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ABOUT THE TRANSFER COEFFICIENTS OF A LINEAR, NON-AUTONOMOUS AND RECIPROCAL GENERAL TWO-PORT, SUPPLIED, IN HARMONIC STEADY-STATE, SIMULTANEOUSLY, AT THE GATES (*1*), (*1′*) AND (*2*), (*2′*)

BY

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Abstract. The transfer coefficients $\underline{k}_{U_{13}}$, $\underline{k}_{U_{23}}$, $\underline{k}_{I_{13}}$, $\underline{k}_{I_{23}}$, $\underline{Z}_{m_{13}}$, $\underline{Z}_{m_{23}}$ of a linear, non-autonomous and reciprocal two-port supplied, in harmonic steady-state, simultaneously, at the gates (*1*), (*1′*) and (*2*), (*2′*), are determined. The curves’ equations which mark the limits of existence domains of these coefficients are determined too, when complex impedance $\underline{Z}_3$ is passive.

Key words: linear, non-autonomous and reciprocal two-ports simultaneously supplied at the gates (*1*), (*1′*) and (*2*), (*2′*); transfer coefficient; existence domains.

1. Introduction

In a previous paper (Rosman, 2007), the transfer coefficients of a linear non-autonomous and general two-port (LNGT – Fig. 1 *a*) in harmonic steady-state, were defined. These transfer coefficients are the followings: the voltage

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transfer coefficients $\underline{k}_{U_{12}} = \underline{U}_2/\underline{U}_1$, $\underline{k}_{U_{13}} = \underline{U}_3/\underline{U}_1$ and $\underline{k}_{U_{23}} = \underline{U}_3/\underline{U}_2$; the current coefficients $\underline{k}_{I_{12}} = \underline{I}_2/\underline{I}_1$, $\underline{k}_{I_{13}} = \underline{I}_3/\underline{I}_1$ and $\underline{k}_{I_{23}} = \underline{I}_3/\underline{I}_2$; the transfer impedances $\underline{Z}_{m_{12}} = \underline{U}_2/\underline{I}_1$, $\underline{Z}_{m_{13}} = \underline{U}_3/\underline{I}_1$ and $\underline{Z}_{m_{23}} = \underline{U}_3/\underline{I}_2$.

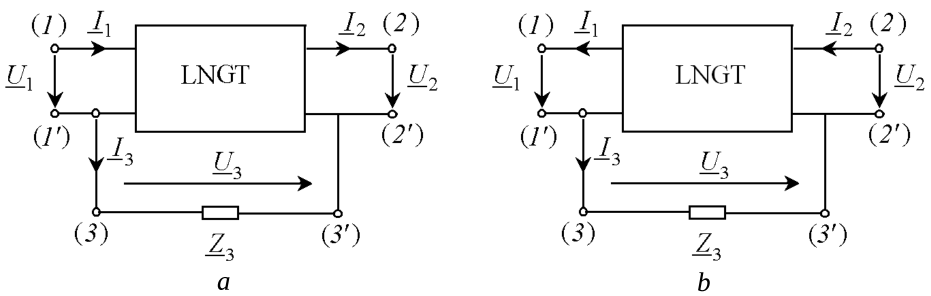

Fig. 1

When the LNGT is supplied, simultaneously, at the gates (*1*), (*1′*) and (*2*), (*2′*) with harmonic voltages having the same frequency (Fig. 1 *b*), the two-ports eqs. are (Sigorsky, 1955)

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \\ \underline{U}_3 \end{bmatrix} = [\underline{A}] \begin{bmatrix} \underline{U}_2 \\ -\underline{I}_2 \\ \underline{I}_3 \end{bmatrix}, \tag{1}$$

where

$$[\underline{A}] = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} & \underline{A}_{13} \\ \underline{A}_{21} & \underline{A}_{22} & \underline{A}_{23} \\ \underline{A}_{31} & \underline{A}_{32} & \underline{A}_{33} \end{bmatrix} \tag{2}$$

is the fundamental matrix of the considered LNGRT (supposed as being reciprocal) and

$$\frac{\underline{U}_3}{\underline{I}_3} = \underline{Z}_3 = R_3 + \mathrm{j}X_3, \quad (R_3 \geq 0). \tag{3}$$

Having in view relations (1) and (3) it is possible to determine the expressions of the before named transfer coefficients namely

$$\underline{k}_{U_{13}} = \frac{\underline{A}_{32}\underline{U}_1 - (\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31})\underline{U}_2}{\underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}} \cdot \frac{\underline{Z}_3}{\underline{U}_1}, \tag{4}$$

$$\underline{k}_{U_{23}} = \frac{\underline{A}_{32}\underline{U}_1 - \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\underline{U}_2}{\underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}} \cdot \frac{\underline{Z}_3}{\underline{U}_2}, \tag{5}$$

$$\underline{k}_{I_{13}} = \frac{\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2}{\left(\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33} + \underline{A}_{22}\underline{Z}_3\right)\underline{U}_1 + \left[\underline{A}_{33} + \left(\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}\right)\underline{Z}_3\right]\underline{U}_2}, \tag{6}$$

$$\underline{k}_{I_{23}} = \frac{\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2}{\left(\underline{A}_{33} - \underline{Z}_3\right)\underline{U}_1 - \left(\underline{A}_{11}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{31} - \underline{A}_{11}\underline{Z}_3\right)\underline{U}_2}, \tag{7}$$

$$\underline{Z}_{m_{13}} = \frac{\left(\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2\right)\underline{Z}_3}{\left(\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33} + \underline{A}_{22}\underline{Z}_3\right)\underline{U}_1 + \left[\underline{A}_{33} - \left(\underline{A}_{11}\underline{A}_{23} - \underline{A}_{12}\underline{A}_{21}\right)\underline{Z}_3\right]\underline{U}_2}, \tag{8}$$

$$\underline{Z}_{m_{23}} = \frac{\left(\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2\right)\underline{Z}_3}{\left(\underline{A}_{33} - \underline{Z}_3\right)\underline{U}_1 + \left(\underline{A}_{12}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{Z}_3\right)\underline{U}_2}. \tag{9}$$

The relations (4),…,(9) are the single transfer coefficients having a physical meaning in the case of a LNGRT supplied, simultaneously, at the gates (*1*), (*1′*) and (*2*), (*2′*), with harmonic voltages having the same frequency. It must be underlined that these expressions were obtained having in view the *reciprocal* character of the considered two-port. In this case the fundamental parameters, $\underline{A}_{ij}$, (i, j = 1, 2, 3), satisfy the relations (Sigorsky, 1955)

$$\left|\underline{A}\right| = \underline{A}_{33},\ \underline{A}_{12}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{22} = \underline{A}_{32},\ \underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31} = \underline{A}_{13}, \tag{10}$$

where $\left|\underline{A}\right| = \det\left|\underline{A}_{ij}\right|,\ (i, j = 1,2,3)$.

Analysing the expressions (4),…, (9) it can be observed that these ones have the general form

$$\underline{k} = \frac{\underline{M} + \underline{N}\underline{Z}_3}{\underline{S} + \underline{T}\underline{Z}_3}, \tag{11}$$

which represents a *conformal transformation* of circles from complex plane (R_3 , jX_3) into circles in complex plane (k', jk''), where $\underline{k} = k' + \mathrm{j}k''$ (Stoilov, 1964). It is possible to determine the curves’ eqs. which mark the limits of existence domains of each transfer coefficient. A simple proceeding is that of solving eq. (11) with respect to $\underline{Z}_3$ and annulling the real part of these one, having in view that this complex impedance is considered passive and consequently $\Re e(\underline{Z}_3) \geq 0$.

2. The Voltage Transfer Coefficient $\underline{k}_{U_{13}}$

If relation (4) is solved with respect to $\underline{Z}_3$ it results

$$\underline{Z}_3 = \frac{\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\underline{k}_{U_{13}}\underline{U}_1}{\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2 - \underline{A}_{12}\underline{k}_{U_{13}}\underline{U}_1}. \tag{12}$$

Introducing the notation

$$\underline{k}_{U_{13}} = k'_{U_{13}} + \mathrm{j}k''_{U_{13}} \tag{13}$$

and annulling the real part of $\underline{Z}_3$ it results

$$a\left(k'^2_{U_{13}} + k''^2_{U_{13}}\right) + bk'_{U_{13}} + ck''_{U_{13}} = 0, \tag{14}$$

where the expressions of coefficients *a*, *b*, *c* are given in the Appendix.

Relation (13) represents the eq. of a circle paasing through the coordinates $k'_{U_{13}}$, $k''_{U_{13}}$ origin. The existence domain of transfer coefficient, $\underline{k}_{U_{13}}$, is situated in the inside and on the frontier of this circle, when the complex impedance, $\underline{Z}_3$ is passive.

3. The Voltage Transfer Coefficient $\underline{k}_{U_{23}}$

Solving relation (5) with respect to $\underline{Z}_3$ one obtains

$$\underline{Z}_3 = \frac{\underline{A}_{32}\underline{U}_1 - \left(\underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31} + \underline{A}_{12}\underline{k}_{U_{13}}\right)\underline{U}_2}{\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\underline{k}_{U_{23}}\underline{U}_2}. \tag{15}$$

If notation

$$\underline{k}_{U_{23}} = k'_{U_{23}} + \mathrm{j}k''_{U_{23}} \tag{16}$$

is used and annulling the real part of $\underline{Z}_3$ it results

$$d\left(k'^2_{U_{23}} + k''^2_{U_{23}}\right) + ek'_{U_{23}} + fk''_{U_{23}} = 0. \tag{17}$$

The expressions of coefficients *d*, *e*, *f* are given in the Appendix.

Relation (17) represents the eq. of a circle which pass through the coordinates $k'_{U_{23}}$, $k''_{U_{23}}$ origin. The existence domain of transfer coefficient, $\underline{k}_{U_{23}}$, is situated in the inside and on the frontier of this circle, the LNGRT's receiver being passive.

4. The Current Transfer Coefficient $\underline{k}_{I_{13}}$

Determining the complex impedance, $\underline{Z}_3$, from relation (6) one obtains

$$\underline{Z}_3 = \frac{\left[\underline{A}_{32} - \left(\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}\right)\underline{k}_{I_{13}}\right]\underline{U}_1 + \left(\underline{A}_{13} - \underline{A}_{33}\underline{k}_{I_{13}}\right)\underline{U}_2}{\left[\underline{A}_{22}\underline{U}_1 - \left(\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}\right)\underline{U}_2\right]\underline{k}_{I_{13}}}. \qquad (18)$$

Introducing the notation

$$\underline{k}_{I_{13}} = k'_{I_{13}} + \mathrm{j}k''_{I_{13}} \qquad (19)$$

and annulling the real part of $\underline{Z}_3$ (s. rel. (18)) it results

$$g\left(k'^2_{I_{13}} + k''^2_{I_{13}}\right) + hk'_{I_{13}} + ik''_{I_{13}} = 0, \qquad (20)$$

where the coefficients *g*, *h*, *i* expressions are reproduced in the Appendix.

Relation (20) represents, in the plane $(k'_{I_{13}},\ k''_{I_{13}})$, the eq. of a circle which pass through the origin. The existence domain of transfer coefficient, $\underline{k}_{I_{13}}$, is situated inside and on the frontier of the circle (20), supposing that the complex impedance $\underline{Z}_3$ is passive.

4. The Current Transfer Coefficient $\underline{k}_{I_{23}}$

Taking into account relation (7) expression

$$\underline{Z}_3 = \frac{\left(\underline{A}_{33}\underline{k}_{I_{23}} - \underline{A}_{32}\right)\underline{U}_1 - \left[\left(\underline{A}_{11}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{31}\right)\underline{k}_{I_{23}} - \underline{A}_{13}\right]\underline{U}_2}{\left(\underline{U}_1 - \underline{A}_{11}\underline{U}_2\right)\underline{k}_{I_{23}}} \qquad (21)$$

can be established. Using the notation

$$\underline{k}_{I_{23}} = k'_{I_{23}} + \mathrm{j}k''_{I_{23}} \qquad (22)$$

and if the real part of $\underline{Z}_3$ (s. rel. (21)) is annulled it results the relation

$$j\left(k_{I_{23}}^{'2}+k_{I_{23}}^{"2}\right)+kk_{I_{23}}^{'}+ik_{I_{23}}^{"}=0, \tag{23}$$

which represents, in the plane $(k_{I_{23}}^{'},\ k_{I_{23}}^{"})$, the eq. of a circle passing through the origin. The existence domain of transfer coefficient, $k_{I_{23}}$, is situated inside and on the frontier of circle (23), when the complex impedance is passive.

The expressions of coefficients *j*, *k*, *l* are given in the Appendix.

6. The Transfer Impedance $\underline{Z}_{m_{13}}$

Using the some proceeding as in the previous sections it may be deduced, from relation (8), the expression

$$\underline{Z}_3=\frac{\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\underline{U}_1+\underline{A}_{13}\underline{U}_2}{\left(\underline{A}_{32}-\underline{A}_{22}\underline{Z}_{m_{13}}\right)\underline{U}_1+\left[\underline{A}_{13}+\left(\underline{A}_{11}\underline{A}_{22}-\underline{A}_{12}\underline{A}_{21}\right)\underline{Z}_{m_{13}}\right]\underline{U}_2}. \tag{24}$$

If the real part of expression (24) is annulled relation

$$m\left(R_{m_{13}}^2+X_{m_{13}}^2\right)+nR_{m_{13}}+oX_{m_{13}}=0 \tag{25}$$

is obtained, where the coefficients *m*, *n*, *o* are given in Appendix, and the relation

$$\underline{Z}_{m_{13}}=R_{m_{13}}+\mathrm{j}X_{m_{13}} \tag{26}$$

is used.

Relation (25) represents, in plane $(R_{m_{13}}, X_{m_{13}})$, the eq. of a circle which pass through the origin.

The existence domain of transfer impedance $\underline{Z}_{m_{13}}$ is situated, when the complex impedance $\underline{Z}_3$ is passive, inside and on the frontier of circle (25).

7. The Transfer Impedance $\underline{Z}_{m_{23}}$

Relation (9) permits to obtain the expression

$$\underline{Z}_3=\frac{\underline{A}_{33}\underline{U}_1-\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\underline{U}_2}{\left(\underline{A}_{32}+\underline{Z}_{m_{23}}\right)\underline{U}_1+\left(\underline{A}_{13}-\underline{A}_{11}\underline{Z}_{m_{13}}\right)\underline{U}_2}. \tag{27}$$

Introducing the notation

$$\underline{Z}_{m_{23}} = R_{m_{23}} + \mathrm{j}X_{m_{23}} \tag{28}$$

and if the real part of expression (27) is annulled relation

$$p\left(R_{m_{23}}^2 + X_{m_{23}}^2\right) + qR_{m_{23}} + rX_{m_{23}} = 0 \tag{29}$$

is obtained, the coefficients *p*, *q*, *r* expressions being given in the Appendix.

Relation (29) represents, in plane $(R_{m_{23}}, X_{m_{23}})$, the equation of a circle passing through the coordinates axis origin. When the complex impedance $\underline{Z}_3$ is passive, the existence domain of transfer impedance $\underline{Z}_{m_{23}}$ is situated inside and on the frontier of this circle.

8. Conclusions

1. The expressions of transfer coefficients $\underline{k}_{U_{13}}$, $\underline{k}_{U_{23}}$, $\underline{k}_{I_{13}}$, $\underline{k}_{I_{23}}$, $\underline{Z}_{m_{13}}$, $\underline{Z}_{m_{23}}$ of a linear, non-autonomous and reciprocal general two-port, supplied simultaneously at the gates (*1*), (*1′*) and (*2*), (*2′*) with harmonic voltages having the same frequency, are determined.

2. In the studied case the curves' eqs. which mark the existence domains of these transfer coefficients are determined too, when the coupling complex impedance $\underline{Z}_3$ is considered passive.

3. The curves which mark these existence domains are, in this case, circles passing through the corresponding coordinates axis origin.

Appendix

The expressions of coefficients *a*, *b*,…, *r* are the followings:

$$a = U_1^2 \Re e\left[\underline{A}_{12}^*(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33})\right],$$

$$b = -U_1^2 \Re e\left[\underline{A}_{12}^*(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33})\right] + \Re e\left[\underline{A}_{13}^*(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33})\underline{U}_1\underline{U}_2^*\right],$$

$$c = U_1^2 \Im m\left[\underline{A}_{12}^*(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33})\right] + \Im m\left[\underline{A}_{13}^*(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33})\underline{U}_1\underline{U}_2^*\right],$$

$$d = U_2^2 \Re e\left[\underline{A}_{12}(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{33}^*)\right],$$

$$e = -\Re e\left[\underline{A}_{32}\left(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{33}^*\right)\underline{U}_1\underline{U}_2^*\right] + U_2^2 \Re e\left[(\underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31})(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{33}^*)\right],$$

$$f = \Im m\left[\underline{A}_{32}\left(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{31}\right)\underline{U}_1\underline{U}_2^*\right] + U_2^2 \Im m\left[(\underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31})(\underline{A}_{13}^*\underline{A}_{32}^* - \underline{A}_{12}^*\underline{A}_{33}^*)\right],$$

$$g = \Re e\left\{\left[\left(\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*\right) - \underline{A}_{22}^*\underline{A}_{33}\right]\underline{U}_1\underline{U}_2^*\right\} +$$

$$+ U_2^2 \Re e\left[\underline{A}_{33}(\underline{A}_{11}^*\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}^*)\right],$$

$$h=-U_1^2\Re e\left[\underline{A}_{12}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{23}\underline{A}_{31}\right)+\underline{A}_{21}\underline{A}_{22}^{*}\right]-\Re e\left[\underline{A}_{22}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\underline{U}_1\underline{U}_2^{*}\right]+$$
$$+\Re e\left(\underline{A}_{13}\underline{A}_{22}^{*}\underline{U}_1^{*}\underline{U}_2\right)-U_2^2\Re e\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\right],$$

$$i=U_1^2\Im m\left[\underline{A}_{12}^{*}\left(\underline{A}_{13}\underline{A}_{32}-\underline{A}_{23}\underline{A}_{31}\right)+\underline{A}_{22}^{*}\underline{A}_{32}\right]-\Im m\left[\underline{A}_{22}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\underline{U}_1\underline{U}_2^{*}\right]+$$
$$+\Im m\left(\underline{A}_{13}\underline{A}_{22}^{*}\underline{U}_1^{*}\underline{U}_2\right)-U_2^2\Im m\left[\underline{A}_{13}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\right],$$

$$j=U_1^2\Re e\left(\underline{A}_{33}\right)-\Re e\left(\underline{A}_{11}^{*}\underline{A}_{23}^{*}\underline{U}_1\underline{U}_2^{*}\right)-\Re e\left[\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\underline{U}_1^{*}\underline{U}_2\right]+$$
$$+U_2^2\Re e\left[\underline{A}_{11}^{*}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\right],$$

$$k=U_1^2\Re e\left(\underline{A}_{33}\right)+\Re e\left(\underline{A}_{11}^{*}\underline{A}_{32}\underline{U}_1\underline{U}_2^{*}\right)-\Re e\left(\underline{A}_{13}\underline{U}_1^{*}\underline{U}_2\right)-U_2^2\Re e\left(\underline{A}_{21}^{*}\underline{A}_{13}\right),$$

$$l=-U_1^2\Im m\left(\underline{A}_{33}\right)+\Im m\left(\underline{A}_{11}^{*}\underline{A}_{32}\underline{U}_1\underline{U}_2^{*}\right)-\Im m\left(\underline{A}_{13}\underline{U}_1^{*}\underline{U}_2\right)-U_2^2\Im m\left(\underline{A}_{21}^{*}\underline{A}_{13}\right),$$

$$m=-U_1^2\Re e\left[\underline{A}_{22}^{*}\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\right]+\Re e\left[\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\underline{U}_1\underline{U}_2^{*}\right]-$$
$$-\Re e\left(\underline{A}_{22}\underline{A}_{33}^{*}\underline{U}_1^{*}\underline{U}_2\right)+U_2^2\Re e\left[\underline{A}_{33}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\right],$$

$$n=U_1^2\Re e\left[\underline{A}_{32}^{*}\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\right]-\Re e\left[\underline{A}_{13}^{*}\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\underline{U}_1\underline{U}_2^{*}\right]+$$
$$+\Re e\left(\underline{A}_{12}^{*}\underline{A}_{33}\underline{U}_1^{*}\underline{U}_2\right)-U_2^2\Re e\left(\underline{A}_{13}^{*}\underline{A}_{33}\right),$$

$$o=-U_1^2\Im m\left[\underline{A}_{32}^{*}\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\right]+\Im m\left[\underline{A}_{13}^{*}\left(\underline{A}_{23}\underline{A}_{32}-\underline{A}_{22}\underline{A}_{33}\right)\underline{U}_1\underline{U}_2^{*}\right]-$$
$$-\Im m\left(\underline{A}_{32}^{*}\underline{A}_{33}\underline{U}_1^{*}\underline{U}_2\right)+U_2^2\Im m\left(\underline{A}_{13}^{*}\underline{A}_{33}\right),$$

$$p=U_1^2\Re e\left(\underline{A}_{33}\right)-\Re e\left(\underline{A}_{11}^{*}\underline{A}_{31}^{*}\underline{U}_1\underline{U}_2^{*}\right)-\Re e\left[\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\underline{U}_1^{*}\underline{U}_2\right]+$$
$$+U_2^2\Re e\left[\underline{A}_{11}^{*}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\right],$$

$$q=U_1^2\Re e\left(\underline{A}_{32}^{*}\underline{A}_{33}\right)+\Re e\left(\underline{A}_{13}^{*}\underline{A}_{31}\underline{U}_1\underline{U}_2^{*}\right)-\Re e\left[\underline{A}_{32}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\underline{U}_1^{*}\underline{U}_2\right]-$$
$$-U_2^2\Re e\left[\underline{A}_{13}^{*}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\right],$$

$$r=-U_1^2\Im m\left(\underline{A}_{32}^{*}\underline{A}_{33}\right)-\Im m\left(\underline{A}_{13}^{*}\underline{A}_{31}\underline{U}_1\underline{U}_2^{*}\right)-\Im m\left[\underline{A}_{32}^{*}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\underline{U}_1^{*}\underline{U}_2\right]+$$
$$+U_2^2\Im m\left[\underline{A}_{13}^{*}\left(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{31}\right)\right].$$

Examining the coefficients *a*, *b*,…,*r* expressions it results that these depend on LNGRT's fundamental parameters, $\underline{A}_{ij}$, (i,j = 1,2,3) and on the complex RMS values of voltages, $\underline{U}_1$, $\underline{U}_2$.

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ASUPRA COEFICIENŢILOR DE TRANSFER AI UNUI CUADRIPOL GENERAL LINIAR, NEAUTONOM ŞI RECIPROC, ALIMENTAT, ÎN REGIM PERMANENT ARMONIC, SIMULTAN, PE LA PORŢILE (*1*), (*1′*) ŞI (*2*), (*2′*)

(Rezumat)

Se stabilesc expresiile coeficienţilor de transfer $\underline{k}_{U_{13}}$, $\underline{k}_{U_{23}}$, $\underline{k}_{I_{13}}$, $\underline{k}_{I_{23}}$, $\underline{Z}_{m_{13}}$, $\underline{Z}_{m_{23}}$ ai unui cuadripol general, liniar, neautonom şi reciproc, alimentat, simultan, pe la porţile (*1*), (*1′*) şi (*2*), (*2′*), cu tensiuni armonice de aceeaşi frecvenţă. Se determină ecuaţiile curbelor care delimitează domeniile de existenţă ale acestor coeficienţi în cazul în care impedanţa $\underline{Z}_3$ este pasivă.