat)

Se prezintă o nouă familie de pulsuri Nyquist îmbunătățite, ce utilizează răspunsuri în frecvență de tip concav–convex. Utilizarea acestor pulsuri în comunicațiile de date fără interferență intersimbol permite diminuarea probabilității de eroare în prezența erorii de generare a momentelor de eşantionare a simbolurilor, comparativ cu pulsul de referință “cosinus ridicat”. Aceste pulsuri sunt adecvate şi operațiilor de extragere a rădăcinii pătrate şi de trunchiere.