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RESONANCES AT THE ACCESS GATES OF A LINEAR, NON-AUTONOMOUS AND PASSIVE TWO-PORT SUPPLYING, IN HARMONIC STEADY-STATE, A PASSIVE NON-LINEAR INERTIAL RECEIVER

BY

HUGO ROSMAN*

“Gheorghe Asachi” Technical University of Iaşi,
Faculty of Electrical Engineering

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Abstract. The possibilities of resonance realization at the access gates of a linear, non-autonomous and passive two-port, are studied when the two-port’s receiver is non-linear inertial. The conditions of resonance realization at the input gate, at the output gate and of the overall resonance are established.

Key words: resonance; linear, non-autonomous and passive two-port; harmonic steady-state; non-linear inertial passive receiver.

1. Introduction

A systematical study of the resonance either at the input gate or at the output gate of a linear, non-autonomous and passive two-port (LNPT), supplying, in harmonic steady-state, a linear and passive receiver, was performed by the author in his Ph. D. dissertation (Rosman, 1968).

Let be

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \end{bmatrix} = \left[\underline{A}\right] \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \end{bmatrix}, \tag{1}$$

* *e-mail*: adi_rotaru2005@yahoo.com

the eqs. of an LNPT (Fig. 1), in harmonic steady-state, where

$$[\underline{A}] = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \tag{2}$$

is the LNPT's fundamental parameters matrix.

Let be, also,

$$\underline{Z}_2 = \frac{\underline{U}_2}{\underline{I}_1} = R_2 + \mathrm{j}X_2 \tag{3}$$

– the complex impedance of the receiver, considered passive ($R_2 \geq 0$).

Initially, two resonance regimes at the gates of an LNPT were considered. The first one is referred to the possibility to realize the resonance at the input gate, when $X_{e1} = 0$, with

$$\underline{Z}_{e1} = \frac{\underline{U}_1}{\underline{I}_1} = R_{e1} + \mathrm{j}X_{e1} \tag{4}$$

the equivalent complex impedance at this gate. The second resonance regime may be realized at the output gate when $X_2 = 0$.

In a previous paper (Rosman, 1969), subsequent to the elaboration of the above mentioned Ph. D. dissertation (Rosman, 1968), an overall resonance at the gates of an LNPT, in harmonic steady-state, was defined, characterized by a null value of the reactive power exchanged, at the ensemble of the two gates, with the outside.

In the first case, concerning the resonance at the input gate, the equivalent parameters of the receiver satisfy the eq. of the circle's arc

$$\begin{gathered} \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left(R_2^2 + X_2^2\right) + \Im m\left(\underline{A}_{11}\underline{A}_{22}^* + \underline{A}_{12}^*\underline{A}_{21}\right)R_2 + \\ + \Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right)R_2 + \Im m\left(\underline{A}_{12}\underline{A}_{22}^*\right) = 0 \end{gathered} \tag{5}$$

situated in half-plane $R_2 \geq 0$ (Rosman, 1968).

In the second case (resonance at the output gate) it is necessarily and sufficiently that the receiver's reactance be null,

$$X_2 = 0. \tag{6}$$

Finally, in the third case, which corresponds to the overall resonance, the receiver's equivalent parameters must satisfy the eq. of the circle's arc (Rosman, 1969)

$$\Im m\left(\underline{A}_{11}\underline{A}_{12}^{*}\right)\left(R_2^2+X_2^2\right)+\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*}+\underline{A}_{12}^{*}\underline{A}_{21}\right)R_2+ \\ +\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*}-\underline{A}_{12}^{*}\underline{A}_{21}\right)+1\right]X_2+\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right)=0, \tag{7}$$

situated in the half-plane $R_2 \geq 0$.

The aim of this paper is to study the three possibilities to realize the above mentioned resonances, when the LNTR's receiver consists in the series connexion of a resistor, a coil and a capacitor (Fig. 1), all three elements being non-linear inertial and passive.

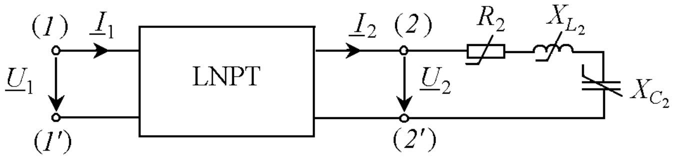

Fig. 1

It is well known (for instance, Philippow, 1963) that, as regards the input–output characteristic of a non-linear inertial element, this one is linear in instantaneous values and nonlinear in RMS values, so if the excitation signal applied to such an element has a harmonical variation in time, the response signal has a same variation in time and, consequently, the steady-state of the considered element is a harmonical one. In these conditions the study of the non-linear inertial element can be performed utilizing the complex symbolic method.

Let be an LNPT, in harmonic steady-state, whose receiver is a non-linear inertial and passive (RNIP) one, obtained by series connexion of a resistor, a coil and a condenser (Fig. 1), all three non-linear inertial. The parameters of these elements depend on the secondary current amplitude, I_{2m} , which flows through the receiver. As it is known (Savin & Rosman, 1973):

a) the resistance of a non-linear inertial resistor has the expression

$$R=R(I_m)=a_1+\frac{3}{4}a_3I_m^2,\ (a_1>0,\ a_3\lesseqgtr 0); \tag{8}$$

b) the reactance of a non-linear inertial coil is given by

$$X_L=X_L(I_m)=b_1\omega-\frac{3}{4}b_3\omega I_m^2,\ (b_1>0,\ b_3\lesseqgtr 0); \tag{9}$$

c) the reactance of a non-linear inertial condenser has a similar form namely

$$X_C = X_C(I_m) = \frac{c_1}{\omega} - \frac{3c_3}{4\omega^3} I_m^2,\ (c_1 > 0,\ c_3 \lesseqgtr 0). \tag{10}$$

In the considered case

$$\begin{cases} R_2 = R_2(I_{2m}) = a_1 + \frac{3}{4} a_3 I_{2m}^2, \\ X_2 = X_2(I_{2m}) = X_L(I_{2m}) - X_C(I_{2m}) = b_1\omega - \frac{c_1}{\omega} - \frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right) I_{2m}^2. \end{cases} \tag{11}$$

2. Resonance at the Input Gate

Determination of conditions which assure realization of the resonance at the input gate of an LNPT, in harmonic steady-state, when the LNPT supplies a receiver having the structure indicated in Fig. 1, implies to replace expressions (11) in eq. (5). Performing the calculus it results eq.

$$\alpha I_{2m}^4 + \beta I_{2m}^2 + \gamma = 0, \tag{12}$$

where

$$\begin{cases} \alpha = \frac{9}{16} \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left[a_3^2 + \left(b_3\omega + \frac{c_3}{\omega^3}\right)^2\right], \\ \beta = \frac{3}{2} \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left[a_1 a_3 - \left(b_1\omega - \frac{c_1}{\omega^3}\right)\left(b_3\omega + \frac{c_3}{\omega^3}\right)\right] + \\ \quad + \frac{3}{4} a_3 \Im m\left(\underline{A}_{11}\underline{A}_{22}^* + \underline{A}_{12}^*\underline{A}_{21}\right) - \frac{3}{4}\left(b_3\omega + \frac{c_3}{\omega^3}\right) \Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right), \\ \gamma = \left[a_1^2 + \left(b_1\omega - \frac{c_1}{\omega^3}\right)^2\right] \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right) + a_1 \Im m\left(\underline{A}_{11}\underline{A}_{22}^* + \underline{A}_{12}^*\underline{A}_{21}\right) + \\ \quad + \left(b_1\omega - \frac{c_1}{\omega^3}\right) \Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right) - \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right). \end{cases} \tag{13}$$

Eq. (12) is an algebraic biquadratic one in, I_{2m}, and, evidently, the resonance at the input gate of the considered LNTP may be obtained only for the real and positive values of hers roots. These ones are of the form

$$I_{2m} = \pm\sqrt{\frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha}}. \tag{14}$$

Evidently, from a physical point of view, only the sign "+" before the external radical must be considered. In (14), α, β, γ are given by relations (13). The roots (14) are real only if

$$\beta^2 - 4\alpha\gamma > 0,\ \operatorname{sign}\left(-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}\right) = \operatorname{sign}\alpha. \tag{15}$$

In this case two real and positive roots of eq. (12) may be discerned and, consequently, two resonance regimes at the input gate of the considered LNPT are possible. The number of these resonance regimes diminishes to only one in the case when relation (15_1) is satisfied while relations

$$\operatorname{sign}\left(-\beta + \sqrt{\beta^2 - 4\alpha\gamma}\right) = \operatorname{sign}\alpha,\ \text{but}\ \operatorname{sign}\left(-\beta - \sqrt{\beta^2 - 4\alpha\gamma}\right) = -\operatorname{sign}\alpha \quad (16')$$

or

$$\operatorname{sign}\left(-\beta + \sqrt{\beta^2 - 4\alpha\gamma}\right) = -\operatorname{sign}\alpha,\ \text{but}\ \operatorname{sign}\left(-\beta - \sqrt{\beta^2 - 4\alpha\gamma}\right) = \operatorname{sign}\alpha \quad (16'')$$

are satisfied too.

In the particular case when

$$\beta^2 = 4\alpha\gamma \tag{17}$$

a single resonance regime at the inpuit gate may be realized, only if the additional relation

$$\operatorname{sign}\beta = -\operatorname{sign}\alpha. \tag{18}$$

is satisfied.

As regards the RNIP's parameters in case(s) when the resonance regime at the input gate is realized, these ones may be obtained replacing in relations (11) the secondary current amplitude, I_{2m} , by the expression (14). Performing the calculus it results

$$\begin{cases} R_2 = a_1 + \dfrac{3\left(-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}\right)}{8\alpha}, \\ X_2 = b_1\omega - \dfrac{c_1}{\omega} - 3\left(b_3\omega + \dfrac{c_3}{\omega^3}\right)\dfrac{-\beta \pm \sqrt{\beta^2 - 4\alpha\gamma}}{8\alpha\gamma}. \end{cases} \tag{19}$$

If relations (17) and (18) are satisfied, expressions (19) become

$$R_2 = a_1 - \frac{3\beta}{8\alpha},\ X_2 = b_1\omega - \frac{c_1}{\omega} + 3\beta\frac{b_3\omega + c_3/\omega^3}{8\alpha}. \tag{20}$$

3. Resonance at the Output Gate

Strictly speaking the possibility of realization of such a regime was studied in a previous paper (Rosman, 1991), where realization of ferroresonance in a non-linear inertial circuit, in harmonic steady-state was investigated. The realization of such a regime imposes that $X_2(I_{2m}) = 0$, that is $X_{2L}(I_{2m}) = X_{2C}(I_{2m})$. Having in view (11) it is necessary that

$$I_{2m} = \frac{2\omega}{\sqrt{3}}\sqrt{\frac{b_1\omega^2 - c_1}{b_3\omega^4 + c_3}}. \tag{21}$$

A resonance regime at the output gate of the LNPT can be obtained only if the relation

$$\operatorname{sign}\left(b_1\omega^2 - c_1\right) = \operatorname{sign}\left(b_1\omega^4 + c_3\right) \tag{22}$$

is satisfied. This relation is equivalent to inequality

$$b_1 b_3\omega^6 - b_3 c_1\omega^4 - b_1 c_3\omega^2 + c_1 c_3 > 0. \tag{23}$$

As regards the receiver's parameters, in this case, it is sufficiently to replace expression (21) of the secondary current amplitude in relations (8),...,(10) obtaining

$$R_2 = a_1 + a_3\omega^2\frac{b_1\omega^2 - c_1}{b_3\omega^4 + c_3},\ X_{2L} = X_{2C} = \omega\frac{b_1 c_3 + b_3 c_1\omega^2}{b_3\omega^4 + c_3}. \tag{24}$$

In brief, it is necessary to underline, that in case of an LNPT which supplies, in harmonic steady-state, an NLIPR having the structure represented in Fig. 1, it is possible to realize, at the most, a single resonance regime at his output gate, when condition (22) is satisfied.

4. Overall Resonance

To establish the conditions which make possible the realization of the overall resonance at the access gates of an LNPT supplying, in harmonic steady-state, an NLIPR, it is necessary to utilize a proceeding similar to those used in § **2**. Namely, the expressions (11) of the considered NLIPR's parameters must be introduced in relation (7), obtaining, finally, the eq.

$$\alpha' I_{2m}^4 + \beta' I_{2m}^2 + \gamma' = 0, \tag{25}$$

where

$$\begin{cases} \alpha' = \alpha = \dfrac{9}{16}\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left[a_3^2 + \left(b_3\omega + \dfrac{c_3}{\omega^3}\right)^2\right], \\ \beta' = \beta - \dfrac{3}{4}\left(b_3\omega + \dfrac{c_3}{\omega^3}\right) = \dfrac{3}{2}\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)\left[a_1 a_3 - \left(b_1\omega - \dfrac{c_1}{\omega^3}\right)\left(b_3\omega + \dfrac{c_3}{\omega^3}\right)\right] + \\ \quad + \dfrac{3}{4}a_3\Im m\left(\underline{A}_{11}\underline{A}_{22}^* + \underline{A}_{12}^*\underline{A}_{21}\right) + \dfrac{3}{4}\left(b_3\omega + \dfrac{c_3}{\omega^3}\right)\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right) + 1\right], \\ \gamma' = \gamma + b_1\omega - \dfrac{c_1}{\omega^3} = \left[a_1^2 + \left(b_1\omega - \dfrac{c_1}{\omega^3}\right)^2\right]\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right) + a_1\Im m\left(\underline{A}_{11}\underline{A}_{22}^* + \underline{A}_{12}^*\underline{A}_{21}\right) + \\ \quad + \left(b_1\omega - \dfrac{c_1}{\omega^3}\right)\left[\Re e\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}^*\underline{A}_{21}\right) + 1\right] + \Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right). \end{cases} \tag{26}$$

Just like (12), eq. (25) is an algebraic biquadratic one, in I_{2m} , and the overall resonance regime can be realized only for real and positive values of this eq.’s roots. These roots are

$$I_{2m} = \pm\sqrt{\frac{-\beta' \pm \sqrt{\beta'^2 - 4\alpha'\gamma'}}{2\alpha'}}, \tag{27}$$

with α', β', γ' given by (26). In this case too only the sign “+” before the external radical may be considered, from an evident physical point of view.

From a formal point of view relations (14) and (27) are analogous (relation (14) representing the eq.’s (12) roots) the analysis of roots (27) leads to conclusions similar to those obtained in § **2** namely:

a) if relation

$$\beta'^2 - 4\alpha'\gamma' > 0,\ \operatorname{sign}\left(-\beta' \pm \sqrt{\beta'^2 - 4\alpha'\gamma'}\right) = -\operatorname{sign}\alpha' \tag{28}$$

are satisfied then two different overall resonance may be realized;

b) if the inequality (28_1) and only one of relations

$$\operatorname{sign}\left(-\beta' + \sqrt{\beta'^2 - 4\alpha'\gamma'}\right) = \operatorname{sign}\alpha',\ \text{but}\ \operatorname{sign}\left(-\beta' - \sqrt{\beta'^2 - 4\alpha'\gamma'}\right) = -\operatorname{sign}\alpha' \tag{29′}$$

or

$$\operatorname{sign}\left(-\beta' + \sqrt{\beta'^2 - 4\alpha'\gamma'}\right) = -\operatorname{sign}\alpha',\ \text{but}\ \operatorname{sign}\left(-\beta' - \sqrt{\beta'^2 - 4\alpha'\gamma'}\right) = \operatorname{sign}\alpha' \tag{29″}$$

are satisfied, then only one overall resonance is possible to realize;

c) in remainder cases isn’t possible to realize any overall resonance.

In the particular case when

$$\beta'^2 = 4\alpha'\gamma' \tag{30}$$

it is possible to realize one single overall resonance; in this last case is necessary to satisfy, in addition, the relation

$$\operatorname{sign}\beta' = -\operatorname{sign}\alpha'. \tag{31}$$

NIPR's equivalent parameters expressions, when the overall resonance(s) is (are) realized, can be obtained substituting in relations (11) the expression (27) of the secondary current's amplitude, I_{2m} . Performing the calculus it results

$$\begin{cases} R_2 = a_1 + \dfrac{3\left(-\beta' \pm \sqrt{\beta'^2 - 4\alpha'\gamma'}\right)}{2\alpha'}, \\ X_2 = b_1\omega - \dfrac{c_1}{\omega} - 3\left(b_3\omega + \dfrac{c_3}{\omega^3}\right)\dfrac{-\beta' \pm \sqrt{\beta'^2 - 4\alpha'\gamma'}}{2\alpha'}. \end{cases} \tag{32}$$

When relation (30) is satisfied expressions (32) become more simply namely

$$R_2 = a_1 - \frac{3\beta'}{2\alpha'};\ X_2 = b_1\omega - \frac{c_1}{\omega} + 3\beta'\frac{b_3\omega + c_3/\omega^3}{8\alpha'}. \tag{33}$$

O b s e r v a t i o n. Restrictions (15), (22) or (23) and (27) indicate the fact that either the resonance at the input gate of an LNPT, or the resonance at the output gate or, also, the overall resonance, may be realized in certain frequency ranges or just at certain frequencies.

5. Conclusions

The possibilities of resonance realization at the access gates of a linear, non-autonomous and passive two-port, in harmonic steady-state, are investigated when the two-port's receiver is a passive non-linear inertial one, as the case represented in Fig. 1. Three different situations are considered: a) the resonance at the input gate; b) the resonance at the output gate and c) the overall resonance. In the first case at most two different resonances are possible, in the second case at most one resonance can be realized and in the third case at most two different resonances may be realized.

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REGIMURILE DE REZONANȚĂ LA PORȚILE DE ACCES ALE UNUI CUADRIPOL DIPORT LINIAR, NEAUTONOM ŞI PASIV, CARE ALIMENTEAZĂ, ÎN REGIM PERMANENT ARMONIC, UN RECEPTOR PASIV NELINIAR INERȚIAL

(Rezumat)

Se studiază posibilitatea realizării rezonanței la porțile de acces ale unui cuadripol diport liniar, neautonom şi pasiv, care alimentează, în regim permanent armonic, un receptor pasiv constituit din gruparea în serie a unui rezistor, unei bobine şi unui condensator, toate trei neliniare inerțiale. Se demonstrează că se pot realiza: a) cel mult două regimuri distincte de rezonanță la poarta de intrare; b) cel mult un singur regim de rezonanță la poarta de ieşire; c) cel mult două regimuri distincte de rezonanță globală.