BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVIII (LXII), Fasc. 4, 2012
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

ADDITIVE NEURAL NETWORK BASED CHAOTIC NOISE GENERATORS

BY

VICTOR GRIGORAŞ[1,*] and CARMEN GRIGORAŞ[2,3]

[1]"Gheorghe Asachi" Technical University of Iaşi,
Faculty of Electronics, Telecommunications
and Information Technology
[2]"Gr.T. Popa" University of Medicine and Pharmacy of Iaşi,
[3]Romanian Academy, Iaşi Branch,
Institute of Computer Science

Received, December 12, 2012
Accepted for publication: December 27, 2012

Abstract. Chaotic circuits exhibit properties of sensitivity to initial condition and system parameters, unpredictible time evolution and ergodicity. These properties are suggesting their usefulness in designing noise generators. The present paper proposes the use of discrete-time additive activation function neural networks (AAFNN) as chaotic noise generators. Theoretic results regarding the proposed class of neural networks highlight their usefulness in the analytic design of discrete-time noise generators. Due to the robustness of the statistical properties for the generated chaotic signals, a parametric analysis is performed in order to identify the value range of the synaptic weights necessary to achieve the desired statistic properties of the designed noise generators. The simulation results presented show that, for a wide range of synaptic weight values, the proposed approach offer satisfactory performance.

Key words: chaotic dynamics; noise generators; neural networks.

1. Introduction

The noise generation field of research is of recent interest due to its important applications in test and measurement equipment design,

---

*Coresponding author: *e-mail*: grigoras@etti.tuiasi.ro

communication security, such as encryption and watermarking, spread spectrum modulation and clock generation. Many research results are reported for chaotic noise generation. They are based on analog implementations (Stojanovsky & Kocarev, 2001; Udaltsov *et al.*, 2002; Yalcin *et al.*, 2004) or fixed point digital systems (Leon *et al.*, 2001; Yang *et al.*, 2004), based on first order discrete-time systems. The first approach is too costly in terms of chip area while the latter lacks flexibility due to the low order of the system and lower number of system parameters.

The present paper proposes higher order noise generators, based on neural networks with additive activation function, implemented in floating point digital arithmetic. In what follows we will review some theoretical results concerning the additive activation function neural networks, present simulation results and extract conclusions on the range of values for the system parameters, to achieve desired statistical properties.

2. AAFNN Noise Generators

The proposed class of discrete-time noise generators is based on additive activation function neural networks. The state description of the AAFNN, given by eqs.

$$\begin{cases} \mathbf{x}[k+1] = r\left(\mathbf{W}\mathbf{x}[k]\right), \\ y[k] = x_1[k]. \end{cases} \tag{1}$$

facilitates the nonlinear dynamics analysis of these discrete-time systems.

In the following we denote the order of the AAFNN by N, thus $\mathbf{x}$ is a column vector of dimension N and $\mathbf{W}$ is the $N \times N$ synaptic weight matrix. The additive nonlinear algebraic activation function, given in eq.

$$r(x) = x - \mathrm{round}(x), \tag{2}$$

is depicted in Fig. 1.

In a previous paper (Grigoraş & Grigoraş, 2000) was demonstrated that, for this particular class of recursive neural networks, the Lyapounov exponents of the discrete-time nonlinear system, λ_n, (n = 1,...N), can be analytically deduced from the eigenvalues, z_n, (n = 1,...N), of the synaptic weights matrix $\mathbf{W}$

$$\lambda_n = \ln\left(\left|z_n\right|\right). \tag{3}$$

This result enables us to separate the parameter range for different nonlinear dynamics. If all eigenvalues of the synaptic weights matrix, z_n, (n = 1,...N), have modulus less than unity, all Lyapounov exponents, λ_n,

(n = 1,...N), are negative, ensuring stable dynamics; if at least one eigenvalue has modulus larger than unity, the corresponding Lyapounov exponent is positive, leading to chaotic dynamics. As such, the theoretical range of the AAFNN parameters must be chosen so that at least one eigenvalue of the synaptic weights matrix has modulus larger than unity for our chaotic noise generation application.

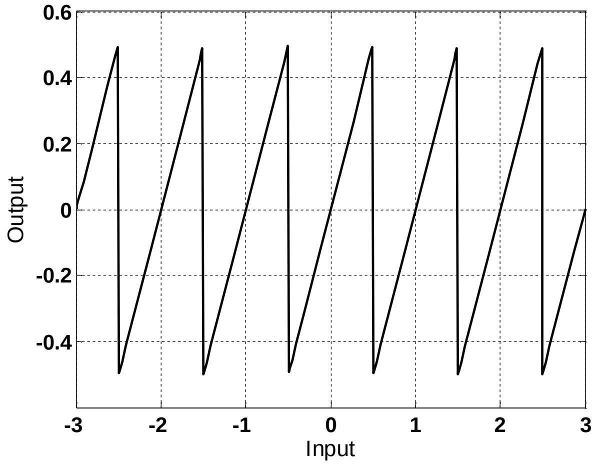

Fig. 1 – Graphical representation of the additive algebraic nonlinear function.

3. Simulation Results

Based on the possibility of a Jordan canonical block-diagonalization representation of the synaptic weights matrix, **W**, we chose to simulate a second order AAFNN block, with complex eigenvalues, $z_n = \rho_n \exp(\pm j\phi)$, given in the standard rotation form

$$W = \rho \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix}, \tag{4}$$

with variable parameters $\rho > 1$ and ϕ.

For nonlinear dynamics evaluation we performed several bifurcation diagrams, for a number of ϕ values and continuous variation of ρ in the range [1,…,3]. In Fig. 2 some of the most relevant bifurcation diagrams are presented. Although the theoretical results are confirmed and the noise generator exhibits chaotic dynamics for all $\rho > 1$, the output values cover the whole [–0.5,..., 0.5] range interval only for larger values of ρ. Moreover, for different values of the angle, ϕ, the threshold value for the modulus of the eigenvalues, ρ, is different, with a maximum of 2 for $\phi = \pi/2$.

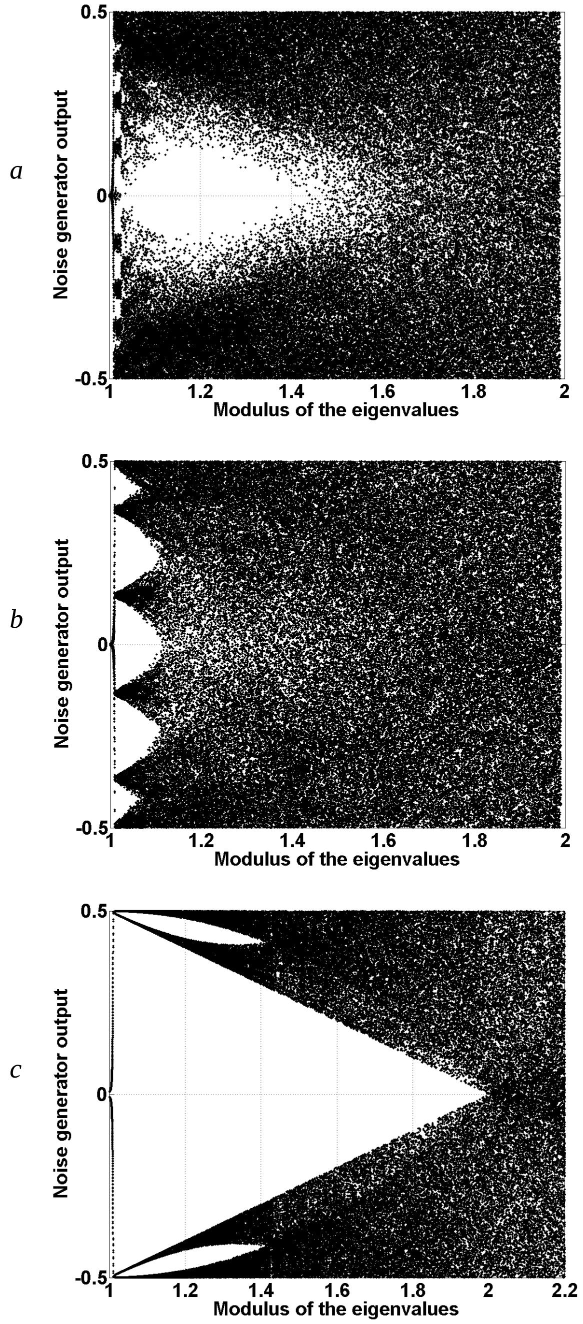

Fig. 2 – Bifurcation diagrams for $\phi = \pi/12$ (*a*), $\phi = \pi/6$ (*b*) and $\phi = \pi/2$ (*c*).

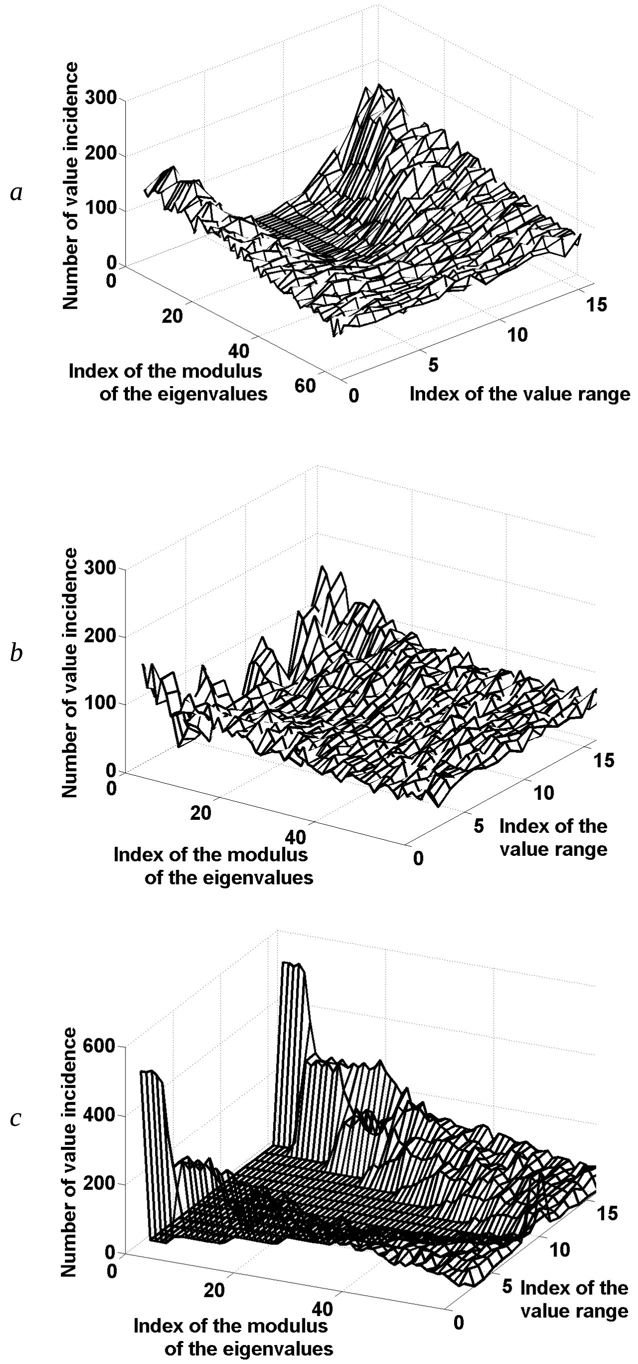

Fig. 3 – Estimated probability distribution of the output signal for $\phi = \pi/12$ (*a*), $\phi = \pi/3$ (*b*) and $\phi = \pi/2$ (*c*).

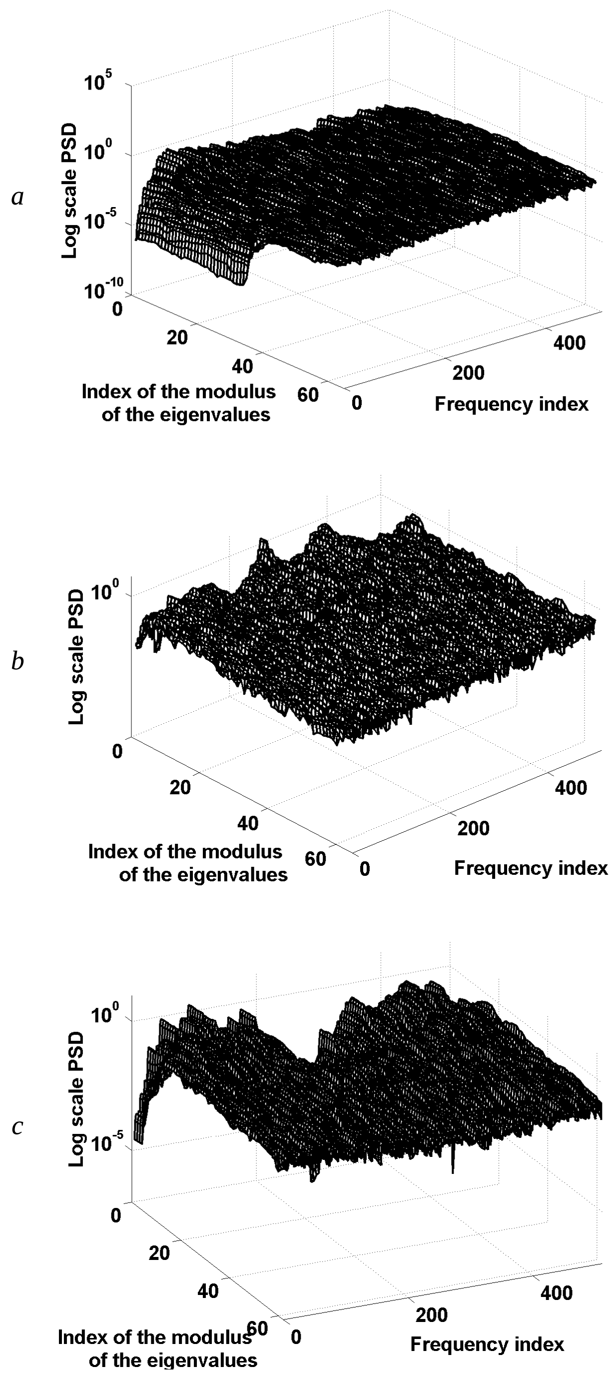

Fig. 4 – Estimated power spectral density of the output signal for $\phi = \pi/12$ (*a*), $\phi = \pi/3$ (*b*) and $\phi = \pi/2$ (*c*).

a

b

c

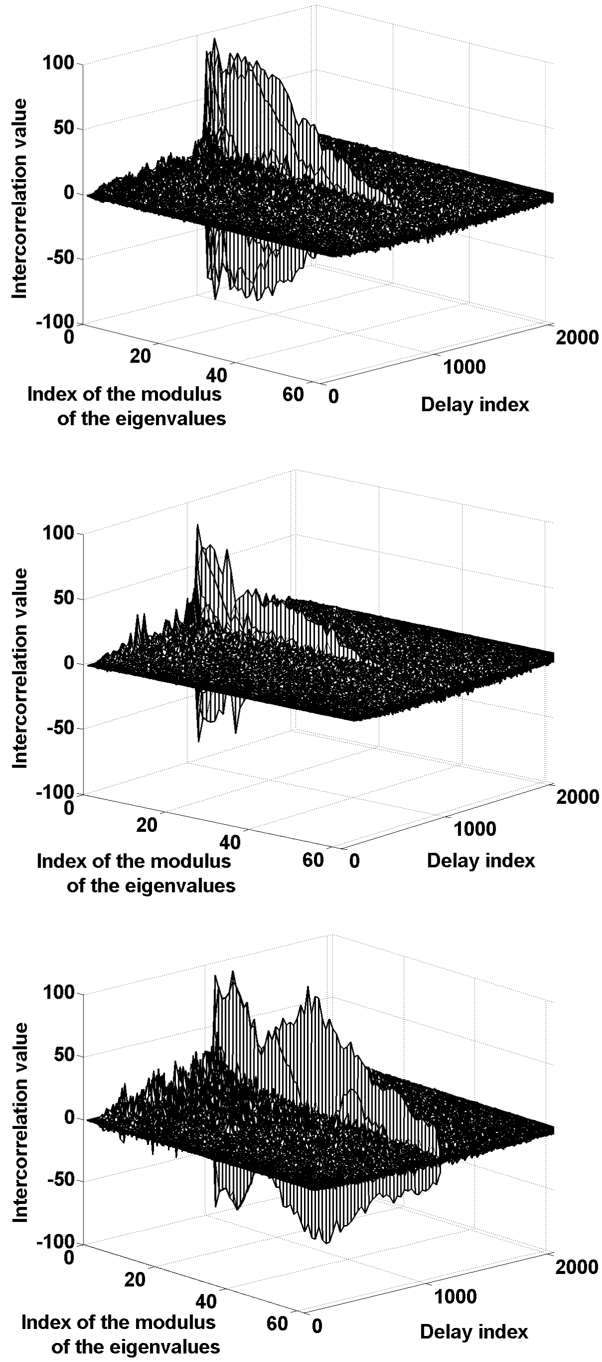

Fig. 5 – Intercorrelation estimation of the AAFNN state variables for $\phi = \pi/12$ (*a*), $\phi = \pi/3$ (*b*) and $\phi = \pi/2$ (*c*).

Once the requirements imposed by the threshold value, identified by means of the previous bifurcation diagrams, are met, it is necessary to test the first order statistics of the generated chaotic noise. We estimated the probability distribution of the output signal from the AAFNN, which is equal to the invariant measure of its chaotic dynamics, using the chaotic noise histogram, calculated over long, non-overlapping time intervals, for variable modulus of the eigenvalues, ρ. Some relevant results, presented in Fig. 3, highlight that, for values of ρ, meeting the requirements imposed by the bifurcation diagram method, the probability distribution is close to a uniform one, consistent with the constant slope of the additive nonlinear algebraic activation function. If the application at hand requires a different type of probability distribution, the uniformly distributed output signal from the AAFNN can be further processed by means of an appropriate algebraic nonlinear function.

From the point of view of the second order statistics, the power spectral density (PSD) of the generated noise is of most importance. As an estimator for the PSD of the chaotic signal, we used the Welch method, based on $k = 2^{10} =$ $= 1{,}024$ samples Short Time Fourier Transform (STFT), with eight spectral averages, as presented in Fig. 4.

If some application requires the generation of two or more uncorrelated noise signals, it is useful to study if the state variables of the chaotic AAFNN meet the uncorrelation criterion. For our case ($N = 2$) we studied the possibility of de-correlating the two state variables, by choosing a large enough modulus of the eigenvalues, ρ. The simulation results, represented in Fig. 5, confirm that, for $\rho > 2$, the state variables of our AAFNN can be estimated as reasonably uncorrelated.

4. Conclusions

A new method of generating discrete-time noise, based on the chaotic dynamics of AAFNN, is proposed. Our goal was to deduce quantitative values for the system parameters in order to satisfy imposed design criteria. Previous theoretical results gave the initial range of values for the synaptic weights matrix of the AAFNN, but our simulation results show that the restriction imposed to the system parameters have to be more stringent if first order statistical properties (such as the invariant measure) and second order ones (power spectral density and state variables intercorrelation) have to be met.

REFERENCES

Grigoraş C., Grigoraş V., *State-Space Description of Additive Non-Linear Systems*. Bul. Inst. Politehnic, Iaşi, s. Electror., Energ., Electron., **XLVI (L)**, *3-4*, 7-12 (2000).

Leon D. *et al*., *Robust Chaotic PN Sequence Generation Techniques*. Proc. of the Internat. Symp. on Circ. a. Syst., ISCAS, Sydney, Australia, May 6-9, 2001, IV-53 – IV-54.

Stojanovsky T., Kocarev L., *Chaos-Based Random Number Generators*. Part I: *Analysis*. IEEE Trans. on Circ. a. Syst. I: Fund. Theory a. Appl., **CAS 48**, *3*, 281 – 288 (2001).

Udaltsov V.S. *et al.*, *Bandpass Chaotic Dynamics of Electronic Oscillator Operating with Delayed Nonlinear Feedback*. IEEE Trans. on Circ. a. Syst. I: Fund. Theory a. Appl., **CAS 49**, *7*, 1006-1009 (2002).

Yalcin M.E., Suykens J.A.K., Vandewalle J., *True Random Bit Generation from a Double Scroll Attractor*. IEEE Trans. on Circ. a. Syst. I: Regular Papers, **CAS 51**, *7*, 1395-1404 (2004).

Yang H.T., Huang R.J., Chang T.I., *A Chaos-Based Fully Digital 120 MHz Pseudo Random Number Generator*. Proc. of the IEEE Asia-Pacific Conf. on Circ. a. Syst., Taiwan, Dec. 6-9, 2004, 357-360.

GENERATOARE DE ZGOMOT HAOTICE BAZATE PE REȚELE NEURALE ADITIVE

(Rezumat)

Circuitele haotice au proprietăți de senzitivitate la condițiile inițiale şi la parametrii sistemului, evoluție temporală nepredictibilă şi ergodicitate. Aceste proprietăți le sugerează utilitatea în proiectarea generatoarelor de zgomot. Se propune utilizarea rețelelor neurale cu funcție de activare aditivă (AAFNN) ca generatoare de zgomot. Rezultatele teoretice referitoare la clasa de rețele neurale subliniază utilitatea lor în proiectarea analitică a generatoarelor de zgomot discrete. Datorită robusteții proprietăților statistice ale semnalelor haotice generate este prezentată o analiză parametrică în scopul identificării domeniului de valori ale ponderilor sinaptice, necesare pentru a atinge proprietățile statistice dorite pentru generatorul de zgomot proiectat. Rezultatele simulărilor prezentate arată că, pentru o gamă dinamică mare a valorilor ponderilor sinaptice, varianta propusă oferă performanțe satisfacatoare.