BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LIX (LXIII), Fasc. 2, 2013
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

COMPONENT RECURSIVE SYSTEMATIC CONVOLUTIONAL CODE ANALYSIS IN A SYMMETRIC TURBO-CODED DECODE-AND-FORWARD RELAY CHANNEL

BY

ALEXANDRA SAVIN[*] **and LUCIAN TRIFINA**

“Gheorghe Asachi” Technical University of Iaşi
Faculty of Electronics, Telecommunications and Information Technology

Received: May 2, 2013
Accepted for publication: June 28, 2013

Abstract. In this paper we present a component code analysis for the turbo-coded decode-and-forward relay system by comparing the use of different generator matrices with both primitive and non-primitive feedback polynomial generators for source and relay, respectively on Additive White Gaussian Noise (AWGN) channels and flat Rayleigh fading channels. For fixed signal to noise (SNR) source-relay and relay destination channels, the primitive feedback polynomial does not always offer better bit error rate (BER) performances at high SNR source destination as it does for the classical turbo coding scheme at high SNR. These performances depend on the memory of the convolutional code, the non-primitive feedback polynomial leading to an increase in the BER performances in error-floor or waterfall region of BER curves. Simulations for codes with memory 2, 3 and 4 enforce this statement.

Key words: turbo code; generator matrices; interleaver; cooperative diversity.

1. Introduction

The need for increased channel capacities in wireless, and not only, networks lead to the development of the technique named *cooperative diversity*.

---

[*] Corresponding author: *e-mail*: asavin@etti.tuiasi.ro

This technique exploits the user diversity for a various range of bandwidths by decoding signals from the relay and the source in a multihop relay network. The difference between a single hop system and a multihop one, is that, in the last case, the receiver decodes the information that comes only from the direct transmission path, the source – destination channel and the relay signal ensuring full diversity.

The classical relay channel system consists of three terminal communication nodes: the source, the destination and the relay (van der Meulen, 1971). Thus for a two-hop relay network the source broadcasts the coded signals to both the destination and the relay. Then the relay decodes the received signals and interleaves them *prior* to encoding. The signals received at the destination consist of coded information symbols transmitted from the source and coded interleaved information symbols transmitted from the relay forming a distributed turbo code (Berrou & Glavieux, 1996).

The decode-and-forward strategy with relay decoding error propagation was chosen as a relaying strategy, because it has the capacity to regenerate the signal by making the relay decode, re-encode and forward the signal (Carleial, 1982). Besides achieving diversity and coding gain, this structure benefits from an interleaving gain due to the turbo code construction and a turbo processing gain given by the iterative decoder (Zhao & Valenti, 2003).

In this paper, we analysed the impact in performance for constituent recursive systematic convolutional codes with both primitive and non-primitive feedback polynomials, when information is transmitted over an Additive White Gaussian Noise (AWGN) channel and a flat Rayleigh fading channel. As a decoding method we used an iterative decoder which does not take into consideration the decoding errors propagated by the relay (Huynh & Aulin, 2012).

The paper is structured in three sections. In section **2** the relay system model is presented, in section **3** we present the recursive systematic convolutional (RSC) codes used for analysis and the simulation results and in section **4** the concluding arguments of this paper are given.

2. The Relay System Model

In Fig. 1 we represent the cooperative diversity scheme.

The system model follows the classical scheme of a cooperative system model that has three terminal communication channels: the source, the relay and the destination. The channels between nodes are AWGN or flat fading Rayleigh channels. In the paper the following notations were made: Signal to Noise Ratios (SNRs) denoted SNR_{sd} for source-destination channel, SNR_{sr} for source-relay channel and SNR_{rd} for relay-destination channel. The system operates in two periods of time: in the first time period the source generates the information bit sequence, i, which goes through an RSC encoder, is Binary Phase Shift Keying (BPSK) modulated and is transmitted to the destination, constituting the first noisy observation sequence, y. In the second time period, the relay demodulates, decodes and generates the detected sequence, *i'*, from the original source sequence, *i*, that is affected by the relay's decoding errors. For decoding, at the relay, the Bahl, Cocke, Jelinek and Raviv (BCJR) decoder was used (Bahl *et al.*, 1974). The destination receives the re-encoded and re-modulated sequence that is represented by the second noise affected observation sequence, *y'*. These two noise affected sequences go through an iterative decoder that uses the Max-Log-MAP decoding algorithm (Robertson *et al.*, 1995), resulting at the destination in the output sequence of data, $\hat{x}$. In the relay system model, π is the permutation that describes the interleaver at relay.

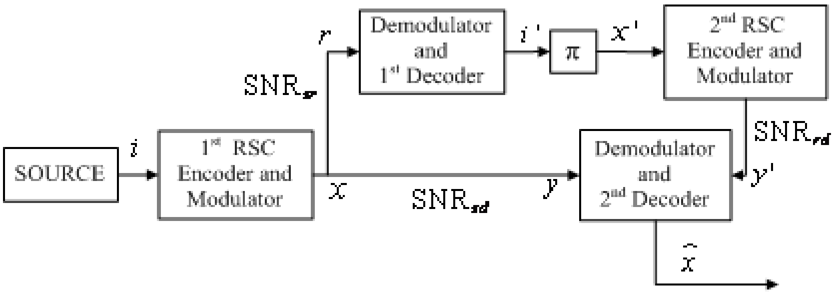


Fig. 1 – The relay system model.

The eqs. for the signals received by the relay and the destination are the following (Sneessens *et al.*, 2008):

$$r = h_{sr}\sqrt{E_s}\,x + z_r\,, \tag{1}$$

$$y = h_{sd}\sqrt{E_s}\,x + z_d\,, \tag{2}$$

where x represents the coded symbols at the source, E_s – the energy per symbol, h_{sr} – the Rayleigh fading coefficient affecting the source-relay signal, h_{sd} – the fading coefficient affecting the source–destination signal, z_r – the AWGN noise sample at the relay and z_d – the AWGN noise sample at the destination. It is assumed that for AWGN channels $h_{sr} = h_{sd} = 1$. The fading coefficients, h_{sr} and h_{sd}, are Rayleigh random variables with variance 1 and the noise signals, z_r and z_d, are random variables of zero mean and two-sided power spectral densities of $N_0/2$.

The encoded and modulated signal is decoded at the relay by the Viterbi or BCJR decoder and then a hard decision is taken. Even if errors occurred after the interleaving the signal is re-encoded and modulated into x'. The information about the state of the source–relay channel can be transmitted by the relay to the destination as channel state information (h_{rd}). The signal received at the destination is given by relation

$$y = h_{rd}\sqrt{E_r}x' + z_d', \tag{3}$$

where: E_r is the average energy per symbol of the signal transmitted by the relay, h_{rd} – the amplitude of the fading, x' is the signal transmitted by the relay and z_d' – the Gaussian noise at the destination. For AWGN channels $h_{rd} = 1$.

Source–destination, source–relay and relay–destination channels are characterized by the signal to noise ratios: SNR_{sd}, SNR_{sr} and SNR_{rd} , respectively.

The decoding algorithm used at the receiver is the Max-log-MAP algorithm (Robertson *et al.*, 1995). The iterative turbo decoding scheme is presented in Fig. 2.

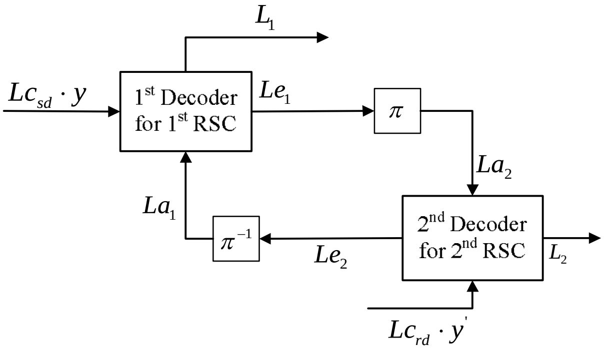


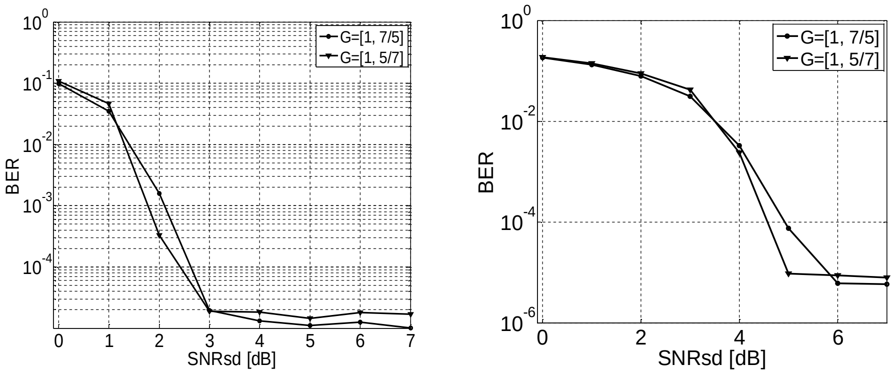
Fig. 2 – The classical iterative turbo decoder for the decode-and-forward relay channel.

The iterative turbo decoder consists of two BCJR decoders that decode the coded information from the two RSC encoders of the source and relay, a random interleaver, π, and its corresponding deinterleaver, π^{-1}. The received sequence corresponding to the information bits of the 1st RSC are different from the received sequence corresponding to the information bits of the 2nd RSC. This fact is justified because of the propagation of decoding errors at the relay, unlike in the classical turbo decoding scheme. In Fig. 2, $Lc_{sd}\, y$ is the received sequence from the source, scaled with source-destination channel reliability and $Lc_{rd}\, y'$ – the received sequence from the relay, scaled with relay–destination channel reliability. La_1 and La_2 are the decoders input *a priori* logarithmic likelihood ratios (LLRs), Le_1 and Le_2 – the decoders outputs LLRs, representing the extrinsic information and the *a posteriori* LLRs. The outputs of each decoder are denoted by L_1 and L_2. After a given number of iterations, for decision making we will use the *a posteriori* LLR output of the first decoder, L_1.

3. RSC Codes Used for Analysis and Simulation Results

In this section we make an analysis of component RSC codes on bit error rate (BER) performances. The simulations were made with the two identical RSC codes for the source and the relay using different code memory and generator matrices for each simulation. The generator matrices have feedback polynomials both primitive and non-primitive. The information block length is equal to 65536 bits and the iteration number is 6.

In Fig. 3 *a*, we plot the BER curves on the AWGN one relay channel for memory two codes with generator matrices: $G = [1, 7/5]$ and $G = [1, 5/7]$. The SNR$_{sr}$ value is 6 dB and the SNR$_{rd}$ value is –7 dB. In Fig. 3 *b* the BER curves on the Rayleigh fading one relay channel using a SNR$_{sr}$ value of 12 dB and a SNR$_{rd}$ value of –7 dB are represented. In Fig. 4 the simulations were done using the RSC codes with the same generator matrices as for Fig. 3 on the AWGN one relay channel and the Rayleigh fading one relay channel, but for the SNR$_{sr}$ value of 6 dB and the SNR$_{rd}$ value of 2 dB.



a *b*

Fig. 3 – BER curves for the memory 2 RSCs with SNR$_{rd}$ = –7 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b* – the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.

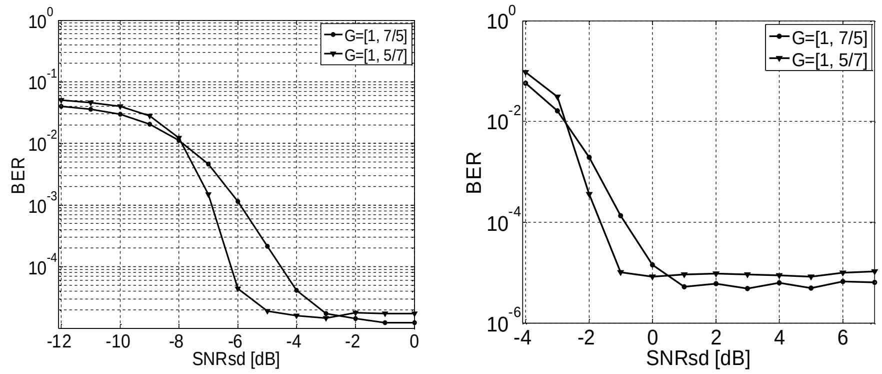
In Figs. 5 and 6 we plot the BER curves for the same SNR$_{sr}$ and SNR$_{rd}$ values used for the two channels, but the memory 3 RSC code generator matrices were $G = [1, 15/17]$, $G = [1, 17/15]$ and $G = [1, 15/13]$, respectively.

In Figs. 7 and 8 we plot the BER curves for the same SNR$_{sr}$ and SNR$_{rd}$ values for the two channels using three RSC codes of memory 4 with generator matrices $G = [1, 21/37]$, $G = [1, 27/31]$ and $G = [1, 35/23]$, respectively.

The primitive polynomials in octal form are 7, 13, 15, 23, 31 and the non-primitive polynomials in octal form are 5, 17, 27 and 37.

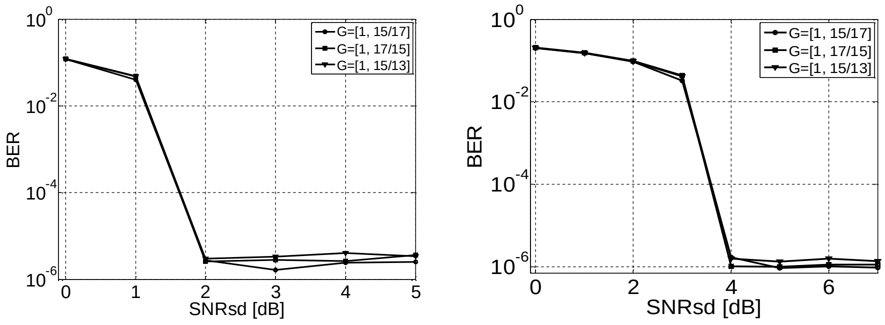
The simulations were run for a sufficiently high value of the SNR$_{sr}$ in order to ensure the propagation of fewer relay decoding errors obtaining a small BER for the given SNR$_{sr}$ after the destination decoding.

From Figs. 3 *a* and 3 *b* it can be seen that for medium SNR$_{sd}$ values (between 1 dB to 3 dB for the AWGN channel and between 3 dB to 5.8 dB for the Rayleigh fading channel) RSCs with the primitive feedback polynomial, 7, have better performances (a supplementary coding gain up to 0.35 dB for AWGN channel and up to 0.7dB for Rayleigh fading channel) than the RSC with the non-primitive feedback polynomial, 5. For higher SNR$_{sd}$ values (greater than 3 dB for AWGN channel and greater than 6 dB for Rayleigh fading channel) performances are better using the RSC with the non-primitive feedback polynomial (lower error-floor or low BER values).



a *b*

Fig. 4 – BER curves for the memory 2 RSCs with SNR$_{sd}$ = 2 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b* – the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.



a *b*

Fig. 5 – BER curves for the memory 3 RSCs with SNR$_{rd}$ = –7 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b*– the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.

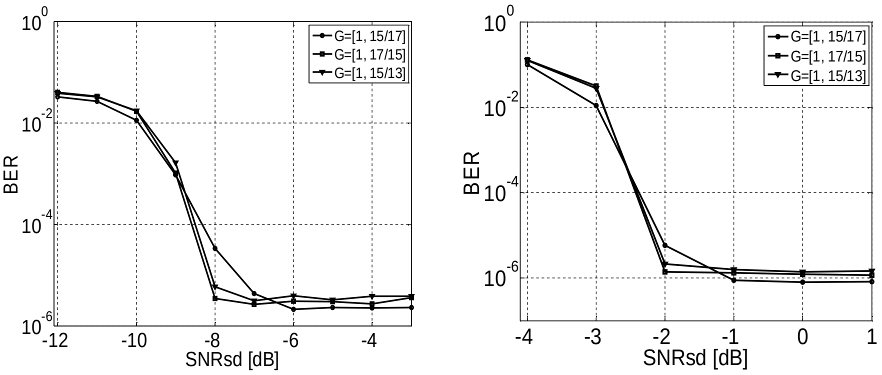
From Fig. 4 it can be seen that for low and high SNR$_{sd}$ values, *i.e.* between –12 dB to –8 dB and from –2.2 dB to 0 dB for the AWGN channel and between –4 dB to –3 dB and from –1.2 dB to 7 dB for the Rayleigh fading channel, RSCs with the non-primitive feedback polynomial, 5, have better performances than the RSC with the primitive feedback polynomial, 7. The first

SNR range corresponds to low SNR and the last SNR range corresponds to high SNR (error-floor region). For medium SNR$_{sd}$ values, *i.e.* –8 dB to –2.3 dB for AWGN channel and –2.5 dB to 0.3 dB for Rayleigh fading channel, performances are better. The supplementary coding gains up to 2 dB for the AWGN channel and up to 1 dB for the Rayleigh fading channel are obtained.

In Figs. 5 and 6 the simulations for memory 3 RSC codes are plotted.

From Fig. 5 it can be seen that for a SNR$_{sd}$ greater than 3 dB for AWGN channels and greater than 4 dB for Rayleigh fading channels, the RSCs with the non-primitive feedback polynomial, 17, have better performances than the RSCs with the primitive feedback polynomial, 15. In the “waterfall” region the coding gain is maintained the same for all generator matrices and for both the AWGN channel and the Rayleigh fading channel.

From Fig. 6 it can be seen that for low and high SNR$_{sd}$ values, *i.e.* between –12 dB to –9 dB and from –6.3 dB to –3 dB for the AWGN channel, between –4 dB to –2.7 dB and from –1.2 dB to 1 dB for the fading Rayleigh channel, RSCs with the non-primitive feedback polynomial, 17, have better performances than the RSC with the primitive feedback polynomials, 15 and 13. For medium SNR$_{sd}$ values, *i.e.* –9 dB to –6.5 dB for AWGN channel and –2.7 dB to –1.2 dB for Rayleigh fading channel, performances are better using the primitive feedback polynomials 15 and 13, resulting in a supplementary coding gain up to 1.1 dB for the AWGN channel and up to 0.6 dB for the Rayleigh fading channel.



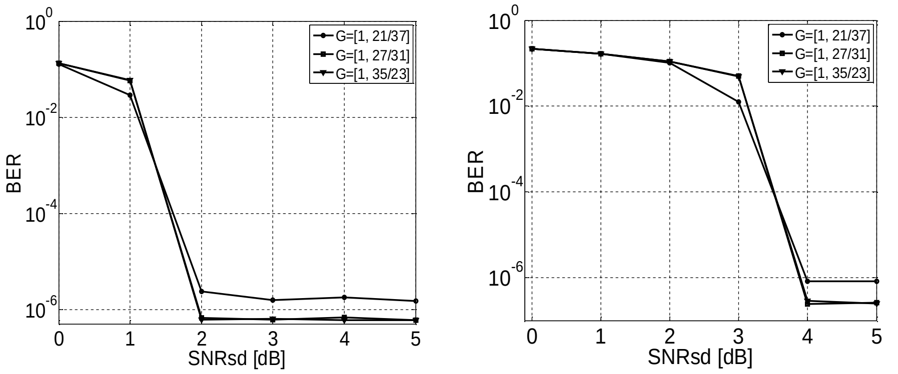
a *b*

Fig. 6 – BER curves for the memory 3 RSCs with SNR$_{rd}$ = 2 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b* – the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.

Simulations for memory 4 RSC codes are plotted in Figs. 7 and 8.

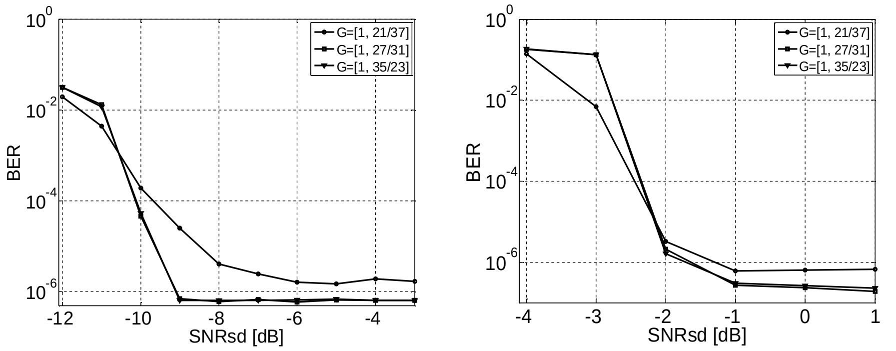
From Fig. 7 it can be seen that for high SNR$_{sd}$ values, *i.e.* greater than 2 dB for AWGN channels and greater than 4 dB for Rayleigh fading channels, the RSCs with the primitive feedback polynomials, 23 and 31, have better performances than the RSC with the non-primitive feedback polynomial, 37.

For medium SNR$_{sd}$ values, *i.e.* 1 dB for AWGN channel and 3 dB for Rayleigh fading channel, performances are better for the RSC with the non-primitive feedback polynomial.



a *b*

Fig. 7 – BER curves for the memory 4 RSCs with SNR$_{rd}$ = –7 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b* – the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.



a *b*

Fig. 8 – BER curves for the memory 4 RSCs with SNR$_{rd}$ =2 dB on *a* – the AWGN for SNR$_{sr}$ = 6 dB and on *b* – the Rayleigh fading channels for SNR$_{sr}$ = 12 dB.

From Fig. 8 it can be seen that for low SNR$_{sd}$ values, *i.e.* between –12 dB to –10.7 dB for the AWGN channel and between –4 dB to –2.3 dB for the Rayleigh fading channel, RSCs with the non-primitive feedback polynomial, 37, have better performances. A supplementary coding gain up to 1.5 dB for the AWGN channel and up to 1 dB for Rayleigh fading channel than the RSC with the primitive feedback polynomials, 31 and 23, is obtained. For higher SNR$_{sd}$ values, *i.e.* greater than –10.5 dB for AWGN channel and greater than –2 dB for Rayleigh fading channel, performances are better using the RSC with the primitive feedback polynomials, 31 and 23.

4. Conclusions

In this paper we present an analysis using RSC codes with generator matrices having specific types of feedback polynomials, namely primitive and non-primitive, at the source and the relay.

We cannot notice the same behavior like in classical turbo codes. BER performances depend on generator matrices, on the memory of the encoder and on SNR values used for the three channels of the relay system.

The simulations showed that for memory 2 and memory 3 component codes at high SNR_{sd} and for a fixed high SNR_{sr} value and a low SNR_{rd} value, performances are better using the RSCs with the non-primitive feedback polynomials, *i.e.* BER is about two times lower. For memory 4 component codes at high SNR_{sd} and the same SNR_{sr} and SNR_{rd} values, performances are better using the RSCs with the primitive feedback polynomials like in the classical turbo codes case, *i.e.* BER is about three-four times lower.

The coding gain in the “waterfall” region for the RSC codes of memory 3 and memory 4 remains the same for both AWGN and Rayleigh fading channel. For memory 2 RSCs with non-primitive feedback polynomials lead to an increase in coding gain of up to 2 dB for the AWGN channel and up to 1 dB for the Rayleigh fading channel.

Acknowledgment. This paper was realized with the support of POSDRU CUANTUMDOC “Doctoral Studies for European Performances in Research and Innovation” ID79407 project funded by the European Social Fund and Romanian Government.

REFERENCES

Bahl L.R., Cocke J., Jelinek F., Raviv J., *Optimal Decoding of Linear Codes for Minimizing Symbol Error Rate*. IEEE Trans. on Inform. Theory, **20**, *2*, 284-287 (1974).

Berrou C., Glavieux A., *Near Optimum Error Correcting Coding and Decoding: Turbo-Codes*. IEEE Trans. Commun., **44**, *10*, 1261-1271 (1996).

Carleial A.B., *Multiple-Access Channels with Different Generalized Feedback Signals.* IEEE Trans. Inform. Theory, **28**, 841-850 (1982).

Huynh K.Q., Tor A., *Improved Iterative Decoders for Turbo-Coded Decode-and-Forward Relay Channels*. IEEE Vehicular Technol. Conf. (VTC Fall), 2012, 1-5, 3-6.

Robertson P., Villebrun E., Hoeher P., *A Comparison of Optimal and Sub-Optimal MAP Decoding Algorithms Operating in the Log Domain.* IEEE Internat. Conf. on Commun., **2**, June 18-22, 1995, 1009-1013.

Sneessens H.H., Louveaux J., Vandendorpe L., *Turbo-Coded Decode-and-Forward Strategy Resilient to Relay Errors*. IEEE Internat. Conf. on Acoustics, Speech and Signal Proc. , March 31 –April 4, 2008, 3213-3216.

Van der Meulen E. C., *Three-Terminal Communication Channels.* Adv. Appl. Prob., **3**, 120-154 (1971).

Zhao B., Valenti M.C., *Distributed Turbo Coded Diversity for Relay Channel.* Electron. Lett., **39**, *10*, 786-787 (2003).

ANALIZA CODURILOR CONVOLUȚIONALE RECURSIVE SISTEMATICE COMPONENTE ÎNTR-UN SISTEM CODAT TURBO DE TIP DECODARE ŞI TRANSMITERE PE UN CANAL CU RELEU

(Rezumat)

Este efectuată o analiză a codurilor componente pentru un sistem de decodare şi transmisie cu releu codat turbo comparând utilizarea diferitelor matrici generatoare atât cu polinoame generatoare primitive, cât şi neprimitive, pentru sursă, respectiv releu, pe canale afectate de zgomot alb aditiv gaussian (AWGN) şi fading Rayleigh. Pentru un raport semnal zgomot (SNR) dat al canalului sursă–releu şi releu–destinație, polinomul de reacție primitiv nu oferă totdeauna cele mai bune performanțe ale ratei erorii de bit (BER) pentru un SNR ridicat al canalului sursă–destinație, aşa cum oferă pentru schema clasică a codării turbo la SNR ridicat. Aceste performanțe depind de memoria codului convoluțional, de polinomul de reacție neprimitiv care duce la o creştere a performanțelor dată de curbele BER în regiunea „error floor” şi „waterfall”. Simulările pentru codurile componente de memorie 2, 3 şi 4 confirmă această afirmație.