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ON THE WORKING CONDITIONS AT EMPTY LOAD AND AT SHORT-CIRCUIT OF A LINEAR, NON-AUTONOMOUS AND RECIPROCAL GENERAL TWO-PORT IN HARMONIC STEADY-STATE

BY

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Abstract. The expressions of equivalent complex impedances at the gate (*1*), (*1′*) of a general linear, non-autonomous and reciprocal two-port are determined when either the (*2*), (*2′*) gate or the (*3*), (*3′*) gate are at empty load or in short-circuit. Thus, the equivalent complex impedances $\underline{Z}_{e10}^{(2)}$, $\underline{Z}_{e1sc}^{(2)}$, respectively $\underline{Z}_{e10}^{(3)}$, $\underline{Z}_{e1sc}^{(3)}$ are defined. The possibility to realize the equalities $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1sc}^{(2)}$ or $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)}$ are studied, determining, in the first case, the corresponding complex impedance $\underline{Z}_3^{'}$, $\underline{Z}_3^{"}$ and in the second case, $\underline{Z}_2^{'}$, $\underline{Z}_2^{"}$. A geometrical interpretation of these situations is also studied.

Key words: linear, non-autonomous and reciprocal two-port; working regime at empty load and in short-circuit; equivalent complex impedances $\underline{Z}_{e10}^{(2)}$, $\underline{Z}_{e1sc}^{(2)}$, $\underline{Z}_{e10}^{(3)}$, $\underline{Z}_{e1sc}^{(3)}$; $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1sc}^{(2)}$, $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)}$ equalities realization.

1. Introduction

In previous two papers (Rosman & Belaus, 1996, 2000), having the

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same title as above (less then the two-port's reciprocal character), were studied the working at empty load and in short-circuit of a linear and non-autonomous general two-port in harmonic steady-state. Especially the condition of equality $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1\mathrm{sc}}^{(2)}$ were established, where $\underline{Z}_{e10}^{(2)}$, $\underline{Z}_{e1\mathrm{sc}}^{(2)}$ represents the complex equivalent impedances at the gate (*1*), (*1′*) when the gate (*2*), (*2′*) is open (empty load) or in short-circuit, the coupling branch between the gates (*1*), (*1′*) and (*2*), (*2′*) being passive.

The aim of this paper is to revise and complete the obtained results concerning the realization of the equalities $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1\mathrm{sc}}^{(2)}$ and to establish the conditions which must be satisfied to realize the equality $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1\mathrm{sc}}^{(3)}$, where $\underline{Z}_{e10}^{(3)}$, $\underline{Z}_{e1\mathrm{sc}}^{(3)}$ represent the equivalent complex impedances at the gate (*1*), (*1′*) when the gate (*3*), (*3′*) is open (empty load), respectively in short-circuit, the receiver's impedance connected at the gate (*2*), (*2′*) being at his turn passive, $\underline{Z}_{e10}^{(2)}$, $\underline{Z}_{e1\mathrm{sc}}^{(2)}$ having the meanings utilized in the cited previous papers. In both cases the two-port is considered as being *reciprocal*.

Let be a linear non-autonomous and reciprocal general two-port (LNRGT – Fig. 1), having, in harmonic steady-state, the eqs., (Sigorsky, 1962):

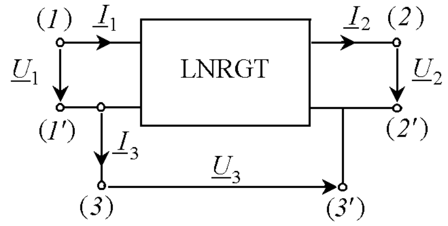

Fig. 1 – Scheme of a linear, non-autonomous and reciprocal general two-port.

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \\ \underline{U}_3 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} & \underline{A}_{13} \\ \underline{A}_{21} & \underline{A}_{22} & \underline{A}_{23} \\ \underline{A}_{31} & \underline{A}_{32} & \underline{A}_{33} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix}, \tag{1}$$

with $\underline{A}_{ij}$, (i, j = 1, 2, 3), his fundamental parameters. At the same time

$$\underline{Z}_2 = \frac{\underline{U}_2}{\underline{I}_2} = R_2 + \mathrm{j}X_2, \quad \underline{Z}_3 = \frac{\underline{U}_3}{\underline{I}_3} = R_3 + \mathrm{j}X_3 \tag{2}$$

represents the complex impedances of passive receivers at the two-port's gates, (*2*), (*2′*) and, respectively, (*3*), (*3′*), while

$$\underline{Z}_{e1} = \frac{\underline{U}_1}{\underline{I}_1} = R_{e1} + \mathrm{j}X_{e1} \tag{3}$$

is the equivalent complex impedance at the two-port's gate (*1*), (*1′*).

If between relations (1),…,(3) are eliminated $\underline{U}_2$, $\underline{I}_2$, $\underline{U}_3$, $\underline{I}_3$ expression

$$\underline{Z}_{e1} = \frac{\left(\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{22}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}} \tag{4}$$

is obtained.

In what follows are called as LNRGT's working regimes at empty load either the one when $\underline{Z}_2 \to \infty$ (and consequently $\underline{I}_2 = 0$), or the one when $\underline{Z}_3 \to \infty$ (and consequently $\underline{I}_3 = 0$). In the first case

$$\underline{Z}_{e10}^{(2)} = \frac{\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}}\underline{A}_{13} \tag{5}$$

and in the second one

$$\underline{Z}_{e10}^{(3)} = \frac{\underline{A}_{11}\underline{Z}_2 + \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_2 + \underline{A}_{22}}. \tag{6}$$

As well the working regimes in short-circuit are either those when $\underline{Z}_2 = 0$ (and consequently $\underline{U}_2 = 0$) or those when $\underline{Z}_3 = 0$ (and consequently $\underline{U}_3 = 0$). In the first case

$$\underline{Z}_{e1sc}^{(2)} = \frac{\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{21}\underline{A}_{33}} \tag{7}$$

and in the second one

$$\underline{Z}_{e1sc}^{(3)} = \frac{\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}. \tag{8}$$

The LNRGT being reciprocal the fundamental parameters satisfy the relations (Sigorsky, 1956)

$$\begin{gathered}\left|\underline{A}\right| = \det \underline{A}_{ij} = \underline{A}_{33},\ (i, j = 1,2,3);\ \underline{A}_{12}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{22} = \underline{A}_{33}, \\ \underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31} = -\underline{A}_{13},\end{gathered} \tag{9}$$

valuable in the case when all the three gates of the considered two-port are access gates with the outside ("hard" general two-port (Rosman, 2008)), or

$$\left|\underline{A}\right| = \det \underline{A}_{ij} = \underline{A}_{33},\ (i, j = 1,2,3);\ \underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21} = 1, \tag{10}$$

in the case when the gate (*3*), (*3′*) is an interior one ("soft" general two-port (Rosman, 2008)).

2. Complex Impedances $\underline{Z}_{e10}^{(2)}$ and $\underline{Z}_{e1sc}^{(2)}$ Equality Conditions

In view to establish the conditions which assure that equality

$$\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1sc}^{(2)} \tag{11}$$

is satisfied, are substituted in (11), $\underline{Z}_{e10}^{(2)}$ and $\underline{Z}_{e1sc}^{(2)}$ with expressions (5), respectively, (7), resulting the equality

$$\frac{\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}} = \frac{\underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\underline{A}_{22}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}, \tag{12}$$

which is equivalent with algebraic eq. of second order

$$\underline{a}\underline{Z}_3^2 + \underline{b}\underline{Z}_3 + \underline{c} = 0. \tag{13}$$

If the case of “hard” LNRGT is considered, having in view (9_1) it results

$$\underline{a} = \underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21},\ \underline{b} = -\underline{A}_{33}\left(1 + \underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}\right),\ \underline{c} = \underline{A}_{33}^2. \tag{14}$$

When the LNRGT is a “soft” one, having in view relations (10) one obtains

$$\underline{a} = 1,\ \underline{b} = -2\underline{A}_{33},\ \underline{c} = \underline{A}_{33}^2. \tag{15}$$

Eq.’s (13) solutions are, in case of “hard” LNRGT

$$Z_{3h}^{'} = \underline{A}_{33},\ Z_{3h}^{''} = \frac{\underline{A}_{33}}{\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}}. \tag{16}$$

where condition

$$\underline{A}_{11}\underline{A}_{22} \neq \underline{A}_{12}\underline{A}_{21} \tag{17}$$

is assumed. In case of “soft” LNRGT eq. (13) has the double root

$$Z_{3s}^{'} = Z_{3s}^{''} = \underline{A}_{33}. \tag{18}$$

Evidently, in order that the studied problem have a physical meaning, having in view that complex impedance $\underline{Z}_3$ is considered, through hypothesis, as being passive, it is necessary that inequalities

$$\Re e\left(\underline{A}_{33}\right) \geq 0,\ \Re e\left[\underline{A}_{33}\left(\underline{A}_{11}^{*}\underline{A}_{22}^{*} - \underline{A}_{12}^{*}\underline{A}_{21}^{*}\right)\right] \geq 0 \tag{19}$$

be satisfied.

The common value of impedances $\underline{Z}_{e10}^{(2)}$ and $\underline{Z}_{e1sc}^{(2)}$ is

$$\underline{Z}_{e1h}^{(2)'} = \frac{\underline{A}_{13}}{\underline{A}_{23}},\ \text{or}\ \underline{Z}_{e1h}^{(2)''} = \frac{\underline{A}_{11}\underline{A}_{33} + \left(\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}\right)}{\underline{A}_{21}\underline{A}_{33} + \left(\underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}\right)}, \tag{20}$$

in the case of "hard" LNRGTs and

$$\underline{Z}_{e1s}^{(2)} = \frac{\underline{A}_{13}}{\underline{A}_{23}}, \tag{21}$$

in the case of "soft" LNRGTs. In order that these expressions have a physical meaning it is necessary, evidently, that the inequalities

$$\Re e\left(\underline{Z}_{e1h}^{(2)'}\right) = \Re e\left(\underline{Z}_{e1s}^{(2)''}\right) \geq 0,\ \Re e\left(\underline{Z}_{e1h}^{(2)''}\right) \geq 0, \tag{22}$$

be satisfied.

3. Complex Impedances $\underline{Z}_{e10}^{(3)}$ and $\underline{Z}_{e1sc}^{(3)}$ Equality Conditions

In this case equality

$$\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)} \tag{23}$$

is satisfied if relation

$$\frac{\underline{A}_{11}\underline{Z}_{2} + \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_{2} + \underline{A}_{22}} = \frac{\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_{2} + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_{2} + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{31}}, \tag{24}$$

occurs, which is obtained through substitution in (23) of complex impedances $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)}$ with expressions (6), respectively (8). Relation (24) leads to algebraic eq. of second order

$$\underline{\alpha}\underline{Z}_{2}^{2} + \underline{\beta}\underline{Z}_{2} + \underline{\gamma} = 0, \tag{25}$$

with

$$\begin{cases} \underline{\alpha} = \underline{A}_{31}\left(\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23}\right), \\ \underline{\beta} = \underline{A}_{32}\left(\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23} - \underline{A}_{31}\right), \\ \underline{\gamma} = -\underline{A}_{32}^{2}, \end{cases} \tag{26}$$

where reciprocity relations (9) were taked into account, considering only the case of "hard" LNRGTs. Eqs. (25) roots are

$$Z_{2h}^{'} = -\frac{\underline{A}_{32}}{\underline{A}_{31}},\ Z_{2h}^{"} = \frac{\underline{A}_{32}}{\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23}}. \tag{27}$$

It was supposed that

$$\underline{A}_{13}\underline{A}_{21} \neq \underline{A}_{11}\underline{A}_{23} \text{ and } \underline{A}_{31} \neq 0. \tag{28}$$

In the particular case when

$$\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23} = -\underline{A}_{31} \tag{29}$$

the roots $\underline{Z}_{2h}^{'}$ and $\underline{Z}_{2h}^{"}$ coincide.

The common value of impedance $\underline{Z}_{e10}^{(3)}$ and $\underline{Z}_{e1\mathrm{sc}}^{(3)}$ is

$$Z_{e1h}^{'} = -\frac{\underline{A}_{13}}{\underline{A}_{22}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{32}},\ \text{respectively}\ Z_{e1h}^{"} = \frac{\underline{A}_{11}\underline{A}_{32} + \underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23}\right)}{\underline{A}_{21}\underline{A}_{32} - \underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{21} - \underline{A}_{11}\underline{A}_{23}\right)}. \tag{30}$$

The considered two-ports being passive it is necessary that

$$\Re e\left(\underline{Z}_{e1h}^{'}\right) > 0,\ \Re e\left(\underline{Z}_{e1h}^{"}\right) > 0. \tag{31}$$

4. Geometrical Interpretations

An analysis of relations (5),…,(8) permits to state that these ones represent *conformal transformations*, (Stoilov, 1964), of circles situated in complex planes $\underline{Z}_2$ or $\underline{Z}_3$ in circles situated in complex planes $\underline{Z}_{e1}$ (strictly speaking $\underline{Z}_{e10}$ or $\underline{Z}_{e1\mathrm{sc}}$). Supposing that the complex impedances $\underline{Z}_2$ and $\underline{Z}_3$ are passive the problem is to determine, using the above mentioned conformal transformastions, of axises $R_2 = 0$, $R_3 = 0$ in circles situated in the complex plane $\underline{Z}_{e1}$, which correspond to complex impedances $\underline{Z}_3 = 0$, respectively,

$\underline{Z}_3 \to \infty$. Thus, in a previous paper (Rosman & Belaus, 2000), were established using this proceeding, circle's equations

$$a'\left(R_{e10}^2 + X_{e10}^2\right) + b'R_{e10} + c'X_{e10} + d' = 0, \tag{32}$$

respectively

$$a''\left(R_{e1sc}^2 + X_{e1sc}^2\right) + b''R_{e1sc} + c''X_{e1sc} + d'' = 0, \tag{33}$$

which correspond to cases when $\underline{Z}_2 \to \infty$, respectively, $\underline{Z}_2 = 0$. The expressions of coefficients a', b', c', d', a'', b'', c'', d'' are given in Appendix.

In the case of "hard" LNRGTs the circles (32), (33) are eventually secant (Fig. 2 *a*) while in the case of "soft" LNRGTs, the respective circles are tangent (Fig. 2 *b*) (exterior or interior).

It exists a domain (hatched in Fig. 2 *a*) where the equivalent complex impedances at empty load and in short circuit may be equal, but for different values of complex impedance $\underline{Z}_3$ (excepting the points *M* and *N*).

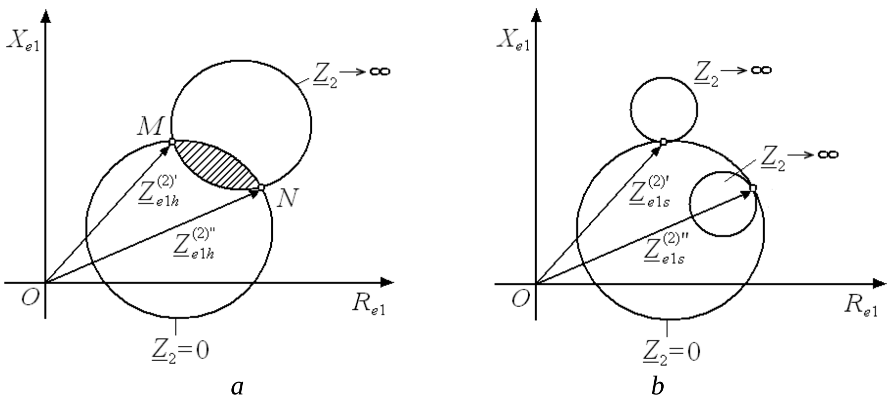

Fig. 2 – Geometrical interpretation of equalities $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1sc}^{(2)}$ and $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)}$.

Analogically may be determined circle's eq. which correspond to cases when $\underline{Z}_3 = 0$ or $\underline{Z}_3 \to \infty$. In this situation relations (6) and (8) which realize the conformal transformation of complex plane $\underline{Z}_3 = R_3 + jX_3$ in to complex plane $\underline{Z}_{e1} = R_{e1} + jX_{e1}$ (more precisely in the complex plane $\underline{Z}_{e10} = = R_{e10} + jX_{e10}$, respectively $\underline{Z}_{e1sc} = R_{e1sc} + jX_{e1sc}$) are utilized. The circle's $\underline{Z}_3 \to \infty$ eq. was established in our Ph. D. dissertation (1968) namely

$$e'\left(R_{e10}^{(2)2} + X_{e10}^{(2)2}\right) + f'R_{e10}^{(2)} + g'X_{e10}^{(2)} + h' = 0. \tag{34}$$

As regards circle's $\underline{Z}_3 = 0$ eq. this one is

$$e''\left(R_{e1sc}^{(2)2}+X_{e1sc}^{(2)2}\right)+f''R_{e1sc}^{(2)}+g''X_{e1sc}^{(2)}+h''=0. \tag{35}$$

Coefficients e', f', g', h', e'', f'', g'', h'' expressions are given in the Appendix.

The circles $\underline{Z}_3 = 0$, $\underline{Z}_3 \to \infty$ are similar to those represented in Fig. 2. Eqs. (32),…,(35) are very useful in the study of resonance regimes at the input gate of an LNRGT working either at empty load or in short circuit. The detailed study of such working regimes will be effectuated in a future work.

5. Conclusions

The study of a linear, non-autonomous and reciprocal general two-port, working at empty load or in short circuit, has allowed

1º The establishing of equivalent complex impedances expressions at the two-port's input gate when either the gate (*2*), (*2′*), or the gate (*3*), (*3′*) is open or in short-circuit ($\underline{Z}_{e10}^{(2)}$, $\underline{Z}_{e1sc}^{(2)}$, $\underline{Z}_{e10}^{(3)}$, $\underline{Z}_{e1sc}^{(3)}$).

2º The establishing of conditions in which either $\underline{Z}_{e10}^{(2)}=\underline{Z}_{e1sc}^{(2)}$ or $\underline{Z}_{e10}^{(3)}=\underline{Z}_{e1sc}^{(3)}$.

3º The sketching of a geometrical interpretation of the obtained results.

Appendix

If the real part of complex impedance

$$\underline{Z}_3=\frac{\left(\underline{A}_{21}\underline{A}_{33}-\underline{A}_{23}\underline{A}_{31}\right)\underline{Z}_{e10}+\underline{A}_{31}\underline{A}_{32}-\underline{A}_{11}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_{e10}-\underline{A}_{11}},\quad\left(\underline{Z}_{e10}\neq\frac{\underline{A}_{21}}{\underline{A}_{11}}\right), \tag{A.1}$$

deduced from relation (5), is annulled, eq. (32) is obtained, where

$$\begin{aligned}
a'&=\Re e\left[\underline{A}_{21}^{*}\left(\underline{A}_{21}\underline{A}_{33}-\underline{A}_{23}\underline{A}_{31}\right)\right],\\
b'&=-2\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)+\Re e\left[\underline{A}_{31}\left(\underline{A}_{11}^{*}\underline{A}_{23}+\underline{A}_{23}\underline{A}_{31}^{*}\right)\right],\\
c'&=2\Re e\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right)+\Im m\left[\underline{A}_{31}\left(\underline{A}_{11}^{*}\underline{A}_{23}+\underline{A}_{23}\underline{A}_{31}^{*}\right)\right],\\
d'&=-\Re e\left[\underline{A}_{11}^{*}\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{11}\underline{A}_{33}\right)\right].
\end{aligned} \tag{A.2}$$

In the same time annulling the real part of complex impedance

$$\underline{Z}_3=\frac{\left(\underline{A}_{13}\underline{A}_{31}-\underline{A}_{23}\underline{A}_{32}\right)\underline{Z}_{e1sc}+\underline{A}_{13}\underline{A}_{32}-\underline{A}_{12}\underline{A}_{31}}{\underline{A}_{22}\underline{Z}_{e1sc}-\underline{A}_{12}},\quad\left(\underline{Z}_{e1sc}\neq\frac{\underline{A}_{12}}{\underline{A}_{22}}\right), \tag{A.3}$$

obtained from relation (6), it results eq. (33) where

$$\begin{aligned}
&a'' = \Re e\left[\underline{A}_{22}^{*}\left(\underline{A}_{22}\underline{A}_{31} - \underline{A}_{23}\underline{A}_{32}\right)\right],\\
&b'' = -2\Re e\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) + \Re e\left[\underline{A}_{32}\left(\underline{A}_{12}^{*}\underline{A}_{23} + \underline{A}_{13}\underline{A}_{22}^{*}\right)\right],\\
&c'' = -2\Re e\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) + \Im m\left[\underline{A}_{32}\left(-\underline{A}_{12}^{*}\underline{A}_{23} + \underline{A}_{13}\underline{A}_{22}^{*}\right)\right],\\
&d'' = -\Re e\left[\underline{A}_{22}^{*}\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right].
\end{aligned} \tag{A.4}$$

As regards the coefficients e', f', g', h', these ones may be determined annulling the real part of complex impedance

$$\underline{Z}_2 = -\frac{\underline{A}_{22}\underline{Z}_{e10}^{(3)} - \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_{e10}^{(3)} - \underline{A}_{11}},\quad \left(\underline{Z}_{e10}^{(3)} \neq \frac{\underline{A}_{11}}{\underline{A}_{21}}\right), \tag{A.5}$$

obtained from (6). It results eq. (34) where

$$\begin{aligned}
&e' = \Re e\left(\underline{A}_{21}\underline{A}_{22}^{*}\right),\ f' = -\Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right),\\
&g' = -\Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} + \underline{A}_{12}\underline{A}_{21}^{*}\right),\ h' = \Re e\left(\underline{A}_{11}\underline{A}_{12}^{*}\right).
\end{aligned} \tag{A.6}$$

It is necessary to underline that coefficients a', b', c', d', a'', b'', c'', d'', were established in a previous work (Rosman & Belaus, 2000), while coefficients e', f', g', h' expressions were determined in the above mentioned author's Ph. D. dissertation.

In view to obtain the coefficients e'', f'', g'', h'' expressions relation (8) is used, from where may be deduced that

$$\underline{Z}_2 = \frac{\left(\underline{A}_{12}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{21}\right)\underline{Z}_{e1sc}^{(3)} + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_{e1sc}^{(3)} + \underline{A}_{11}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{31}}. \tag{A.7}$$

Annuling the real part of expression (A.7), eq. (35) is obtained where

$$\begin{aligned}
&e'' = \Re e\left[\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\left(\underline{A}_{13}^{*}\underline{A}_{31}^{*} - \underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right],\\
&f'' = 2\Re e\left[\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\left(\underline{A}_{11}^{*}\underline{A}_{33}^{*} - \underline{A}_{13}^{*}\underline{A}_{31}^{*}\right) + \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*} - \underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right],\\
&g'' = \Im m\left[-\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\left(\underline{A}_{11}^{*}\underline{A}_{33}^{*} - \underline{A}_{13}^{*}\underline{A}_{31}^{*}\right) + \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}\right)\left(\underline{A}_{23}^{*}\underline{A}_{31}^{*} - \underline{A}_{21}^{*}\underline{A}_{33}^{*}\right)\right],\\
&h'' = \Re e\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{11}^{*}\underline{A}_{33}^{*} - \underline{A}_{13}^{*}\underline{A}_{31}^{*}\right)\right].
\end{aligned} \tag{A.8}$$

It is necessary to observe that coefficients a', b',…, h'' expressions depend exclusively on LNRGT's fundamental parameters $\underline{A}_{ij}$, (i, j = 1,2,3).

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ASUPRA FUNCȚIONĂRII LA GOL ŞI ÎN SCURTCIRCUIT A UNUI CUADRIPOL GENERAL LINIAR, NEAUTONOM ŞI RECIPROC, ÎN REGIM PERMANENT ARMONIC

(Rezumat)

Se determină expresiile impedanțelor echivalente complexe la poarta de intrare a unui cuadripol liniar, neautonom şi reciproc atunci când fie poarta (*2*), (*2′*), fie poarta (*3*), (*3′*) este deschisă sau în scurtcircuit. Se definesc astfel impedanțele echivalente complexe $\underline{Z}_{e10}^{(2)}, \underline{Z}_{e1sc}^{(2)}, \underline{Z}_{e10}^{(3)}, \underline{Z}_{e1sc}^{(3)}$. Se studiază posibilitatea de a fi satisfăcută egalitatea $\underline{Z}_{e10}^{(2)} = \underline{Z}_{e1sc}^{(2)}$ sau $\underline{Z}_{e10}^{(3)} = \underline{Z}_{e1sc}^{(3)}$, determinând, în primul caz, impedanțele complexe (passive) $\underline{Z}_3^{'}$, $\underline{Z}_3^{''}$ iar în cel de al doilea caz impedanțele complexe (de asemenea passive), $\underline{Z}_2^{'}$, $\underline{Z}_2^{''}$, care fac posibile aceste egalități. Se propune o interpretare geometric a acestor cazuri.