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## AN APPROACH REGARDING THE PLACEMENT OF DISTRIBUTED GENERATION SOURCES IN ELECTRIC DISTRIBUTION SYSTEMS USING HURWITZ CRITERION

BY

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**Abstract.** Over the past several decades, the electric power system in most countries, around the world, has been undergoing a fundamental transformation due to deregulation. This transformation has been accompanied by an increase in the number of market participants and changes to the electricity flow patterns due to new distributed generation sources (DG) installed in electrical networks. In this sense, this paper develops a decision making methodology, that uses Hurwitz criterion, taking into account the time-variable generation and load, to optimal sizing of given types of DG sources placed in the nodes of a distribution network, to minimizing the power losses and improving the voltage profile. The validity of the method is observed through tests into a 20 kV electric energy distribution network.

**Key words:** DG sources; decision making; Hurwitz criterion; power losses; voltage levels.

### 1. Introduction

The development of competitive electric markets has introduced significant uncertainties in transmission and distribution expansion planning.

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Since methods of modeling random uncertainties (decision making strategies with *prior* probability distributions), non-random uncertainties (decision making without *prior* probability distributions), and vagueness (decision making strategies with *posterior* probability distributions) are different, the power system uncertainties and vagueness must be identified and classified clearly before planning, (Ravi Ravindran, 2009).

The uncertainty can be the result from a prediction or can be caused by the difficulty to measure a certain parameter. Typically, this is translated in probability distributions with a considerable variance. Several sources of uncertainty can be considered in the electrical networks: the load profile over the next hour, over the whole year, the load distribution among buses and generation dispatch, system and equipment parameter values, equipment outage rates (lines, generators and transformers), fault types and locations, ambient conditions.

A fundamental decision making tool for power system engineers is the optimal power flow control, in order to optimize an objective function while satisfying a set of nonlinear equality and inequality constraints (Wisniewski, 2006).

Regarding the decision making process, there are a lot of criteria in the literature that can be used for the planning of any system. These criteria can be successfully used in different cases, like transmission expansion planning (Ravi Ravindran, 2009), in the security assessment analysis (Iowa State University, 2012), choosing of the best alternative in the planning DG sources in electrical distribution networks, etc.

Generally, the decision-making strategies can be classified into strategies without *prior* probability distributions, strategies with *prior* probability distributions, and strategies with *posterior* probability distributions (Ionescu *et al.*, 1999). The first type of decision-making is characterized by completely ignoring any probabilistic nature of the phenomena/processes. The decision making with a *prior* probability distribution is characterized by the decision maker having either partial or complete knowledge of the probability distribution on the state of nature. This type of decision making under uncertainty, can be viewed as including a probability distribution to obtain the maximum expected value of gain. The last decision making is characterized by the possibility of obtaining additional information or data before a decision is rendered. The decision is then made between the available actions by finding the maximum expected value for each action, with the *posterior* probabilities, the revised *prior* probability.

In this paper, a new approach is presented to reduce the losses and to improve the voltage profile into a distribution network by means of appropriate DG planning. This is based on a strategy without *prior* probability distributions based on the Hurwitz criterion.

## 2. Method

### 2.1. Objective Function

The main goal of this paper is to develop a decision making methodology based on the Hurwitz criterion, taking into account the time-variable generation and load, to optimal sizing of given types of DG sources placed in the nodes of a distribution network, minimizing the power losses and improving the voltage profile.

Let us consider two typical objectives for distribution systems:

1. Energy losses

$$F_{\text{Loss}}(\mathbf{x}) = \sum_{h=1}^H \sum_{b=1}^B \Delta P_h^{(b)}(\mathbf{x}) \Delta t_h, \quad (1)$$

where:  $\Delta P_h^{(b)}$  is the active power losses in branch  $b = 1, \dots, B$  at time interval  $h = 1, \dots, H$  and  $\Delta t_h$  represent the time intervals duration for  $h = 1, \dots, H$ .

2. Voltage profile

$$F_{\text{Voltage}}(\mathbf{x}) = \sum_{h=1}^H \sum_{i=2}^N \left| U_h^{(i)}(\mathbf{x}) - U^{(r)} \right|, \quad (2)$$

where:  $U_h^{(i)}$  is the voltage magnitude at node  $i = 2, \dots, N$  (excluding the slack node) and  $U^{(r)}$  hour  $h = 1, \dots, H$ , and is the rated voltage of the system.

The column vector,  $\mathbf{x}$ , whose length is equal to the number of nodes,  $N$ , include the data referring to the power capacity of the local generators placed in specified positions.

These two objectives typically are of non-conflicting nature, thus, the multi-objective problem can be easily transformed into a single objective problem. In this case, the optimization problem minimizes the objective function,  $C(\mathbf{x})$ , with the following formulation:

$$\min \left\{ C(\mathbf{x}) = \alpha \frac{F_{\text{Loss}}(\mathbf{x})}{F_{\text{Loss}}(\mathbf{x}^{(\text{base})})} + (1 - \alpha) \frac{F_{\text{Voltage}}(\mathbf{x})}{F_{\text{Voltage}}(\mathbf{x}^{(\text{base})})} \right\}, \quad (3)$$

where the introduction of the parameter,  $\alpha$ , makes it possible to give a prevailing role to the term referring to the energy losses (for  $\alpha$  tending to unity) or to the one referring to the voltage profile (for  $\alpha$  tending to zero).  $F_{\text{Loss}}(\mathbf{x}^{(\text{base})})$  is the power losses value in the base case, without DG and  $F_{\text{Voltage}}(\mathbf{x}^{(\text{base})})$  is the nominal voltage value of the analysed network). Corresponding to Hurwitz criterion the parameter  $\alpha$  is called *index of optimism* and  $1 - \alpha$ , *index of pessimism*.

## 2.2. Constraints

The equality constraints, given by the classical power flow eqs. and the inequality constraints, are of different types: voltage magnitude limits, branch thermal limits, generation limits for DG sources and constraints for reactive power, (Rotaru *et al.*, 2012; Ramalingaiah *et al.*, 2009).

## 2.3. Hurwitz Criterion

This criterion is developed to mitigate the extremes and to allow for a range of attitudes of the decision maker. The Hurwitz criterion is designed to model a range of decisions making attitudes from the most conservative to the most optimistic. Hurwitz suggests examining some weighted combination of the maximum and minimum gain and then taking the action which has the most desirable weighted value. The relationship that forms the basis on this criterion is

$$\left\{ C(\mathbf{x}) = \alpha \min F_{\text{Loss}}(\mathbf{x}^{(\text{base})}) + (1 - \alpha) \max F_{\text{Voltage}}(\mathbf{x}^{(\text{base})}) \right\}. \quad (4)$$

This approach is based on an index of optimism given by  $\alpha$  and an index of pessimism,  $1 - \alpha$ , that are ranged between zero and one (Wisniewski, 2006). If  $\alpha = 0$ , this criterion is simply the minimax criterion (*i.e.* the minimum gain is maximized, a conservative criterion). If  $\alpha = 1$ , then the criterion seeks the maximum possible payoff (an optimistic criterion). A typical value for  $\alpha$  is 0.5.

## 3. Decision Making Methodology

The decision making methodology based on the Hurwitz criterion, to optimal sizing of different types of DG sources placed in the nodes of a distribution network, to minimize the power losses and to improve the voltage profile is presented below namely

a) Regime calculations – calculate the real and reactive power loads, the real and reactive power losses in nodes and branch and the variations of voltage value in nodes.

b) Determine the loss sensitivity factors (LSFs) in nodes of electrical system analysed (Rotaru *et al.*, 2012) and normalize LSFs and voltages values (El-Khattam *et al.*, 2004).

c) Use the clustering methods for grouping the nodes from the viewpoint of the operation characteristics normalized LSFs and voltages values, (Grigoraş *et al.*, 2010; Grigoraş *et al.*, 2009; Cârţină *et al.*, 2005).

d) Select the pilot node where DG sources can be located for each cluster resulted in the clustering process.

e) Use the exhaustive search to find the optimal size of DG sources that were installed in every pilot node of clusters and to analyse the evolution of the objective function for the distribution network analysed.

f) Use the Hurwitz criterion to find the optimal case, regarding the optimization problem which minimizes the objective function,  $C(\mathbf{x})$ . Was taking into account the variation of the parameter  $\alpha$  in the range  $[0, 1]$  in order to determine the optimal solution for the planning DG sources into a distribution system.

g) Select the best combination of the objective function.

In decision analysis, planners try to find the most flexible plan that satisfies the optimization problem requirements and also the constraints. The procedure of finding the optimal decision over the entire planning period is a classical stochastic dynamic programming.

#### 4. Study Case

The methodology proposed was tested on a 20 kV distribution network with 24 nodes. The schematic diagram for the test system is represented in Fig. 1. Fig. 2 shows the hourly evolution of the real power in the load nodes in the initial condition (without DG).

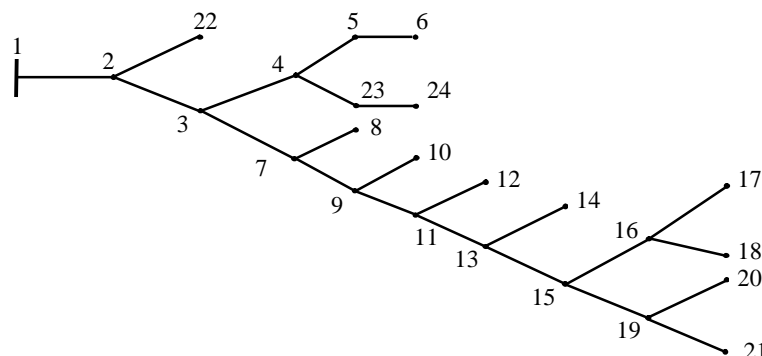


Fig. 1 – The schematic diagram for the test network.

To determine the normalized values of the loss sensitivity factors (LSFs) and voltages in the network nodes was used the expressions established by Scarlatache *et al.* (2012). Using these normalized parameters in the clustering techniques was selected the pilot node where DG sources can be located (Grigoraș *et al.*, 2009). After that, based on the exhaustive search, was find the optimal size of DG sources that were installed in every pilot node of clusters and was analysed the evolution of the objective function.

The best solution is the combination with 40 kW injected in node 4 (PV), 100 kW in node 7 (PV), 150 kW in node 9 (PV), SH of 100 kW in node 11 and 400 kW injected in node 13 (SH).

The optimal case of the objective function,  $C(\mathbf{x})$ , was obtained using the Hurwitz criterion. We considered a variation of the parameter  $\alpha$  in the range  $[0, 1]$  in order to determine the optimal solution for the planning DG sources in the distribution system analysed (Table 1).

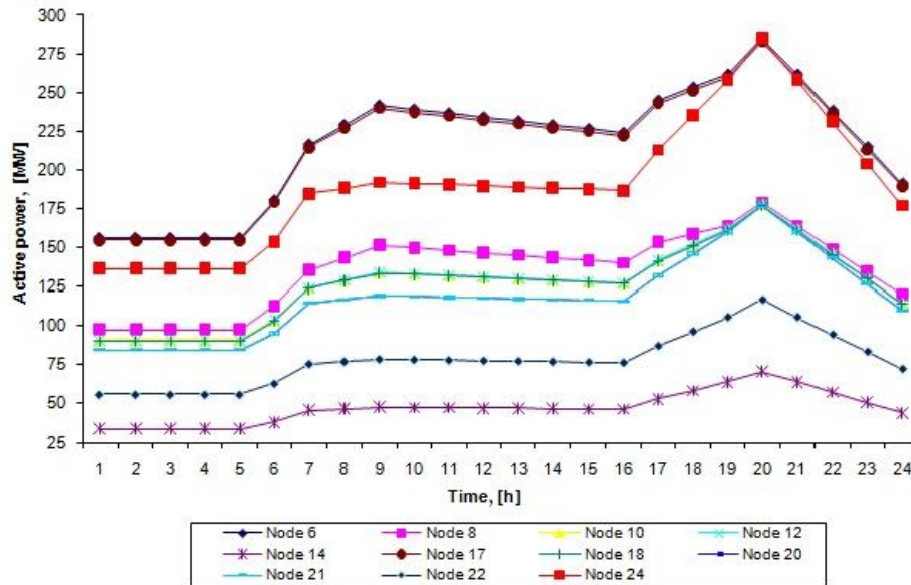


Fig. 2 – The active power profiles in the nodes of network; the DG types considered in the area are combined heat and power (CHP), photovoltaic systems (PV), and small hydro plants (SH).

**Table 1**  
The Objective Function,  $C(\mathbf{x})$ , Evolution in the Variation Process of the Parameter  $\alpha$

$\alpha$	Iterations no.	$\alpha F_{\text{Loss}}(\mathbf{x})$	$(1 - \alpha)F_{\text{Voltage}}(\mathbf{x})$	$C(\mathbf{x})$
0.1	14	0.045377	0.886172	0.931549
0.2	14	0.090754	0.787708	0.878462
0.3	14	0.136131	0.689245	0.825376
0.4	14	0.204196	0.590781	0.794978
0.5	14	0.226885	0.492318	0.719202
0.6	14	0.272262	0.393854	0.666116
0.7	14	0.317639	0.295391	0.613029
0.8	14	0.363016	0.196927	0.559943
0.9	14	0.408393	0.098464	0.506856

For determining the optimal value of the coefficient  $\alpha$  of the objective function,  $C(\mathbf{x})$ , it was represented the variations of the functions:  $\alpha F_{\text{Loss}}(\mathbf{x})$  – variation of power loss and  $(1 - \alpha)F_{\text{Voltage}}(\mathbf{x})$  – voltage variation in network nodes analysed (Fig. 3). The intersection point of this two functions was considered the optimum value of the coefficient  $\alpha$ , ( $\alpha = 0.7$ ) (Fig. 4).

Analysing the objective function,  $C(\mathbf{x})$ , evolution, following conclusions can be highlighted:

1. When the coefficient  $\alpha$  tends to a minimum value ( $\alpha \approx 0$ ), the loss is

minimization function,  $F_{\text{Loss}}(\mathbf{x})$ , has an insignificant contribution and implicitly a coefficient of optimism,  $\alpha$ , insignificant. The voltage maximization function,  $F_{\text{Voltage}}(\mathbf{x})$ , has a special importance in evaluation of the objective function,  $C(\mathbf{x})$ ; the coefficient of pessimism,  $(1 - \alpha)$ , dominates the objective function,  $C(\mathbf{x})$ .

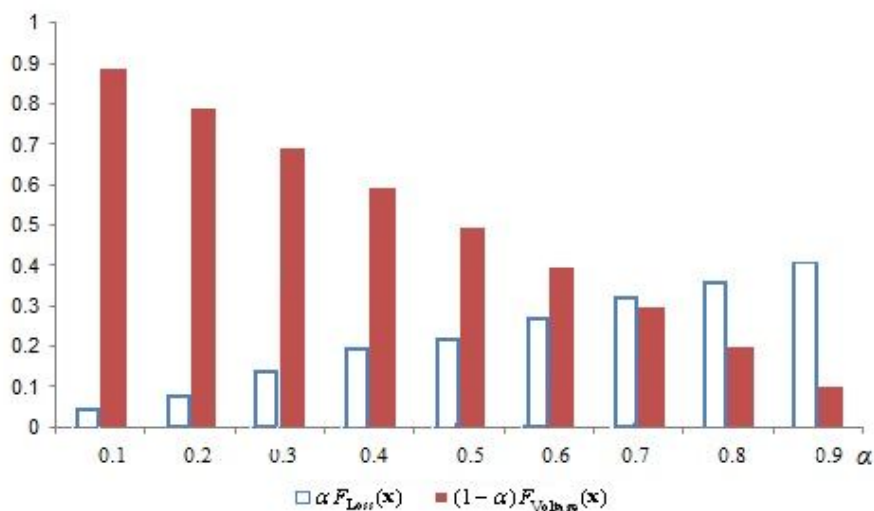


Fig. 3 – Variations of the  $\alpha F_{\text{Loss}}(\mathbf{x})$  and  $(1 - \alpha) F_{\text{Voltage}}(\mathbf{x})$  functions.

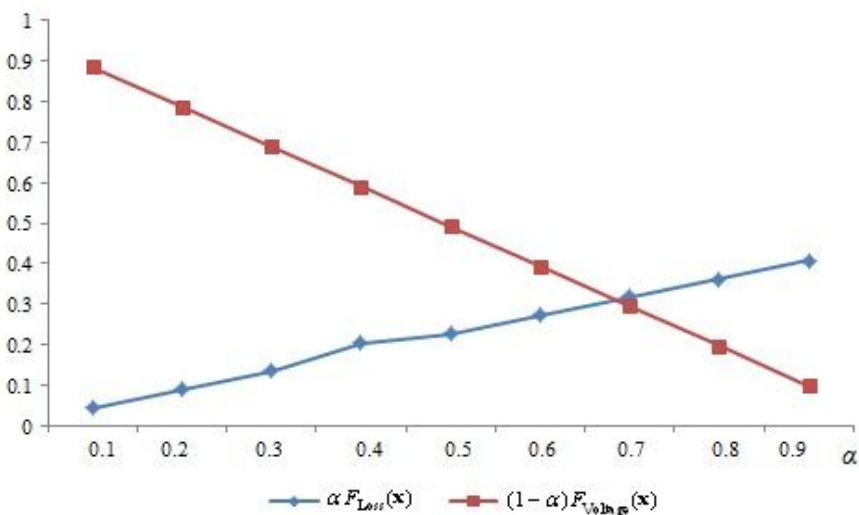


Fig. 4 – Variations of the  $\alpha F_{\text{Loss}}(\mathbf{x})$  and  $(1 - \alpha) F_{\text{Voltage}}(\mathbf{x})$  functions (intersection point ( $\alpha = 0.7$ )).

2. When  $\alpha$  tends to a maximum value ( $\alpha \approx 1$ ), the loss minimization function,  $F_{\text{Loss}}(\mathbf{x})$ , dominates the objective function,  $C(\mathbf{x})$ , and the coefficient of

optimism has a significantly. The voltage maximization function,  $F_{\text{voltage}}(\mathbf{x})$ , is less significant in the objective function,  $C(\mathbf{x})$ , evolution.

3. If  $\alpha = 0$  then the objective function,  $C(\mathbf{x})$ , is transformed into a voltage maximization function,  $F_{\text{voltage}}(\mathbf{x})$ , (total pessimism). If  $\alpha = 1$  then the objective function,  $C(\mathbf{x})$ , is transformed into a loss minimization function,  $F_{\text{Loss}}(\mathbf{x})$ , (total optimism).

4. In the optimal case when the functions intersects (in our case  $\alpha = 0.7$ ) the loss minimizing functions and the voltage maximizing functions have approximately equal values, and a coefficient of optimism,  $\alpha$ , higher than that of the coefficient of pessimism,  $(1 - \alpha)$ , in the evolution of the objective function,  $C(\mathbf{x})$ .

## 5. Conclusions

In this paper a new approach is presented to reduce the power losses and improving the voltage profile by using a decision making methodology, in the context of the optimal sizing of DG sources placed in the nodes of a distribution network. The method is based on the Hurwitz criterion for determining the optimal case of the objective function,  $C(\mathbf{x})$ .

The obtained results show the evident benefits of using the proposed method in terms of reducing the computational burden to obtain an optimal solution. In the same time, the decision analysis process become most flexible and for the planners is more easily to find the plan that satisfies the optimization problem requirements and also the constraints.

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O ABORDARE PRIVIND AMPLASAREA SURSELOR DE GENERARE  
DISTRIBUITĂ ÎN SISTEMELE ELECTRICE DE DISTRIBUȚIE FOLOSIND  
CRITERIUL HURWITZ

(Rezumat)

Se dezvoltă o nouă metodologie de luare a deciziilor bazată pe criteriul Hurwitz, în scopul reducerii pierderilor de putere și îmbunătățirii profilului de tensiune într-o rețea de distribuție prin intermediul unei planificări adecvate a amplasării surselor de generare distribuită (DG). Analiza se efectuează ținând cont de faptul că, în prealabil, a fost realizată o dimensionare optimă a surselor de DG amplasate în nodurile rețelei. Metoda a fost testată pe o rețea de 20 kV reală, iar rezultatele au demonstrat validitatea metodologiei propuse.

