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# CURRENT TRANSFER COEFFICIENT OF A LINEAR TWO-PORT WITH NONLINEAR INERTIAL RECEIVER

BY

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**Abstract.** The differential equation satisfied by the function $X_2(R_2)$ is established, where $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ represents the complex impedance of a nonlinear, inertial and passive receiver of a linear and non-autonomous, two-port, in harmonic steady-state, so that the modulus of the current transfer coefficient of the two-port have an extreme value. This equation is nonlinear, of first order, with separate variables.

**Key words:** linear and non-autonomous two-port; nonlinear, inertial and passive receiver; current transfer coefficients.

## 1. Introduction

In a previous paper (Rosman, 2006) were studied some problems concerning the *voltage transfer coefficient* of a linear, non-autonomous two-port having a nonlinear inertial and passive receiver. The aim of this paper is to study some analogous problems concerning the *current transfer coefficient* in the same considered case.

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Let be a linear and non-autonomous two-port (LNATP) supplied with a harmonic signal, whose receiver is nonlinear inertial and passive (NIPR) (Fig. 1). The eqs. of such an LNRTP are

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = [\underline{A}] \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}, \tag{1}$$

where

$$[\underline{A}] = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \tag{2}$$

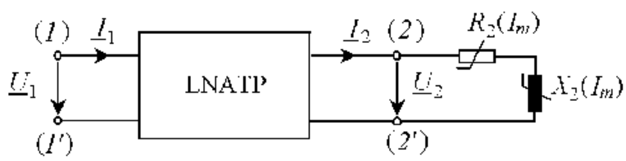


Fig. 1

is the fundamental parameters matrix of the LNATP. The complex impedance of the NIPR is

$$\underline{Z}_2(I_m) = R_2(I_m) + \mathrm{j}X_2(I_m)\,, \tag{3}$$

where it was admitted the hypothesis that for any structure of this one his equivalent resistance, $R_2(I_m)$, as well his equivalent reactance, $X_2(I_m)$, are functions of the amplitude, $I_m$ , of an harmonic arbitrary current, $I_m$ , called *reference current* (Rosman, 2005). It was considered the property of inertial nonlinear elements to have the response – excitation characteristic linear in instantaneous values and nonlinear in RMS ones (Philippow, 1971), so that the circuit's analysis, represented in Fig. 1, may be performed utilizing the symbolic method of complex values.

The current transfer coefficient being defined by the ratio

$$\underline{k}_I = \frac{\underline{I}_2}{\underline{I}_1} \tag{4}$$

and having in view that the receiver's complex impedance is

$$\underline{Z}_2 = \frac{\underline{U}_2}{\underline{I}_2} \tag{5}$$

considering at the same time the eqs. (1) of the LNATP, expression (4) can be written

$$\underline{k}_I = \frac{1}{\underline{A}_{21}\underline{Z}_2\left(I_m\right)+\underline{A}_{22}}. \tag{6}$$

The aim of this paper is to determine the function $X_2(R_2)$ which corresponds to extreme values of $k_I$ , that is to determine the geometric-locus diagram of the receiver's complex impedance, $\underline{Z}_2(I_m)$, corresponding to extreme values of $k_I$, provided that these extreme values exist.

## 2. The Differential Equation Satisfied by the Function $X_2(R_2)$

Having in view relation (3), expression (6) becomes

$$\underline{k}_I\left(I_m\right) = \frac{1}{\underline{A}_{21}\left[R_2\left(I_m\right)+\mathrm{j}X_2\left(I_m\right)\right]+\underline{A}_{22}}, \tag{7}$$

whose modulus is

$$k_I(I_m) = \frac{1}{\sqrt{A_{21}^2\left[R_2^2(I_m)+X_2^2(I_m)\right]+2\Re e\left(\underline{A}_{21}\underline{A}_{12}^*\right)R_2(I_m)-2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)X_2(I_m)+A_{22}^2}}. \tag{8}$$

The study of function $k_I(I_m)$ implies the necessity to perform the derivative

$$\begin{aligned}\frac{\mathrm{d}k_I(I_m)}{\mathrm{d}I_m} = &-\Big\{A_{21}^2\left[R_2^2(I_m)+X_2^2(I_m)\right]+2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)R_2(I_m)- \\ &-2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)X_2(I_m)+A_{22}^2\Big\}^{-3/2}\Big\{\left[A_{21}^2R_2(I_m)+\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]\frac{\mathrm{d}R_2}{\mathrm{d}I_m}+ \\ &+\left[A_{21}^2X_2(I_m)-\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]\frac{\mathrm{d}R_2}{\mathrm{d}I_m}\Big\}.\end{aligned} \tag{9}$$

If the derivative (9) is annulled then the differential eq.

$$\frac{\mathrm{d}X_2(I_m)}{\mathrm{d}R_2(I_m)} = \frac{A_{21}^2R_2(I_m)+\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{-A_{21}^2X_2(I_m)+\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)} \tag{10}$$

results. Using the notations

$$X_2(I_m)=y,\ R_2(I_m)=x,\ \Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)=d,\ \Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)=e, A_{21}^2=f \tag{11}$$

the differential eq. (10) becomes

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{fx+d}{-fy+e}. \tag{12}$$

### 3. Solution of Differential Equation (10)

The differential eq. (12) is nonlinear, of first order, with separate variables (Corduneanu, 1981), whose solution is

$$\frac{f}{2}\left(x^2+y^2\right)+ex-dy=C, \tag{13}$$

where $C$ is a constant of integration. Having in view notations (11) the solution (13) may be written

$$x^2+y^2+\frac{2\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}x-\frac{2\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}y=\frac{C}{A_{21}^2}, \tag{14}$$

which represents the eq. of a family of concentric circles having the center in $\left(\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)/A_{21}^2,\ \Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)/A_{21}^2\right)$ and the radius $\sqrt{2C-A_{22}^2}/A_{21}$. Taking into account that

$$\frac{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}=R_{e20}>0,\quad \frac{\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)}{A_{21}^2}=X_{e20}\lesseqgtr 0 \tag{15}$$

eq. (14) becomes

$$\left(x+X_{e20}\right)^2+\left(y-R_{e20}^2\right)=\frac{2C-A_{22}^2}{A_{21}^2}. \tag{16}$$

It may be noticed that as regards the constant of integration, $C$, this one must to satisfy the inequality

$$C\geq\frac{A_{22}^2}{2}>0. \tag{17}$$

The circle's (16) center is situated in semi-plane $R_2 \geq 0$ if $X_{e20} < 0$ or in semi-plane $R_2 \leq 0$ in opposite case. In Fig. 2 is represented this circle when $X_{e20} < 0$; the mentioned circle intersects the ordinates axis in two points having the ordinates

$$\begin{cases} X_{21} = \dfrac{1}{A_{21}^2}\left\{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right) + \sqrt{\left[\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]^2 + 2C}\right\} > 0, \\ X_{22} = \dfrac{1}{A_{21}^2}\left\{\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right) - \sqrt{\left[\Re e\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]^2 + 2C}\right\} < 0. \end{cases} \tag{18}$$

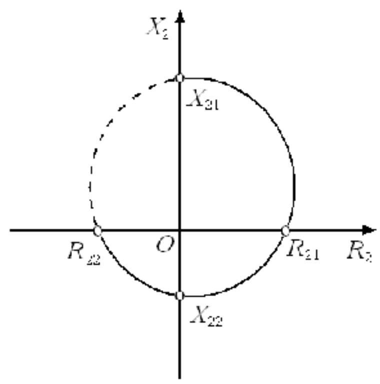


Fig. 2

In the same time the circle (14) intersects the abscissa axis in two points having the abscissae

$$\begin{cases} R_{21} = \dfrac{1}{A_{21}^2}\left\{-\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right) + \sqrt{\left[\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]^2 + 2C}\right\} > 0, \\ R_{22} = \dfrac{1}{A_{21}^2}\left\{-\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right) - \sqrt{\left[\Im m\left(\underline{A}_{21}\underline{A}_{22}^*\right)\right]^2 + 2C}\right\} < 0. \end{cases} \tag{19}$$

The circles (14) are situated in the semi-plane $R_2 \geq 0$ and may be considered as representing the geometric diagram-locus of the complex impedance $\underline{Z}_2(I_m) = R_2(I_m) + \mathrm{j}X_2(I_m)$ for different extreme values of the current transfer coefficient modulus, $k_I$ , when the RMS value, $U_1$, of the LNATP's primary voltage (and consequently of amplitude, $I_m$ , of the reference current) varies. Evidently these extreme values may represent maximum or minimum ones, as the sign of the second derivative, $\mathrm{d}^2 k_I(I_m)/\mathrm{d}I_m^2$ is negative, respectively positive for the values of $k_I$ which annule the first derivative, $\mathrm{d}k_I(I_m)/\mathrm{d}I_m$ .

## 4. Conclusions

1. The differential eq. satisfied by the function $X_2(R_2)$ is established, where $R_2(I_m)$, $X_2(I_m)$ represent the equivalent parameters of a nonlinear inertial and passive receiver of a linear and non-autonomous two-ports, in harmonic steady-state, when the current transfer coefficient modulus of the considered two-ports has an extreme value. It is supposed that $R_2$ and $X_2$ are functions of the amplitude, $I_m$ , of a same, arbitrary, harmonic current.

2. The established differential eq. is nonlinear, of first order, with separate variables; their solution represents the equation of a concentric circles family in plane $(R_2, X_2)$. These circles are situated in the semi-plane $R_2 \geq 0$ and represent the geometric diagram-locus of the complex impedance $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ for different values of the current transfer coefficient modulus, when the primary voltage varies.

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## COEFICIENTUL DE TRANSFER AL CURENTULUI AL UNUI CUADRIPOL LINEAR AVÂND UN RECEPTOR NELINIAR INERȚIAL

(Rezumat)

Se stabileşte ecuația diferențială satisfăcută de funcția $X_2(R_2)$, unde $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ reprezintă impedanța complexă a unui receptor neliniar inerțial pasiv a unui cuadripol liniar neautonom în regim permanent armonic, astfel încât modulul coeficientului de transfer a curentului să aibe o valoare extremă. Această ecuație diferențială este neliniară, de ordinal întâi, cu variabile separate. Soluția ei este o familie de cercuri concentrice situate în planul $(R_2, X_2)$; arcul unui astfel de cerc situat în semiplanul $R_2 \geq 0$ constituie diagrama-loc geometric a impedanței complexe $\underline{Z}_2 = R_2 + \mathrm{j}X_2$ pentru diferite valori ale modulului coeficientului de transfer al curentului, atunci când valoarea efectivă, $U_1$ , a tensiunii primare variază.