BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVIII (LXII), Fasc. 4, 2012
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

ABOUT POWER EVALUATION OF INDUCTION MACHINE IN STEADY-STATE AND TRANSIENT

BY

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Received, October 5, 2012
Accepted for publication: November 9, 2012

Abstract. The two-phase model is widely used in the study of electrical machines, including the three-phase induction machine. When the machine is supplied, in steady-state, by sinusoidal voltages in quadrature, all electromagnetic quantities describing the two-phase model are also sinusoidal in quadrature. In these circumstances, it is relatively easy to determine, in each time moment, the value of active and reactive powers using adequate formulas. However, in transient, the use of same formulas implies errors. The paper aims to determine the acceleration limit up to which the results obtained by using two specific calculation quantities, P^{tr} and Q^{tr}, can be extrapolated to the active and reactive powers with no major errors.

Key words: active power; reactive power; induction machine; two-phase model; PSpice.

1. Introduction

Consider the two-phase model of an induction machine with the stator supplied by quadrature sinusoidal voltages. In steady-state, all quantities characterizing the electromagnetic operation of the machine (voltages, currents,

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magnetic fluxes) are also sinusoidal, and the corresponding quantities on the two axes are in quadrature. Fig. 1 presents the case of a certain branch of the equivalent circuit of an induction machine and shows the voltages and currents corresponding to the two axes.

If the voltage corresponding to the *d*-axis is considered the phase origin, the expressions of the voltages and currents of interest become

$$\begin{cases} v_{xd} = \sqrt{2}V_x \cos\omega t; \quad i_{xd} = \sqrt{2}I_x \cos\left(\omega t - \varphi\right); \\ v_{xq} = \sqrt{2}V_x \sin\omega t; \quad i_{xq} = \sqrt{2}I_x \sin\left(\omega t - \varphi\right). \end{cases} \tag{1}$$

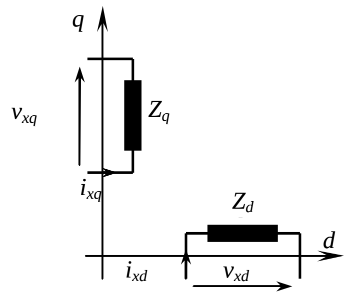

Fig. 1 – Two-phase induction machine circuit components.

The corresponding phasor diagram is presented in Fig. 2. It is shown, on one hand, that the corresponding quantities on the two axes are sinusoidal, with the same characteristic values but in quadrature and, on the other hand, the fact that the phase shift between voltage and current is the same on the two axes.

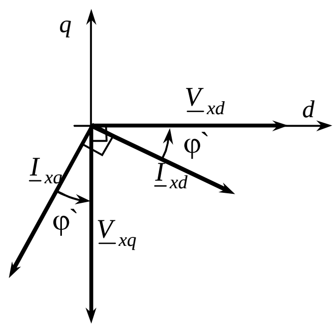

Fig. 2 – Phasor diagram of two-phase quantities.

Considering the above observations the maximum value or the effective value can be determined based on the knowledge of the instantaneous values at a given moment

$$\begin{cases} \sqrt{2}V_x = \sqrt{v_{xd}^2(t) + v_{xq}^2(t)}, \\ \sqrt{2}I_x = \sqrt{i_{xd}^2(t) + i_{xq}^2(t)}. \end{cases} \tag{2}$$

Knowing the instantaneous values of voltages and currents on the two phases, it is possible to determine the active power at a given moment, *i.e.*

$$\begin{aligned} e_1(t) &= v_{xd}i_{xd} + v_{xq}i_{xq} = \\ &= 2V_xI_x\left[\cos\omega t\cos\left(\omega t - \varphi\right) + \sin\omega t\sin\left(\omega t - \varphi\right)\right] = \\ &= 2V_xI_x\cos\varphi, \end{aligned} \tag{3}$$

$$P = 2V_xI_x\cos\varphi = v_{xd}i_{xd} + v_{xq}i_{xq}\,. \tag{4}$$

Similarly, the reactive power, in a given moment, can be determined namely

$$\begin{aligned} e_2(t) &= v_{xq}i_{xd} - v_{xd}i_{xq} = \\ &= 2V_xI_x\left[\sin\omega t\cos\left(\omega t - \varphi\right) - \cos\omega t\sin\left(\omega t - \varphi\right)\right] = \\ &= 2V_xI_x\,sin\varphi, \end{aligned} \tag{5}$$

$$Q = 2V_xI_x\sin\varphi = v_{xq}i_{xd} - v_{xd}i_{xq}\,. \tag{6}$$

Using relations (4) and (6), the active and reactive powers at a given moment can be calculated, and they can be used in the computer aided study of the electrical machines, especially of the asynchronous machine. These formulas are valid only in steady-state operation.

2. Electric Power Evaluation in Transient

In transient the matter is more complex. Consider the induction machine supplied by quadrate sinusoidal voltages and the rotor speed increasing at constant acceleration. Changing the speed leads to a transient state with forced components due to the quadrate sinusoidal voltages supply and natural components that usually decay exponentially.

Since the speed change is considered of ramp type (constant acceleration) the natural components magnitude is proportional to the rate of speed change. If the acceleration is reduced, the magnitude of the natural components is negligible and the operation is close to the steady-state, *i.e.*

quasi-stationary state. For larger accelerations, the natural components take important values that cannot be neglected.

In what follows we intend to find out to what extent the formulas to calculate active and reactive power are acceptable in transient. Correspondingly to (4) and (6) we define the following calculation quantities:

$$P^{\mathrm{tr}} = v_{xd} i_{xd} + v_{xq} i_{xq} \,, \tag{7}$$

$$Q^{\mathrm{tr}} = v_{xq} i_{xd} - v_{xd} i_{xq} \,. \tag{8}$$

In quasi-stationary state, up to a certain speed slope limit, we expect that the values obtained using (7) and (8) to be similar to the values of the active and reactive powers defined in steady-state at the respective speed

$$P^{\mathrm{tr}} \approx P, \quad Q^{\mathrm{tr}} \approx Q \,. \tag{9}$$

In the case of large accelerations unacceptable differences between those values may occur. The paper aims to determine the acceleration limit up to which the results obtained by calculating P^{tr} and Q^{tr} can be extrapolated to P and Q with no major errors.

The induction machine behavior is extremely complex and can be described by an intricate non-linear eqs. system. The transient operation is generally a complex problem because there are many time constants related to the angular speed and to the slope of the angular speed. If the speed of the machine is variable, we are dealing with a complicate transient operation. In these difficult situations, an analytical approach is quite impossible. Only numerical data analysis is possible and simulation results can be obtained.

To calculate the active and reactive power a two-phase model of an induction machine was considered. The characteristic parameters of the machine were

$$\begin{array}{llll} P_n = 5\ \mathrm{kW}; & R_1 = 1.4\ \Omega; & L_{\sigma 1} = 7.5\ \mathrm{mH}; & p = 1, \\ U_{1ln} = 380\ \mathrm{V}; & R_2^{'} = 1.5\ \Omega; & L_{\sigma 2}^{'} = 8.0\ \mathrm{mH}; & J = 40\ \mathrm{g.m^2}. \\ f_1 = 50\ \mathrm{Hz}; & \text{Y-connection}; & L_m = 117.5\ \mathrm{mH}; & \end{array}$$

In the following, a study of the possibilities to obtain the active and reactive power values are presented. The transients determined by the rotor speed variation are considered. The simulations were carried out using PSpice software as presented by Cociu & Cociu (1997, 2005), Justus (1997).

To determine the influence of the slope of angular rotor speed on the power evaluation, different dynamic situations were considered, like in Fig. 3. The moment t = 2 s is the end of steady-state and the start of the transient. Two typical values for the slope, 500 rad/s^2 and 1,000 rad/s^2, were taken into account. The value of 2,000 rad/s^2 is very difficult to obtain with such a machine. On the other hand, the value of 100 rad/s^2 is considered as a reference value replacing the steady-state. In fact, this is a quasi-stationary operation, near the steady-state.

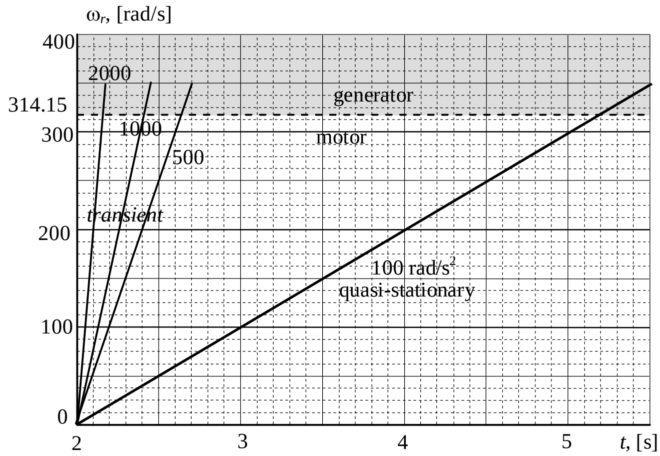

Fig. 3 – External rotor speed variation.

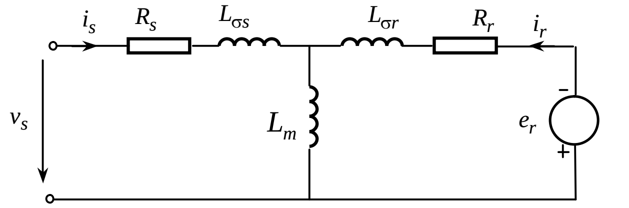

Fig. 4 – Equivalent circuit of the two-phase model of the induction machine.

All notations used below refer to the elements of the equivalent circuit presented in Fig. 4.

3. Active Power Evaluation in Transient

First, the active power evaluation at the stator supply terminals is considered using the calculation quantity

$$P_s^{\mathrm{tr}} = v_{sd} i_{sd} + v_{sq} i_{sq} . \tag{10}$$

The evaluated values are presented in Fig. 5. We can see that in quasi-stationary operation, the results are similar to the steady-state. At synchronous angular speed, the active power is zero, due to the zero value of rotor current and very low value of the stator current.

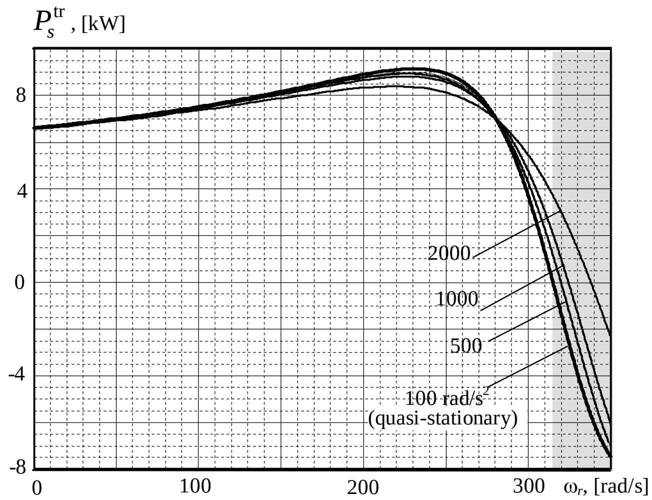

Fig. 5 – Stator calculation-active power evaluation.

Then, the Joule losses in stator and rotor windings are evaluated and corresponding results are presented in Figs. 6 and 7.

$$p_{js}^{tr} = v_{Rsd} i_{sd} + v_{Rsq} i_{sq}, \tag{11}$$

$$p_{jr}^{tr} = v_{Rrd} i_{rd} + v_{Rrq} i_{rq}. \tag{12}$$

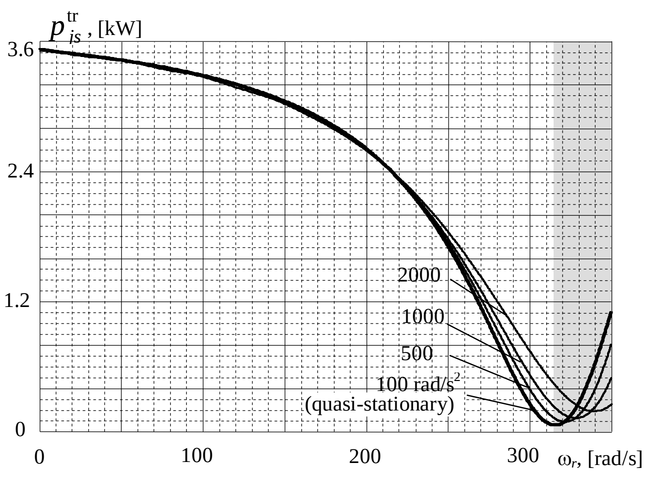

Fig. 6 – Stator Joule losses evaluation.

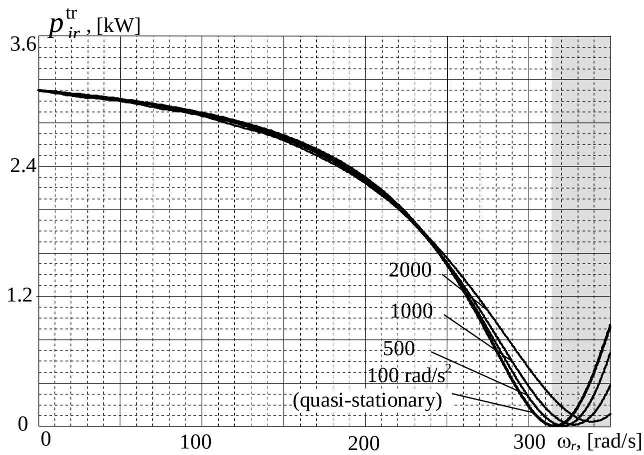

Fig. 7 – Rotor Joule losses evaluation.

To verify a similar relation to the balance of active power, the evaluation of the mechanical output power is also presented in Fig. 8,

$$P_m^{tr} = T_e \omega_r . \tag{13}$$

Fig. 9 presents the electromagnetic torque evaluation, necessary for a better understanding of the mechanical power variations.

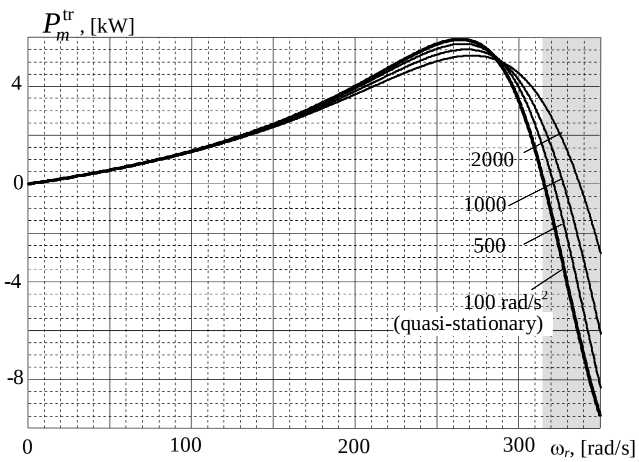

Fig. 8 – Mechanical output power *versus* speed.

For a small value of the slope (100 rad/s²) the mechanical power is zero at zero rotor speed and at synchronous speed. At this slope, the torque values are similar to those at the steady-state. For greater values of the slope, the maximum torque value decreases.

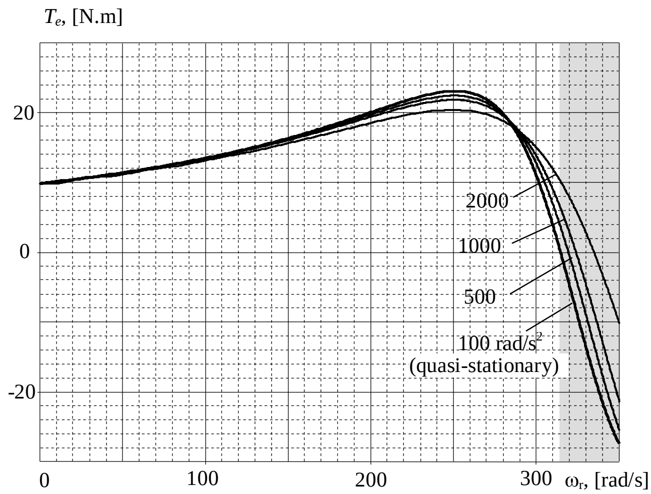

Fig. 9 – Electromagnetic torque variation.

In all situations, we have the relation

$$P_s^{\text{tr}} \approx p_{js}^{\text{tr}} + p_{jr}^{\text{tr}} + P_m^{\text{tr}}, \tag{14}$$

the error increasing from 0.25% for a slope of 100 rad/s^2 to 4% for a slop of 2000 rad/s^2.

4. Reactive Power Evaluation in Transient

First, the reactive power evaluation at the stator supply terminals is considered using the relation

$$Q_s^{\text{tr}} = v_{sq} i_{sd} - v_{sd} i_{sq}. \tag{15}$$

The variation of the total pseudo-reactive power at the stator supply terminals is presented in Fig. 10. Figs. 11,…,13 show the pseudo-reactive power related to the stator, rotor and magnetizing inductances. Similar relation are used for any evaluation,

$$Q_{\sigma s}^{\text{tr}} = v_{L\sigma sq} i_{sd} - v_{L\sigma sd} i_{sq}, \tag{16}$$

$$Q_{\sigma r}^{\text{tr}} = v_{L\sigma rq} i_{rd} - v_{L\sigma rd} i_{rq}, \tag{17}$$

$$Q_m^{\text{tr}} = v_{Lmq} i_{md} - v_{Lmd} i_{mq}. \tag{18}$$

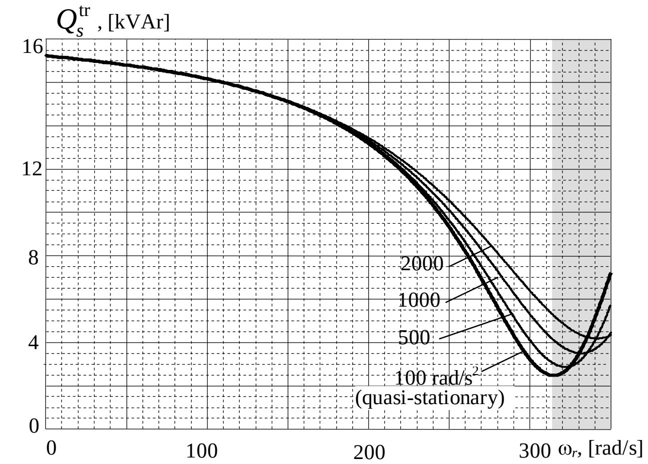

Fig. 10 – Total calculation-reactive power evaluation.

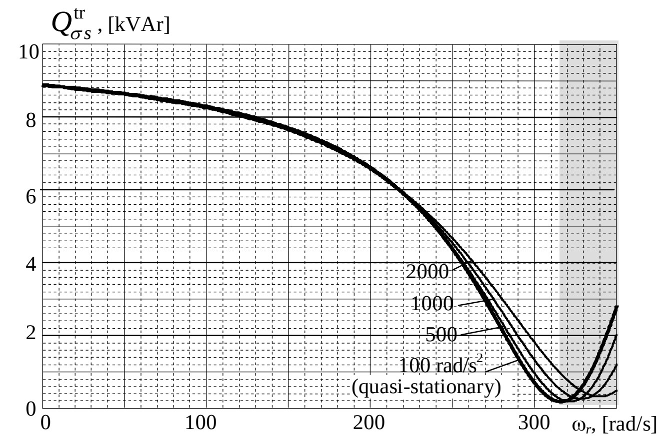

Fig. 11 – Stator pseudo-reactive power evaluation.

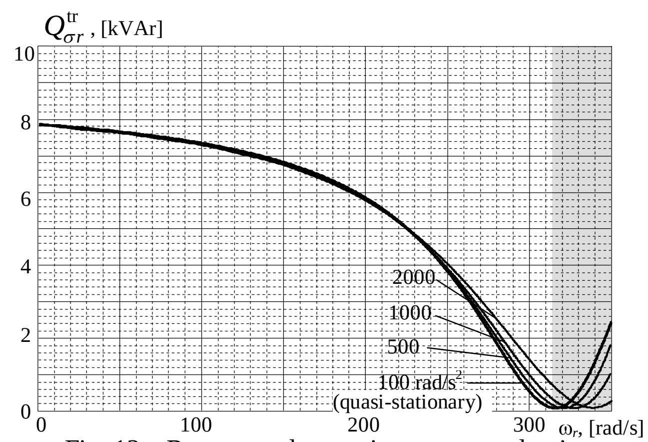

Fig. 12 – Rotor pseudo-reactive power evaluation.

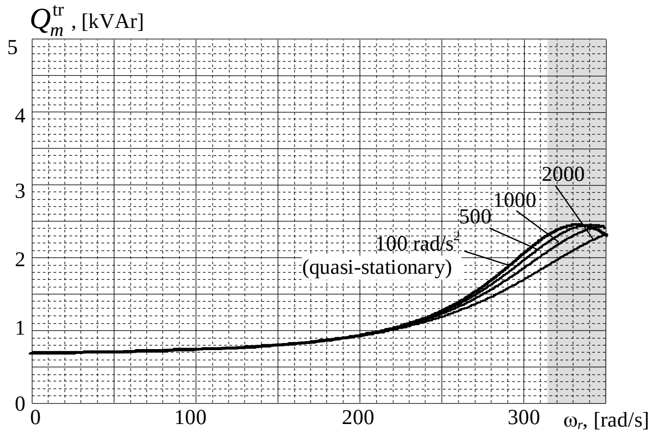

Fig. 13 – Magnetizing calculation-reactive power evaluation.

5. Conclusions

Using relations (4) and (6), the active and reactive powers at a given moment can be calculated. They can be used in the computer aided study of the electrical machines, especially of the induction machine. These relations are valid only in steady-state operation.

The case of the transient is quite different. Even if the machine is supplied with sinusoidal voltages, the other voltages, currents or fluxes are not sinusoidal. In the system response, natural components appear, generally damping exponentially, that lead to the distortion of the quantities of interest.

Correspondingly to (4) and (6) we can define two calculation quantities (7) and (8) to evaluate the calculation-active and reactive powers using the transient of the induction machine.

The study highlights the influence of the angular rotor speed slope on the power evaluation, in different dynamic situations. If the value of the slope of the angular rotor speed is low (50…200 rad/s^2), we have a quasi-stationary operation, similar to the steady-state.

For greater slope (over 500 rad/s^2), unacceptable difference occurs between the calculation-active and reactive powers evaluated with (7) and (8) in PSpice and the real powers.

Note that for the induction machine in the example above, in starting operation the slope is of 200…800 rad/s^2, corresponding to the load value. Greater values result in no-load starting whereas smaller values result in starting at rated load. Therefore this approach can be used only about the rated load operation.

An expression similar to the active power balance is nearly true in transient as well no matter the value of rotor speed, for low and medium speed slope up to 1,000 rad/s^2. At 2,000 rad/s^2 the errors are still acceptable. On the contrary an expression similar to the reactive power balance can be considered only for low angular rotor speed slope.

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ASUPRA EVALUĂRII PUTERII ÎN REGIM PERMANENT ŞI TRANZITORIU LA MAŞINA DE INDUCȚIE

(Rezumat)

Modelul bifazat este utilizat pe scară largă în studiul maşinilor electrice, în special al celor de inducție. În regim permanent, dacă modelul maşinii se consideră alimentat cu tensiuni în cuadratură, toate mărimile de natură electromagnetică rezultă de asemenea în cuadratură. În aceste condiții este relativ uşor de determinat, la un moment dat, puterea activă şi reactivă utilizând formule adecvate. Utilizarea aceloraşi formule şi în regim tranzitoriu implică însă apariția anumitor erori. Scopul lucrării este să evidențieze limita accelerației până la care utilizarea celor două mărimi de calcul specifice, P^{tr} şi Q^{tr}, permite extrapolarea rezultatelor obținute către puterile active şi reactive, fără erori majore.