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CRITICAL ASPECTS OF THE SIMPLIFIED ORTHOGONAL MODEL IN TERMS OF FLUXES

BY

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Abstract. Based on the orthogonal model of the induction machine in terms of fluxes, through adequate transformations of the eqs. and considering the equality between a stator voltage and a rotor voltage, a simplified equivalent schematic is attained. The paper aims to analyse the relationship between the two voltages and the manner in which the consideration of this equality introduces errors in simulation results, when the stator and rotor resistances are not equal.

Key words: induction machine; two-phase model; equivalent circuit; PSpice.

1. Introduction

The behavior of the induction machines can be described using different mathematical models with distributed or concentrated parameters. The two-phase (orthogonal) model is a very useful model when single-phase, two-phase or even three-phase induction machines are involved.

Different forms of mathematical models were considered. The voltage eqs. use currents and fluxes in order to model the electromagnetic behavior of a winding. But it is possible to express the voltage eqs. only in terms of currents or in terms of fluxes using the well known relation between these quantities. The first approach is the most common in literature (Ong, 1997), while the

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second, that retains only the total fluxes, is newer (Simion, 2008). In each case, an equivalent circuit has been established and different implementations of the model have been suggested in order to simulate them with the aid of different computer programs (Justus,1993).

The approach presented by Cociu & Cociu, (2012) shows the differences between the two types of modelling. A comparison of the voltage eqs. results in a similar format, except for two derivatives in the classical form *versus* only one derivative when total fluxes are used. In order to implement the system of eqs. in PSpice it is necessary to establish the equivalent electrical circuits in both cases (Cociu & Cociu, 1997, 2012). The two circuits give similar but not identical topologies. The model using only currents has one less controlled-voltage source and one less node. In this case, the simulation is a little faster and the output data file a bit smaller.

It is possible to simplify the equivalent schematic as shown by Cociu & Cociu, (2013). In this case, even if the model using only the total flux is employed, the topology of the new equivalent circuit is identical to the classical case of the currents-only model, with the same number of nodes and controlled sources. The output files have the same size and the speed simulation increases.

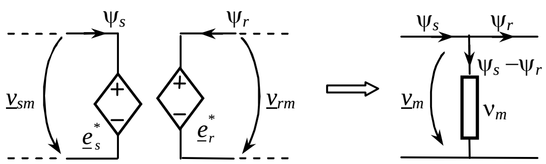

Fig. 1 – The principle of simplifying the equivalent circuit.

The principle of the equivalent circuit simplification is shown in Fig. 1. Using the notations:

$$v_{sm} = \frac{R_s L_m}{\Delta_L},\; v_{rm} = \frac{R_r' L_m}{\Delta_L}, \tag{1}$$

$$\Delta_L = \frac{L_s L_r}{L_m^2},$$

the other notations being the usual ones.

We get a new form of the voltage eqs., in terms of fluxes:

$$\begin{cases} \underline{v}_s = v_{s\sigma r}\underline{\psi}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} + \underline{e}_s^*, \\ \underline{v}_r' = v_{r\sigma s}\underline{\psi}_r' + \dfrac{\mathrm{d}\underline{\psi}_r'}{\mathrm{d}t} + \underline{e}_r^* - \underline{e}_{r\omega}^*. \end{cases} \tag{2}$$

where

$$\underline{e}_s^* = v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \quad \underline{e}_r^* = v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right), \quad \underline{e}_{r\omega}^* = \mathrm{j}\omega_r \underline{\psi}_r^{'} . \tag{3}$$

In order to simplify the equivalent circuit, we must assume the equality of the two voltages:

$$\begin{aligned} \underline{v}_{sm} &= v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \\ \underline{v}_{rm} &= v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right). \end{aligned} \tag{4}$$

If the values of the two resistances, R_s and $R_r^{'}$, are close, then we can consider

$$R_s \approx R_r^{'} \;\rightarrow\; \underline{e}_s^* \approx \underline{e}_s^* \;\rightarrow\; \underline{v}_{sm} \approx \underline{v}_{rm} . \tag{5}$$

Accepting the $\underline{v}_{rm} \cong \underline{v}_{sm} \cong \underline{v}_m$ equality approximation, even when $R_s \neq R_r^{'}$, is similar to accepting errors in final results when using the simplified equivalent circuit.

The paper aims to evaluate the errors introduced by using the simplified equivalent circuit and to analyse the possibilities to control the size of these errors in different circumstances. In the following, we try to establish the operating conditions of the induction machine when the errors introduced by the simplified model are tolerable.

2. Evaluation of the Voltage Difference

Using the simplified model implies accepting the approximation $\underline{\Delta v}_m \approx 0$. In this case, the absolute voltage difference is

$$\begin{aligned} \underline{\Delta v}_m &= v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right) - v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right) = \left(v_{sm} - v_{rm}\right)\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \\ \underline{\Delta v}_m &= \left(R_s - R_r^{'}\right)\frac{L_m}{\Delta_L}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \end{aligned} \tag{6}$$

and the relative voltage difference becomes

$$\frac{\underline{\Delta v}_m}{\underline{v}_{sm}} = \left(R_s - R_r^{'}\right)\frac{L_m}{\Delta_L}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right)\frac{1}{R_s}\cdot\frac{\Delta_L}{L_m}\cdot\frac{1}{\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right)} = \varepsilon_R , \tag{7}$$

where the notation $\varepsilon_R = (R_s - R_r^{'})/R_s$ was used.

Therefore, the relative difference of the stator and rotor resistance, ε_R , is directly and entirely found in the relative voltage difference, $\underline{\Delta v}_m$. For all practical purposes, we are not interested in the relative voltage difference, but in the extent to which this difference influences the machine processes. It is natural to normalize the voltage difference by the stator supply voltage according to the stator voltage eq.:

$$\frac{\Delta V_m}{V_s} = \frac{\left(R_s - R_r^{'}\right)}{R_s} \cdot \frac{R_s L_m}{\Delta_L} \cdot \frac{\left|\underline{\psi}_s - \underline{\psi}_r^{'}\right|}{V_s} \approx \varepsilon_R \frac{R_s L_m}{\Delta_L} \frac{\left|\underline{\Delta\psi}\right|}{V_s}. \tag{8}$$

For correct evaluation of the error introduced by the simplified model, we refer to the well known total flux expressions, in order to calculate $\underline{\Delta\psi}$:

$$\begin{aligned} \underline{\psi}_s &= \underline{\psi}_m + \underline{\psi}_{\sigma s} = \underline{\psi}_s + L_{\sigma s}\underline{i}_s, \\ \underline{\psi}_r^{'} &= \underline{\psi}_m + \underline{\psi}_{\sigma r} = \underline{\psi}_s + L_{\sigma r}^{'}\underline{i}_r^{'}. \end{aligned} \tag{9}$$

The evaluation of the errors must be performed taking the operation conditions of the induction machine into account.

2.1. No Load Operation

For easy understanding of the following reasoning, Fig. 2 shows the flux phasor diagram in two different cases: no load and load operation.

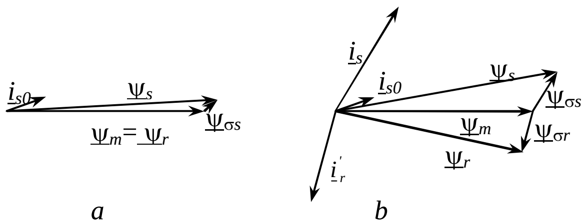

Fig. 2 – Phasor diagram for quantities of interest:
a – no load operation; *b* – load operation.

In no load operation, the rotor current is zero, so:

$$\underline{i}_{r0}^{'} = 0 \quad \rightarrow \quad \underline{\psi}_{\sigma r}^{'} = 0, \tag{10}$$

and the two flux expressions become:

$$\begin{aligned} \underline{\psi}_s &= \underline{\psi}_m + \underline{\psi}_{\sigma s} = \underline{\psi}_s + L_{\sigma s}\underline{i}_{s0}, \\ \underline{\psi}_r^{'} &= \underline{\psi}_m, \end{aligned} \tag{11}$$

where $\underline{i}_{s0}$ is the no load stator current. The difference between the fluxes is

$$\underline{\psi}_s - \underline{\psi}_r^{'} = \underline{\psi}_{\sigma s} = L_{\sigma s}\underline{i}_{s0}. \tag{12}$$

Therefore, the relative difference of the voltages becomes

$$\underline{\Delta v}_m = \varepsilon_R \frac{R_s L_m}{\Delta_L} \underline{\psi}_{\sigma s} . \tag{13}$$

Because the stator current is reduced without a load, the dominant term in the stator voltage eq. is due to the voltage induced by the magnetizing flux. The other terms are noticeably smaller and can be neglected. We assume the approximations

$$V_s \approx \omega \Psi_m, \quad \Psi_{\sigma s} << \Psi_m . \tag{14}$$

Therefore

$$\frac{\Delta V_m}{V_s} = \varepsilon_R R_s \frac{L_m}{\omega \Delta_L} \cdot \frac{\Psi_{\sigma s}}{\Psi_m} . \tag{15}$$

Replacing the Δ_L expression and considering reasonable approximations we get

$$\frac{\Delta V_m}{V_s} = \varepsilon_R \frac{R_s}{\omega(L_{\sigma s} + L'_{\sigma r})} \cdot \frac{\Psi_{\sigma s}}{\Psi_m} . \tag{16}$$

For a typical induction machine, we have: $X_\sigma = 4...6R_s$ and $\Psi_m = 10...25\Psi_{\sigma s}$. In these cases, the relative voltage difference value is 50…150 times smaller than the error introduced by considering the equality of the resistances. In no load operation, the general error of simulation results is negligible when using the simplifying model.

2.2. Full Load Operation

In full load operation we can neglect the magnetizing current towards the stator and rotor currents

$$\underline{i}_s + \underline{i}'_r = \underline{i}_{s0} \approx 0 \quad \rightarrow \quad \underline{i}'_r \approx -\underline{i}_s . \tag{17}$$

The difference between the two fluxes becomes

$$\underline{\Delta\psi} = \underline{\psi}_s - \underline{\psi}'_r = \underline{\psi}_{\sigma s} - \underline{\psi}'_{\sigma r} = L_{\sigma s}\underline{i}_s - L'_{\sigma r}\underline{i}'_r = \left(L_{\sigma s} + L'_{\sigma r}\right)\underline{i}_s . \tag{18}$$

The voltage difference is

$$\underline{\Delta v}_m = \varepsilon_R \frac{R_s L_m}{\Delta_L} \left(L_{\sigma s} + L'_{\sigma r}\right) \underline{i}_s = \varepsilon_R R_s \, \underline{i}_s \tag{19}$$

and the relative voltage difference

$$\frac{\Delta V_m}{V_s} = \varepsilon_R \frac{R_s\, I_s}{V_s}\,. \tag{20}$$

For full load operation of an induction machine, without overload, the voltage drop, R_sI_s, differs from V_s only by a few percents. We expect that, also in this case, making the assumption $R_s \approx R_r^{'}$ wouldn't introduce important errors in numerical simulation.

2.3. Start-Up Operation

The stator and rotor currents are greater than in full load operation, so the magnetizing current is also negligible. Eq. (20) is also valid but the stator current is in this case the short-circuit current

$$I_s = I_{sk} = \frac{V_s}{Z_k}\,, \tag{21}$$

where $\underline{Z}_k$ is the short-circuit complex impedance of the induction machine working in start-up conditions (s = 1). The relative voltage difference becomes

$$\frac{\Delta V_m}{V_s} = \varepsilon_R \frac{R_s}{Z_k}\,. \tag{22}$$

In this situation, the voltage difference, $\Delta \underline{v}_m$, is no longer negligible. Considering the equality between the two voltage drops, $\underline{v}_{rm} \cong \underline{v}_{sm}$ (or resistances, $R_s \approx R_r^{'}$), brings us to a simplified equivalent circuit but introduces important errors in simulation results.

3. Evaluating and Comparing Voltage Differences

We can extend the validity of eq. (20) in all operating conditions without altering the essence of the phenomena. In order to adequately compare the results of numerical simulations, the same orthogonal model of an induction machine as the one utilized by Cociu & Cociu, (2013), has been performed, rated as follows:

$P_n = 5$ kW; $R_s = 1\,\Omega$; $L_{\sigma s} = 8$ mH; $J = 30$ g.m^2;
$U_{1\text{ln}} = 380$ V; $R_r^{'} = 1\,\Omega$; $L_{\sigma r}^{'} = 8$ mH; $F_\alpha = 30$ N.m.s/rad;
$f_1 = 50$ Hz; Y connection; $L_m = 120$ mH; $p = 1$.

First, we try to determine the way in which the machine operating mode influences the values of the voltage drops involved in model simplification. To this end, we considered a slip variation corresponding to angular speed values of 0…350 rad/s. This range is large enough to cover the most important of the operating states encountered in practice, from start-up to generator operation. In Fig. 3 a comparison between the voltage drops, V_m and R_sI_s, is presented.

As we can see, the two curves have similar shapes. Except for the small values of the slip around synchronism speed, the ratio of the two voltage drops seems to be constant. The stator current can be considered a measure of the degree in which the relative difference between the stator and rotor resistance, ε_R , is found in the relative voltage difference, $\Delta V_m / V_s$.

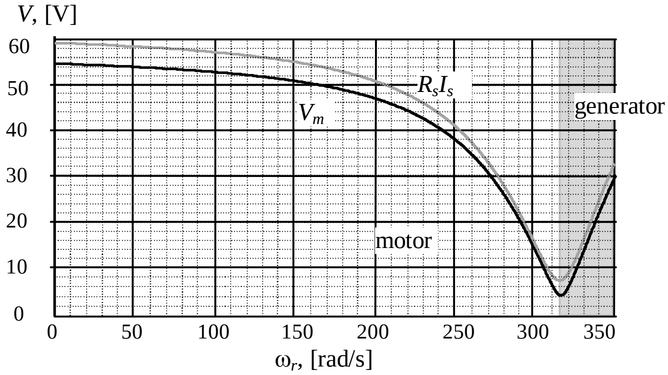

Fig. 3 – Comparison between voltage V_m and drop voltage R_sI_s for different angular rotor speed.

Fig. 4 shows the relative voltage difference *versus* angular rotor speed, for different values of ε_R.

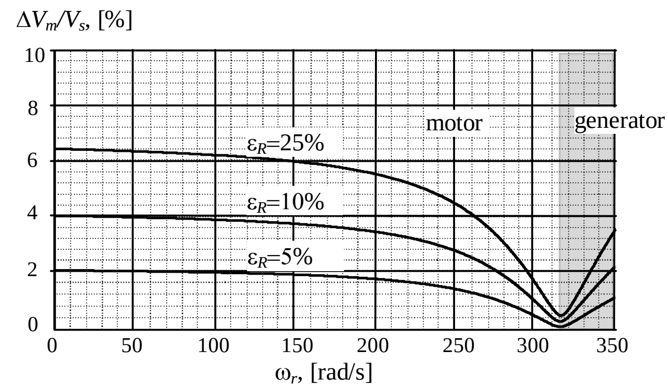

Fig. 4 – The relative voltage difference *versus* angular rotor speed for different values of ε_R.

As expected, the relative voltage difference is similar in shape with the voltage drop, V_m. Smaller values are founded around synchronism and greater values appear when greater values of slip are involved. We can also see that the curves scale with the relative difference between the stator and rotor resistance, ε_R.

4. Conclusions

The PSpice implementation of the two-phase (orthogonal) model in terms of fluxes is very useful for analysing the behaviour of the induction machine. The simplified model based on the simplified equivalent circuit (Cociu & Cociu, 2013) brings clear advantages in the simulation of machine behaviour, concerning simulation speed and output data file size. On the other hand, it can-not be applied without introducing errors, except if the two stator and rotor resistances are close.

The present analysis has emphasized two different situations:

a) If the induction machine operates in normal load (without overload), for small slip values, around the synchronism speed, the difference between the two voltages, $\underline{v}_{sm}$ and $\underline{v}_{rm}$, in the complete equivalent circuit, is insignificant. In this case, even for great values of relative difference between the stator and rotor resistance, ε_R, we can assume the approximation $\underline{v}_{rm} \cong \underline{v}_{sm}$ and use the simplified equivalent circuit with minor simulation errors.

b) For overload operation or start-up operation, when the slip has greater values, it is not possible to assume $\underline{v}_{rm} \cong \underline{v}_{sm}$ without errors in final simulation results. In this situation, we can use the simplified model based on the simplified equivalent circuit, only for equal or close values of the stator and rotor resistances.

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ASPECTE CRITICE PRIVIND MODELUL SIMPLIFICAT AL MAŞINII DE INDUCȚIE UTILIZÂND FLUXURI TOTALE

(Rezumat)

Pe baza modelului ortogonal al maşinii asincrone utilizând exprimarea în fluxuri totale, trecând prin transformări adecvate ale ecuațiilor şi considerând egalitatea între două tensiuni, una statorică şi alta rotorică, s-a stabilit o schemă electrică simplificată. Lucrarea de față efectuează o analiză a relațiilor care se stabilesc între cele două tensiuni şi a modului în care considerarea egalității acestora introduce erori în rezultatele finale ale simulării, în condițiile în care rezistențele statorică şi rotorică nu sunt egale.