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PSEUDO-QUANTITIES IN ELECTRIC NETWORKS AND IN ELECTROMAGNETIC FIELDS

BY

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Abstract. Tellegen’s theorem permits to define the notions of instantaneous pseudo-powers, complex apparent pseudo-powers, active and reactive pseudo-powers. Generalized Manley-Rowe-Kontorovich formula permits to define a series of pseudo-quantities as: stored pseudo-energy in the electrostatic, respectively magnetostatic field, radiated active and reactive pseudo-powers in an electromagnetic field, the generalized complex apparent pseudo-power and the generalized stored active and reactive pseudo-powers in an electromagnetic field, the volumetric densities of generalized electromagnetic pseudo-impulse, respectively of Lorentz generalized type magnetic pseudo-force in an electromagnetic field.

Key words: pseudo-powers; stored pseudo-energies in an electrostatic, respectively magnetostatic field; volumetric density of generalized electromagnetic pseudo-impulse and of Lorentz type magnetic pseudo-force.

1. Introduction

Tellegen’s theorem, established initially in the case of electric networks (1952), was afterwards extended, by Ţugulea (1986), to the case of an

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electromagnetic field and by the author of this paper (Rosman, 2000) to the case of capacitors or permanent magnets circuits.

As regards Tellegen's theorem, for the case of electric networks, this one concerns two insulated active or passive, linear, non-linear or parametric circuits, (I) and (II), having identical topological oriented graphs. If $u_k^{(\mathrm{I})}(t')$ represents the voltage between the nodes incident to *k*-branch of network (I), at a moment *t′*, and $i_k^{(\mathrm{II})}(t'')$ represents the current which flows through *k*-branch of network (II), at a moment *t″*, where $k = 1,2,...,B$ (*B* – the branches number), the working regime of the two considered networks being quasi-stationary, of anelectric type, it is possible to demonstrate that in this case relation

$$\sum_{k=1}^{B} u_k^{(\mathrm{I})}(t')\, i_k^{(\mathrm{II})}(t'') = 0 \tag{1}$$

is satisfied, which represents Tellegen's theorem (in case of electric networks).

This theorem leads to definition of a series of *pseudo-quantities*, the aim of this paper being to review such quantities and to introduce others pseudo-quantities, non-signaled in the speciality literature. These pseudo-quantities are specific either in electric, electrostatic and magnetostatic networks, or in electromagnetic field.

2. Pseudo-Quantities in Electric Networks

If two electric networks, having the properties indicated in the previous section, are considered, it is evidently that expression

$$p_k^{(\mathrm{I}),(\mathrm{II})}(t',t'') = u_k^{(\mathrm{I})}(t')i_k^{(\mathrm{II})}(t''), \tag{2}$$

represents an *instantaneous pseudo-power*, as was designated by Penfield, Spence and Duinker (1970). In the particular case when $t' = t'' = t$, relation (2) becomes

$$p_k^{(\mathrm{I}),(\mathrm{II})}(t) = u_k^{(\mathrm{I})}(t)i_k^{(\mathrm{II})}(t), \tag{3}$$

having the mean value

$$P_k = \frac{1}{T}\int_0^T p_k^{(\mathrm{I}),(\mathrm{II})}(t)\mathrm{d}t, \tag{4}$$

which may be considered as representing an *active pseudo-power*, if the signals $u_k^{(\mathrm{I})}(t)$, $i_k^{(\mathrm{II})}(t)$ are periodic. Evidently, when this signals are harmonic, using the symbolic representation of such signals by complex quantities, relations

$$\underline{s}_k^{(\mathrm{I}),(\mathrm{II})} = \underline{U}_k^{(\mathrm{I})} \underline{I}_k^{(\mathrm{II})*},\ P_k^{(\mathrm{I}),(\mathrm{II})} = \Re e\left(\underline{U}_k^{(\mathrm{I})} \underline{I}_k^{(\mathrm{II})*}\right),\ Q_k^{(\mathrm{I}),(\mathrm{II})} = \Im m\left(\underline{U}_k^{(\mathrm{I})} \underline{I}_k^{(\mathrm{II})*}\right) \tag{5}$$

may be obtained, where $\underline{U}_k^{(\mathrm{I})}$, $\underline{I}_k^{(\mathrm{II})}$ are the complex RMS values attached to harmonic signals $u_k^{(\mathrm{I})}(t)$, $i_k^{(\mathrm{II})}(t)$. It is natural to consider $\underline{s}_k^{(\mathrm{I}),(\mathrm{II})}$ as representing a *complex apparent pseudo-power*, $P^{(\mathrm{I}),(\mathrm{II})}$ – an *active pseudo-power* and $Q^{(\mathrm{I}),(\mathrm{II})}$ – a *reactive pseudo-power*.

The pseudo-powers may be considered as having a connection not only with Tellegen's theorem but also with Manley-Rowe relations, established in 1956. Thus, in a previous paper (Rosman, 2004) was demonstrated that considering a non-linear electric network, constituted of non-dissipative elements, without hysteresis, in which not periodical process not occur, supplied by two harmonic electromotive voltages having the frequencies f_1, respectively f_2, the voltage, $u(t)$, at the terminals of any network's element and the current, $i(t)$, which flows through the respectively element, are double-periodic functions with respect to time. Developing the signals $u(t)$, $i(t)$ in Fourier complex double series and proceeding as in the cited paper, the relations

$$\begin{cases} \displaystyle\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\psi(p,q)\Re e\left(\underline{U}_{p+\mu,q+\nu} \underline{I}^*_{p+\sigma,q+\tau}\right)}{p\omega_1 + q\omega_2} = 0, \\ \displaystyle\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{\varphi(p,q)\Im m\left(\underline{U}_{p+\mu,q+\nu} \underline{I}^*_{p+\sigma,q+\tau}\right)}{p\omega_1 + q\omega_2} = 0, \end{cases}, \tag{6}$$

are satisfied, where

$$\varphi(p,q) = \varphi(-p,-q),\ \ \psi(p,q) = -\psi(-p,-q) \tag{7}$$

are two arbitrary functions, the first even and the second odd with respect to p and q. Also, μ, ν, σ, $\tau \in \mathbb{N}$ and (Tolstov, 1955)

$$\begin{cases} \underline{U}_{p,q} = \dfrac{1}{4\pi^2} \displaystyle\int_0^{2\pi} \mathrm{d}(\omega_2 t) \int_0^{2\pi} u(t) \mathrm{e}^{-\mathrm{j}(p\omega_1 + q\omega_2)t} \mathrm{d}(\omega_1 t), \\ \underline{I}_{p,q} = \dfrac{1}{4\pi^2} \displaystyle\int_0^{2\pi} \mathrm{d}(\omega_1 t) \int_0^{2\pi} i(t) \mathrm{e}^{-\mathrm{j}(p\omega_1 + q\omega_2)t} \mathrm{d}(\omega_2 t). \end{cases} \tag{8}$$

The passing from $\underline{U}_{p,q}$, $\underline{I}^*_{p,q}$ to $\underline{U}_{p+\mu,q+\nu}$, $\underline{I}^*_{p+\sigma,q+\tau}$ is realized using the proceeding utilized by the author in the cited previous paper (Rosman, 2004).

Evidently

$$\begin{cases} \Re e\left(\underline{U}_{p+\mu,q+\nu}\underline{I}^*_{p+\sigma,q+\tau}\right) = P_{p+\mu,q+\nu;p+\sigma,q+\tau}, \\ \Im m\left(\underline{U}_{p+\mu,q+\nu}\underline{I}^*_{p+\sigma,q+\tau}\right) = Q_{p+\mu,q+\nu;p+\sigma,q+\tau} \end{cases} \tag{9}$$

represent a *generalized active pseudo-power*, respectively a *generalized reactive pseudo-power* and

$$\underline{s}_{p+\mu,q+\nu;p+\sigma,q+\tau} = \underline{U}_{p+\mu,q+\nu;p+\sigma,q+\tau}\underline{I}^*_{p+\mu,q+\nu;p+\sigma,q+\tau} \tag{10}$$

– a *generalized complex apparent pseudo-power*.

In an other paper (Rosman, 2000) was established that in case of two electrostatic networks, constituted exclusively by condensers and generators (for instance, voltage generators), insulated, named with (I) and (II), having identical topological oriented graphs, the circuits being linear, non-linear or parametric, their regime being an *electrostatic* one, relation

$$\sum_{j=1}^{B} W_{ej}^{(\mathrm{I}),(\mathrm{II})} = \sum_{u=1}^{N} q_{u0}^{(\mathrm{II})} \sum_{u=1}^{N} V_u^{(\mathrm{I})}, \tag{11}$$

was established, where

$$\sum_{j \in u} q_j^{(\mathrm{II})} = q_{u0}^{(\mathrm{II})},\ (u = 1,2,\ldots,N), \tag{12}$$

represents the first Kirchhoff's theorem applied at the node, *u*, of circuit (II) with $q_j^{(\mathrm{II})}$ – the electric charge of the condenser situated in branch *j*, (*j* = = 1,2,…,*B*), of the considered electrostatic network (II) and

$$W_{ej}^{(\mathrm{I}),(\mathrm{II})} = U_j^{(\mathrm{I})} q_j^{(\mathrm{II})}. \tag{13}$$

Relation (11) represents Tellegen's theorem applied to an electrostatic network, as was established in a previous paper (Rosman, 2000). $W_{ej}^{(\mathrm{I}),(\mathrm{II})}$ constitutes the *stored pseudo-energy in the electrostatic field*, of each of the electrostatic network's *j*-branches. In relations (11) and (12) *N* represents the number of nodes and B – the number of branches of each considered electrostatic network.

In a similar manner, considering two circuits, (I) and (II), constituted of permanent magnets (linear, non-linear or parametric) and having identical topological oriented graphs, in *magnetostatic* working regime, relation

$$\sum_{j=1}^{B} W_{mj}^{(\mathrm{I}),(\mathrm{II})} = 0 \tag{14}$$

is satisfied (Rosman, 2000), where

$$W_{mj}^{(\mathrm{I}),(\mathrm{II})} = U_{mj}^{(\mathrm{I})} \psi_{j}^{(\mathrm{II})}, \tag{15}$$

with U_{mj} – the magnetic voltage (tension) at the j-branche's incident nodes of the first magnetic circuit and ψ_j – the magnetic flux which flows through the j-branche, (j = 1,2,…,B), of the second one.

Expression (14) represents Tellegen's theorem applied to a circuit constituted of permanent magnets (Rosman, 2000). $W_{mj}^{(\mathrm{I}),(\mathrm{II})}$ can be called *stored pseudo-energy in j-branche's magnetic field.*

3. Pseudo-Quantities in Electromagnetic Field

A generalization of Tellegen's theorem in case of an electromagnetic field was performed by Ţugulea (1986) implying the introduction of some vectorial pseudo-quantities, which are essential when the energetic balance in an electromagnetic field is followed. It is a matter of *Poynting*'s *pseudo-vector*, which may be defined, utilizing the terminology used by the cited author, with relations

$$\begin{cases} \mathbf{S}^{(\mathrm{I}),(\mathrm{II})}(\mathbf{r},t',t'') = \mathbf{E}^{(\mathrm{I})}(\mathbf{r},t') \times \mathbf{H}^{(\mathrm{II})}(\mathbf{r},t''), \\ \mathbf{S}^{(\mathrm{II}),(\mathrm{I})}(\mathbf{r},t',t'') = \mathbf{E}^{(\mathrm{II})}(\mathbf{r},t'') \times \mathbf{H}^{(\mathrm{I})}(\mathbf{r},t'), \end{cases} \tag{16}$$

which characterize each pair of electromagnetic systems topological equivalents multitudes (Ţugulea, 1986). With help of pseudo-Poynting vectors may be defined the *instantaneous electromagnetic powers*

$$\begin{cases} p_{\Sigma}^{(\mathrm{I}),(\mathrm{II})}(t',t'') = \iint_{\Sigma} \mathbf{S}^{(\mathrm{I}),(\mathrm{II})}(\mathbf{r},t',t'')\mathbf{dA}, \\ p_{\Sigma}^{(\mathrm{II}),(\mathrm{I})}(t',t'') = \iint_{\Sigma} \mathbf{S}^{(\mathrm{II}),(\mathrm{I})}(\mathbf{r},t',t'')\mathbf{dA}, \end{cases} \tag{17}$$

through the closed surface, Σ.

As in the case of electric networks, the electromagnetic pseudopowers may be connected, in the case of an electromagnetic field, with some generalizations of Manley-Rowe relations. Thus, in a previous paper (Rosman, 2000), the relations

$$\begin{cases} \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\dfrac{\psi(p,q)\Re e\left(\underline{\mathbf{E}}_{p+\mu,q+\nu}\times\underline{\mathbf{H}}^{*}_{p+\sigma,q+\tau}\right)}{p\omega_1+q\omega_2}=0, \\ \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\dfrac{\varphi(p,q)\Im m\left(\underline{\mathbf{E}}_{p+\mu,q+\nu}\,\underline{\mathbf{H}}^{*}_{p+\sigma,q+\tau}\right)}{p\omega_1+q\omega_2}=0, \end{cases} \tag{18}$$

were established, valid in an electromagnetic field created by two harmonic sources, having the frequencies f_1, f_2, which evolves in a motionless, non-linear, homogeneous and isotropic medium, considered non-polarized and non-magnetized permanently, non-dissipatif, without hereditary properties, in which not occur non-periodic phenomena.

Functions $\psi(p,q)$ and $\varphi(p,q)$ expressions are given by relations (7) and

$$\underline{\mathbf{E}}_{p+\mu,q+\nu}\times\underline{\mathbf{H}}^{*}_{p+\sigma,q+\tau}=\underline{\mathbf{S}}_{p+\mu,q+\nu;p+\sigma,q+\tau} \tag{19}$$

represents the *generalized pseudo-Poynting complex vector*. This definition is in agreement with those proposed by Ţugulea (1986), with the specification that, she is referred to the particular case of an unique electromagnetic system.

In relations (18)

$$\begin{cases} \Re e\left(\underline{\mathbf{E}}_{p+\mu,q+\nu}\times\underline{\mathbf{H}}^{*}_{p+\sigma,q+\tau}\right)=P_{r_{p+\mu,q+\nu;p+\sigma,q+\tau}}, \\ \Im m\left(\underline{\mathbf{E}}_{p+\mu,q+\nu}\times\underline{\mathbf{H}}^{*}_{p+\sigma,q+\tau}\right)=Q_{r_{p+\mu,q+\nu;p+\sigma,q+\tau}} \end{cases} \tag{20}$$

represent the *generalized active*, respectively *generalized reactive pseudo-powers radiated* in the considered electromagnetic field through a closed surface situated in this field.

Having in view the electromagnetic energy theorem (Mocanu, 1981), applied in case of studied electromagnetic field,

$$\frac{\partial w}{\partial t}+\nabla\mathbf{S}=0, \tag{21}$$

where w represents the volumetric density of the in field stored energy and **S** – the Poynting vector, were established (Rosman, 2011) the relations

$$\begin{cases} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \dfrac{\psi(p,q)\mathrm{d}\left[\Re\mathrm{e}\left(\underline{s}_{p+\mu,q+\nu;p+\sigma,q+\tau}\right)\right]/\mathrm{d}v}{p\omega_1+q\omega_2}=0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \dfrac{\psi(p,q)\mathrm{d}\left[\Im m\left(\underline{s}_{p+\mu,q+\nu;p+\sigma,q+\tau}\right)\right]/\mathrm{d}v}{p\omega_1+q\omega_2}=0, \end{cases} \tag{22}$$

where $\psi(p,q)$ is given by (7_1) and $\mathrm{d}\underline{s}_{p+\mu,q+\nu;\ p+\sigma,q+\tau}$ represents the *generalized apparent complex pseudo-power stored in volume unit of the electromagnetic field*, while $\mathrm{d}\left[\Re e\left(\underline{s}_{p+\mu,q+\nu;p+\sigma,q+\tau}\right)\right]/\mathrm{d}v$, $\mathrm{d}\left[\Im m\left(\underline{s}_{p+\mu,q+\nu;p+\sigma,q+\tau}\right)\right]/\mathrm{d}v$ represent *the generalized active*, respectively *the generalized reactive pseudo-powers stored in volume's unit of the electromagnetic field*.

Having in view that Poynting vector, **S**, electromagnetic impulse's volumetric density, **G**, and Lorentz type magnetic force volumetric density, $\mathbf{f}_m$, satisfy relation (Mocanu, 1981)

$$\mathbf{f}_m=\frac{\sigma}{\varepsilon}\mathbf{G}=\sigma\mu\mathbf{S}, \tag{23}$$

where σ, ε and μ are material constants of the medium in which evolves the field, it is possible to conceive the existence of following pseudo-quantities:

$$\underline{\mathbf{G}}_{p+\mu,q+\nu;p+\sigma,q+\tau}=\Re e\left(\mathbf{D}_{p+\mu,q+\nu}\times\mathbf{B}^*_{p+\sigma,q+\tau}\right)+\mathrm{j}\Im m\left(\mathbf{D}_{p+\mu,q+\nu}\times\mathbf{B}^*_{p+\sigma,q+\tau}\right), \tag{24}$$

which can be considered as representing the *generalized electromagnetic pseudo-impulse's volumetric density;*

$$\mathbf{f}_{m_{p+\mu,q+\nu;p+\sigma,q+\tau}}=\Re e\left(\underline{\mathbf{J}}_{p+\mu,q+\nu}\times\mathbf{B}^*_{p+\sigma,q+\tau}\right)+\mathrm{j}\Im m\left(\underline{\mathbf{J}}_{p+\mu,q+\nu}\times\mathbf{B}^*_{p+\sigma,q+\tau}\right), \tag{25}$$

which, analogously, may be considered as representing *generalized Lorentz type magnetic pseudo-force's volumetric density*.

These results can be considered in connection with some papers published by the author in 2004.

4. Conclusions

Tellegen's theorem and Manley-Rowe relations permit to define a number of pseudo-quantities, some of these known from the speciality literature and the others established by the author of this paper. From the last relations may be cited: the pseudo-energy stored in the electric field of a condensers network; the pseudo-energy stored in the magnetic field of a circuit constituted of permanent magnets; active and reactive pseudo-powers radiated through a closed surface situated in the electromagnetic field, the generalized complex apparent, active and reactive pseudo-powers stored in the electromagnetic field, the generalized volumetric densities of the electromagnetic pseudo-impulse and of the Lorentz type magnetic pseudo-force.

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PSEUDOMĂRIMI ÎN CIRCUITE ELECTRICE ŞI ÎN CÂMP ELECTROMAGNETIC

(Rezumat)

Teorema lui Tellegen a permis introducerea noțiunilor de pseudoputere instantanee, pseudoputere aparentă complexă, pseudoputere activă şi pseudoputere reactivă.

În lucrare se stabilesc noțiunile de pseudoenergie înmagazinată în câmpul electrostatic al unui circuit de condensatoare, pseudoenergie înmagazinată în câmpul magnetic al unui circuit de magneți permanenți, pseudoputeri active şi reactive radiate în câmpul electromagnetic. De asemenea relațiile lui Manlez-Rowe-Kontorovici permit la rîndul lor introducerea noțiunilor de pseudoputere aparentă complexă generalizată, pseudoputere activă generalizată şi pseudoputere reactivă generalizată, înmagazinate în câmpul electromagnetic, densitate volumetrică generalizată a pseudoimpulsului electromagnetic, densitate volumetrică generalizată a pseudoforței magnetice de tip Lorentz.