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OPTIMIZATION POSSIBILITIES FOR RADIAL ELECTRIC ENERGY DISTRIBUTION NETWORK ROUTES

BY

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Abstract. The rational structures or configurations setting are one of the most complex levels in the distribution networks design and operation. The evolution of computing technology at remarkable performances directly influenced the mathematical methods and algorithms for electric energy distribution systems design and operation. Thus, taking into account the distribution networks topology or structure, the paper proposes a methodology for the radial distribution network route optimization. In this regard it is proposed an algorithm that, starting from the set of nodes defined by loads, allows the optimal route construction based on a successive search method and the introduction of additional nodes branch. Finally, to highlighten the usefulness of proposed methodology, a real rural distribution network from our country was analysed.

Key words: optimization; Steiner point; distribution network; graph theory.

1. Introduction

Public distribution networks consist of substation and medium (MV) or low voltage (LV) overhead and cable lines which supply with electric energy

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households, tertiary and low power industrial consumers (*i.e.* commercial, social-cultural, public services, small industrial consumers), located in a rural or urban habitable area (Ionescu *et al*., 2006). The consistent shifting of electric energy consumption from HV (high voltage) to MV and LV imposed for the specialists a more attention to the complex processes of design, reconstruction, refurbishment and rational operation of public electric energy distribution systems from technical and economical points of view (Georgescu, 2007).

In literature, the “network structure” term does not have a sufficiently precise definition, referring to “network diagram” or “network configuration”, although its significance is much broader. Generally, network structures represent internal construction, that is, nodes and lines (Eremia *et al*., 2006). Also, the network structure means the combined electric power sources, consuming nodes (PQ nodes) and branching lines between them. In public electric energy distribution networks the branching lines are represented by overhead or cable lines and transformer installations (Georgescu *et al*., 2012).

According to Avella *et al*., (2005), can be said that a distribution network has a radial or tree configuration when it does not allow cycles or closed contours, and each node (station, substation, consumer) is supplied with electric energy through an only way. It should be noted that, usually, in steady state electric energy operation, the distribution networks presents radial configuration, allowing a relatively simple operation, and the faults can be easily detected and quickly restored.

At the electric networks development, the establishing of rational or optimal radial or tree networks structures is one of the most complex stages of the design process. In what follows a particular approach to determine the optimal route (structure) and optimal route reconfiguration of a real distribution network are presented. This approach is based on a combination of algorithms that use graph theory.

2. Mathematical Formulation for Route Optimization of Radial or Arborescence Distribution Networks

On the basis of numerous methods for the synthesis of the optimal networks configuration can be formulated the problem of a minimum spanning tree constructing (minimum length distribution network). To solve these problem, in the literature (Sudhakar *et al*., 2010) are proposed both classical algorithms sufficiently simple and effective such as Prim and Kruskal algorithm, and artificial intelligence algorithms such as any colony and particle swarm algorithms, etc.

In order to optimize the radial networks routes, the mathematical model proposed in this paper includes two steps, namely, a first step in determining the minimum length complete graph using all network nodes (a single source node and the all consuming nodes), and a second step corresponds to a length graph optimization (power network route) resulting minimum from the first step by introducing an arbitrary number of additional nodes.

Below, based on graph theory, the two steps are listed, and for the optimization method the goal function is considered as the total minimum length of the network route.

2.1. Step 1 – Prim Algorithm

The algorithm was developed in 1930 by Czech mathematician, Vojtech Jarnik, and developed later (1957) by Robert C. Prim. Prim algorithm has the following mathematical model: from a lot of indicated points, two characterized by a minimum distance between them is searching; the two points must be connected by a straight segment, forming a branch. Further, from the remaining nodes should be chosen the closest node of one from the two retained nodes connected to it by a new branch. In a similar way all the other nodes are treated, and finally a radial or tree network is obtained. If the power supply node is known, the algorithm is carried out starting from this one. Fig. 1 presents an example of the minimum length radial network construction, and can easily observe that the introduction of new additional branching nodes allows to obtain a tree with a total length less than Prim's tree, according to Fig. 2, which illustrate a tree formed by three nodes, arranged in an equilateral triangle vertices.

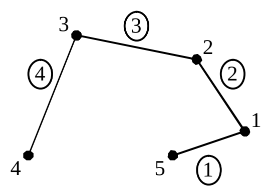

Fig. 1 – Example for minimum length network.

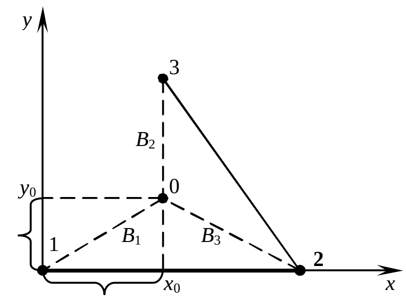

Fig. 2 – Steiner tree construction for three nodes.

As shown in Fig. 2, by using Prim algorithm a minimum tree with total length 2*a* (*a* representing the equilateral triangle branch length) is obtained. Similarly, if additional node with coordinates (x_0, y_0) is considered, it results a network length equal with $\sqrt{3}a$ (represented in Fig. 2 by dashed lines).

At each Prim algorithm step, by examining all the informations and choosing the best solution a new branch which represents a network fragment was added. If the previously described completely graph construction (complete graph means *m* nodes and $m - 1$ branches) is possible, the calculations volume required will be $2m^2 + 1$.

2.2. Step 2 – Steiner Algorithm

In the literature, the minimum network length determining through union of the system formed by initial nodes (generally known) and an arbitrary number of nodes newly introduced is known as the generalized Steiner problem. Considering these aspects, the minimum length tree from all trees with additional nodes is called *minimal Steiner tree*, which results at the searching process among the all trees that can be obtained based on all possible combinations of initial and additional nodes. Of course, these trees types are numerous, their number is $(n + l)^{n + l + 2}$, where n represent the initial number nodes and l is the additional number nodes. Thus, for the minimum network length search the mathematical methods based on analysed variables number reduction, are used.

The minimum network length construction methods, using the Steiner points, form a whole class of methods based on the following properties of Steiner trees, namely:

a) The branches that connect the initial nodes with the Steiner points are arranged at 120 degrees angles.

b) One Steiner point corresponds to three vertices (nodes); theoretically, the number of Steiner points is unlimited ($0 \leq k \leq n - 2$).

c) Best solution is for a network with least three nodes and one Steiner point.

Hereinafter the Steiner tree construction using the so-called Euclidean constructions is presented. For the network with initial nodes a_1, a_2, a_3 is necessary an additional point, b_1. This point will coincide with one of the given points if any defined angle by the nodes is greater or equal with 120º (if the angle $a_2a_1a_3 \geq 120º$, then b_1 coincides with a_1); if all angles are less than 120º, then b_1 is found within the triangle formed by a_1, a_2, a_3 vertices.

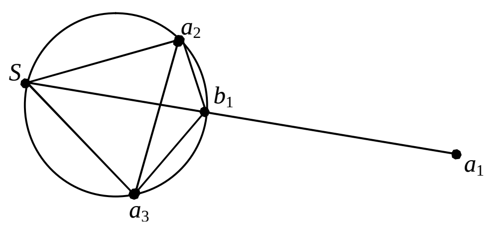

Fig. 3 – Euclidian construction of the Steiner tree.

The Euclidean construction is obtained as follows: by using one of the branches, for example a_2a_3, an equilateral triangle is formed and the peak S is situated in opposition with a_1 node in the a_2a_3 edge. Point b_1 will be situated on the circumscribed circle of the triangle. In this case, the Steiner point is situated at the intersection of the circle with the right point Sa_1 called the *equivalent of the* a_2 *and* a_3 *nodes*. The main steps of the minimum length network construction (the optimal route from the economic point of view) in all Steiner method variants are the following:

a) The initial set of nodes is decomposed into subsets which allow Steiner tree construction (for problem size reduction).

b) For each subset of nodes, using the described procedure a Steiner tree topology is obtained.

c) Minimum tree length is obtained by aggregating the separate subsets.

3. Algorithm and Software for Distribution Network Route Optimization

The choice of economic indicators for various variants estimation of networks routes depends of the nature problem to be solved and the particular characteristics of each design level. Here can also be included the values of total updated expenses, total investment, energy losses, operating costs voltage drops, damages caused by the unpowered consumers, etc.

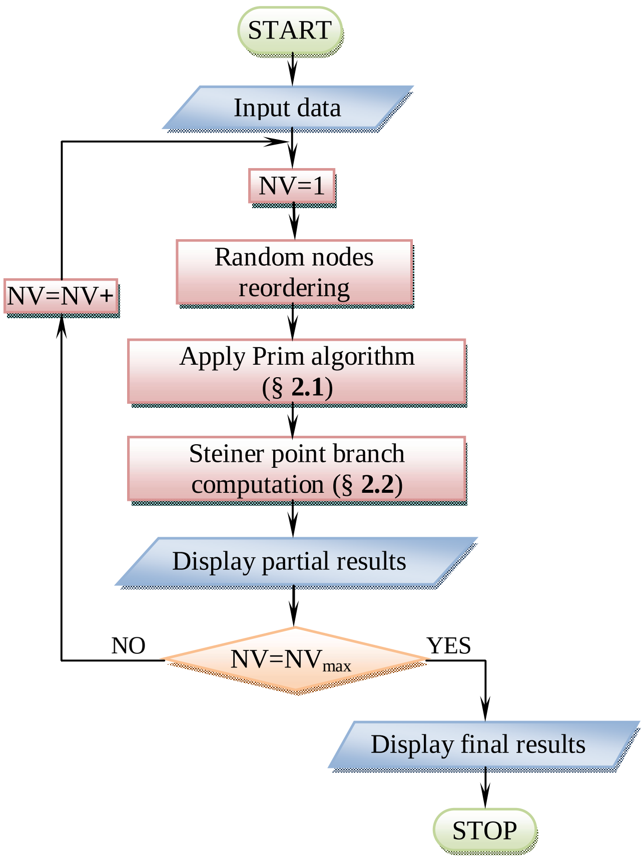

Fig. 4 – The radial or arborescence route network optimization flowchart.

Radial network synthesis problem is divided in two stages: one uses Prim algorithm building the minimum length network and the second improves the network by adding supplementary branch nodes. The diagram or route network improvement can be achieved by shifting the source nodes to the ends and *vice versa*, with the particularity that the latter would be preferable because the ends power flows are known.

Table 1
Cartesian Coordinates of the Radial Distribution Network

Node	*x*	*y*	Node	*x*	*y*	Node	*x*	*y*	Node	*x*	*y*
1	0	210	43	1,330	210	85	2,030	490	127	2,730	140
2	70	210	44	1,330	280	86	2,100	630	128	2,730	70
3	140	210	45	1,400	210	87	2,170	210	129	2,660	70
4	210	210	46	1,400	140	88	2,170	280	130	2,730	0
5	280	210	47	1,400	70	89	2,170	350	131	2,800	210
6	280	280	48	1,470	210	90	2,170	420	132	2,800	140
7	280	350	49	1,470	280	91	2,240	420	133	2,870	210
8	350	210	50	1,540	210	92	2,170	490	134	2,870	140
9	350	280	51	1,610	210	93	2,240	490	135	2,870	70
10	420	210	52	1,610	140	94	2,310	490	136	2,940	70
11	420	280	53	1,680	140	95	2,170	560	137	3,010	140
12	490	210	54	1,610	70	96	2,100	560	138	3,010	70
13	560	210	55	1,680	210	97	2,170	630	139	3,080	70
14	630	210	56	1,680	280	98	2,240	630	140	3,150	70
15	630	280	57	1,750	210	99	2,240	560	141	3,220	70
16	700	280	58	1,750	280	100	2,170	700	142	3,220	140
17	770	280	59	1,750	350	101	2,100	700	143	3,290	70
18	630	350	60	1,750	420	102	2,100	770	144	3,290	140
19	700	210	61	1,820	420	103	2,030	700	145	3,360	70
20	770	210	62	1,820	490	104	2,030	770	146	3,430	70
21	770	140	63	1,820	560	105	1,960	700	147	3,500	70
22	840	210	64	1,820	630	106	1,890	700	148	3,500	0
23	840	140	65	1,820	350	107	1,890	770	149	2,940	210
24	910	210	66	1,820	280	108	1,820	700	150	2,940	140
25	910	140	67	1,820	210	109	1,750	700	151	3,010	210
26	910	70	68	1,890	210	110	1,750	770	152	3,080	210
27	910	0	69	1,890	140	111	2,240	210	153	3,080	280
28	980	210	70	1,960	210	112	2,240	280	154	3,010	280
29	980	280	71	1,960	140	113	2,310	280	155	3,080	350
30	910	280	72	1,960	70	114	2,240	350	156	3,010	350
31	910	350	73	2,030	70	115	2,310	210	157	3,080	420
32	840	280	74	1,890	70	116	2,400	210	158	3,080	490
33	840	350	75	1,890	0	117	2,400	140	159	3,010	420
34	1,050	210	76	1,820	70	118	2,470	210	160	3,010	490
35	1,050	140	77	1,750	0	119	2,470	280	161	2,940	420
36	1,050	70	78	1,750	70	120	2,520	210	162	2,940	190
37	1,120	210	79	2,030	210	121	2,590	210	163	2,870	420
38	1,120	140	80	2,100	210	122	2,660	210	164	2,870	490
39	1,190	210	81	2,100	280	123	2,660	280	165	2,800	420
40	1,190	280	82	2,100	350	124	2,660	350	166	2,800	490
41	1,260	210	83	2,100	420	125	2,730	350	167	3,150	210
42	1,260	280	84	2,100	490	126	2,730	210	168	3,150	140

By using the mathematical model described in a previous section, a radial or arborescence distribution networks route optimization application was developed. The application uses a combination of Prim and Steiner algorithms, and optimal criterion or goal function is the total minimum network length without technical restrictions. Fig. 4 shows the flowchart that contains the following main steps:

a) Input data: general data (source and PQ nodes); consumers data (Cartesian coordinates, input nodes name); the source node is always node 1.

b) Minimum length tree construction with Prim algorithm.

c) Steiner algorithm application for network built in the previous step, by additional branch nodes introduction, called *Steiner points*.

d) Display the partial results (total network length) for the current version, to select the optimal variant. Then, display the network topology and the global minimum network length.

3. Case Example

To highlighten the utility of the application based on the algorithm shown in Fig. 4, proposed in this paper, in order to optimal route determination of radial or arborescence distribution networks, several medium voltage rural distribution networks located in Moldova area were analysed. In this context, Table 1 presents the input data of an only rural real distribution network with 168 nodes (load points).

The developed application input data were the nodes defined by load, namely, name and Cartesian coordinates of the node. Based on the presented methodology with a successive search technique, through application of the first step (Prim algorithm), the network length results of 11.620 km. Also, by optimizing minimum graph length obtained using Prim algorithm, in a second step, to allow the new route construction, are needed 60 additional branch nodes (Steiner points), and the minimum length resulted is of 10.765 km. The additional branch nodes (Steiner points) are summarized in Table 2 such as: added node number, the Cartesian coordinates and the three nodes which could form the Steiner branch point.

The software application developed has a friendly interface and allows user to upload data both the keyboard and from a .txt file, and display the analysed network from graphical point of view. Fig. 5 shows a network zone (network route between 23 and 49 nodes), in order to view the two steps for route optimization of the electric energy distribution networks, in a radial or arborescence configuration.

Table 2
Additional Branch Points (Steiner Points) Necessary in the Analysed Network

No of Steiner point	Steiner point coordinates		Nodes which form the Steiner point			No of Steiner point	Steiner point coordinates		Nodes which form the Steiner point		
	x	*y*					*x*	*y*			
169	303	303	6	9	7	201	2,263	467	93	91	94
170	327	233	8	9	5	202	2,123	583	96	86	95
171	327	257	9	6	8	203	2,217	537	99	95	93
172	607	233	14	15	13	204	2,077	723	101	103	102
173	677	257	16	19	15	205	2,077	747	102	104	101
174	747	257	17	16	20	206	1,867	723	106	108	107
175	793	163	21	20	23	207	1,797	677	108	64	109
176	1,003	233	28	29	34	208	1,773	723	109	110	108
177	933	257	30	24	29	209	2,263	303	112	114	113
178	817	257	32	17	22	210	2,287	257	113	115	112
179	863	327	33	31	32	211	2,287	233	115	113	111
180	1,073	117	35	38	36	212	2,423	187	116	118	117
181	1,143	187	37	39	38	213	2,487	233	118	120	119
182	1,097	163	38	35	37	214	2,683	233	122	126	123
183	1,213	257	40	39	42	215	2,683	327	124	125	123
184	1,423	187	45	46	48	216	2,753	117	127	132	128
185	1,447	233	48	45	49	217	2,777	187	131	126	132
186	1,633	187	51	52	55	218	2,777	163	132	131	127
187	1,633	163	52	53	51	219	2,917	93	136	150	135
188	1,703	233	55	57	56	220	3,033	93	138	137	139
189	1,703	257	56	55	58	221	3,173	93	140	168	141
190	1,727	303	58	56	59	222	3,243	93	141	143	142
191	1,797	303	66	58	65	223	3,267	93	143	141	144
192	1,867	187	68	69	67	224	3,477	47	147	148	146
193	1,913	163	69	68	71	225	2,963	203	149	162	151
194	1,867	47	74	76	75	226	2,963	157	150	162	137
195	1,773	47	78	76	77	227	3,103	233	152	167	153
196	2,077	513	84	85	96	228	3,057	327	155	156	153
197	2,123	607	86	97	96	229	3,057	443	157	158	159
198	2,147	233	87	80	88	230	3,033	397	159	157	156
199	2,193	373	89	114	90	231	2,823	443	165	166	163
200	2,147	397	90	89	83	232	3,127	187	167	168	152

Following the distribution networks analysis it is found that through the proposed methodology usage, the route of the analysed network is reduced to 855 m. Taking into account such considerations, it should be noted that for rural distribution electrical networks from our country, which may have long lengths (tens of kilometers), the algorithm proposed in this paper proves to be an effective tool in design process, leading to a minimization of the route length.

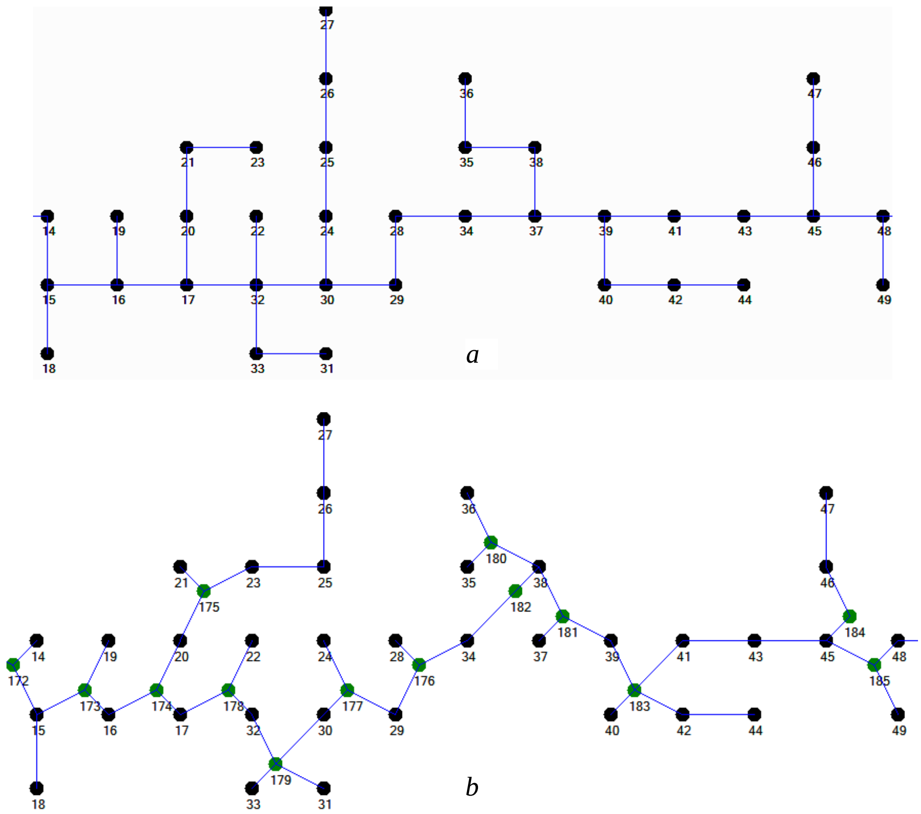

Fig. 5 – Radial distribution route network optimization: *a* – Prim algorithm (step 1); *b* – Steiner branch nodes introduction (step 2).

4. Conclusions

The consistent shifting of electric energy consumption from high voltage to medium and low voltage, imposed to the specialists a more attention to the complex processes of design, reconstruction, refurbishment and rational operation of public electric energy distribution systems from technical and economical points of view.

In order to optimize the radial networks routes, the mathematical model proposed in this paper includes two steps, namely: a first step consists in determining the minimum length complete graph using all network nodes (a single source node and the all consuming nodes), and a second step corresponds to a length graph optimization (power network route) resulting minimum from the first step by introducing an arbitrary number of additional nodes or Steiner points.

The proposed mathematical model, algorithm and software application represent an effective tool in power networks design for optimal route determination of the rural medium voltage distribution networks.

Obviously, the proposed mathematical models and algorithms in this paper must be completed by various restrictions, such as: network routes problems, geographical and ecological barriers, proximity to low current lines (telecommunications), etc.

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POSIBILITĂȚI DE OPTIMIZARE A TRASEELOR REȚELELOR ELECTRICE DE DISTRIBUȚIE RADIALE

(Rezumat)

Stabilirea structurilor sau configurațiilor raționale ale rețelelor de distribuție reprezintă una din cele mai complexe etape în procesul de proiectare şi exploatare al acestora. Evoluția tehnicii de calcul în direcția unor performanțe remarcabile a influențat în mod direct perfecționarea modelelor şi metodelor matematice şi a algoritmilor de calcul privind proiectarea şi exploatarea sistemelor de distribuție a energiei electrice. Astfel, ținând seama de topologia sau structura rețelelor de distribuție, în lucrare se propune o metodologie de optimizare a traseului rețelelor de distribuție având o configurație radială sau arborescentă. În acest sens se propune un algoritm care, plecând de la mulțimea nodurilor definite de sarcini, permite construirea traseului optim, pe baza unei metode de căutări succesive şi a introducerii unor noduri suplimentare de ramificație. În final, pentru a evidenția utilitatea metodologiei de calcul amintite anterior, s-a analizat o rețea reală de distribuție rurală din țara noastră.