BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LIX (LXIII), Fasc. 3, 2013
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

A NEW APPROACH FOR ELECTRIC ENERGY DISTRIBUTION NETWORK ROUTES OPTIMIZATION

BY

BOGDAN NEAGU[*], GHEORGHE GEORGESCU and OVIDIU IVANOV

“Gheorghe Asachi” Technical University of Iaşi
Faculty of Electrical Engineering

Received: May 3, 2013
Accepted for publication: June 29, 2013

Abstract. The optimal routes setting are one of the most complex levels in tree distribution networks design. Taking into account the distribution networks topology, a methodology for the tree distribution network route optimization is proposed. Namely it is proposed an algorithm that, starting from the set of nodes, allows the optimal route construction based on heuristic method (ant colony) completed by a classical method (Steiner method) that optimize the route by added supplementary branch nodes. Finally, in order to highlight the usefulness of proposed methodology, a real distribution network was analysed.

Key words: ant colony optimization; Steiner point; graph theory.

1. Introduction

The “network structure” term does not have a sufficiently precise definition, referring to “network diagram” or “network configuration”, although its significance is much broader (Neagu *et al.*, 2013). Generally, network structures represent internal construction, that is, nodes and lines (Eremia *et al.*, 2006). Also, network structure means the combined electric power sources, consuming nodes (PQ nodes) and branching lines. In public electric energy distribution networks the branching lines are represented by overhead or cable lines and transformer installations (Georgescu *et al.*, 2012). According to the

[*]Corresponding author: *e-mail*: bogdan.neagu@ee.tuiasi.ro

results obtained by Avella *et al*., (2005), can be said that a distribution network has a tree configuration when it does not allow cycles or closed contours, and each node (station, substation or consumer) is supplied with energy through an only way. Usually, distribution networks presents radial configuration, allowing a simple operation for easily faults detected and quickly restored (Neagu *et al.*, 2012).

At the electric networks development, optimal tree networks structures establishing is a complex stage of the design process. In this work a particular approach to optimal route determination and optimal route reconfiguration of an electric energy distribution network (EDN) are presented. This approach is based on a combination of classical and met-heuristic algorithms that use graph theory.

2. Problem Formulation

The proposed problem in this paper is the optimal topology determination among a various alternatives (Hamdaoui *et al.*, 2008). This problem is known in literature as a problem of total cost minimization or minimum spanning tree (MST). A minimalist formulation of the mathematical model for tree networks route optimization problem is: by knowing locations of supply and consuming nodes (Cartesian coordinates) must be determined the network route starting from supply node (first node), then moving only once at each consumer node without cycles taking into account the minimum route length. On the basis of numerous methods for the synthesis of the optimal networks configuration can be formulated the MST problem (minimum length distribution network). In literature, the MST setting is a typical combinatorial optimization problem, and to solve this there are various deterministic methods (Greenberg, 1998; Sudhakar *et al*., 2010) or based on artificial intelligence algorithms (Zhou *et al*., 2006). In order to optimize the tree networks routes, the proposed mathematical model includes two steps, namely:

I. First step is determining the optimal network route construction using all network nodes (a single source node and the all consuming nodes).

II. Second step optimizes length network route resulting minimum from first step by introducing an arbitrary number of additional nodes.

Below, based on graph theory, the two steps are listed, and for the optimization method the goal function is considered as the total minimum length of the network route. The newness of the paper consists in the use of ant colony optimization (met-heuristic) algorithm adapted to graph theory.

2.1. Optimal Network Route Construction Using the Ant Colony Optimization Method Adapted to Graph Theory

Ant colony optimization (ACO) paradigm is included in relatively recent intelligent agents, based on ant’s biological inspiration (Gavrilaş, 2002). By tracking the ants behavior in nature, is found that they can find the shortest path from the anthill to a food source in absence of visual information without direct communication between them; the same ants can be adapted to environmental changes and ACO tries to use real ant skills to solve the optimization problems.

In this section is presented an ACO particular approach (Fig. 1), adapted from graph theory for EDN optimal route determination. The solution uses a graph with n vertex and all edges between these. Each edge (i,k) of the complete graph is associated with a pheromone concentration (τ_{ik}), used for the route choosing by the ant from the colony. Initially, the τ_{ik} are set to small positive values (*i.e.* 0.01). In the algorithm shown in Fig. 1 the minimum route length was initialized with a high value (symbolically denoted $L_{\min} = \infty$). The n_a ants will be distributed as evenly as possible between the graph vertices. In step 2.1 is admitted that the number of vertices and the number of ants is chosen such that $n_a = mn$ (m – integer value), while in each node will distribute $m = n_a/n$ ants. Also, must note that in step 3, Nod_j represents the place where each ant (j) is located at one time.

According to the proposed problem the optimization process contains the restriction that an ant must pass through each node without forming cycles. Each ant route selection is done in tabu list, which contains the elements that describe the sequence of visited nodes (vertices). After the ant's distribution in the graph nodes, the tabu list assigned to each ant will initialize the first position with the order number of the node where that ant was distributed.

Further, the ants should move in different graph nodes, until the tabu lists are complete, each ant making a complete graph tour. In step 6 (Fig. 1), for every ant, j, the starting node, i = Nod$_j$ and destination node, k,* are considered. The k^* node should not be included in tabu list, being determined by computing probabilities using the following expressions

$$P_{ik} = \begin{cases} \dfrac{(\tau_{ik})^{\alpha}(1/d_{ik})^{\beta}}{\sum\limits_{p \notin \text{Tabu}_j} (\tau_{ip})(1/d_{ip})^{\beta}}, & k \notin \text{Tabu}_j, \\ 0, \ k \in \text{Tabu}_j. \end{cases} \tag{1}$$

From the previous relation it can be seen that from all k nodes where is allowed the movement from the node i, will be select node k^*, where the probability (P_{ik}^*) becomes the highest. Therefore, the ant j will move to node k^* (Nod$_j$ = k^*) and k^* will be introduced in tabu list (Tabu$_j$(s) = k^*). Regarding α and β from (1), they control the percentage of pheromone concentration (τ_{ik}) and visibility (1/d_{ik}) to establish the probability. If $\beta = 0$, the P_{ik} probabilities depend only on the concentration pheromone. Also, if $\alpha = 0$ the P_{ik} probabilities depend only on the nodes visibility (distance between nodes).

When all the ants have passed through all the graph nodes, each ant route is closed without returning to the origin node. Practically, this aspect is the ACO algorithm adaptation to the studied problem. Further, according to the

1. **Initial data:** n – nodes; na – ants; (x_i, y_i), $i = 1..n$ – Cartesian coordinate; c – initial pheromone concentration; Q – total pheromones quantity of an ant; ρ - pheromone evaporation rate; $T_{\max}$ – iteration numbers.
2. **General initialization**
   - **2.1.** Set the ants in the nodes: $\boldsymbol{m = na / n}$
   - **2.2.** Set the minimum length route to a very high value: $\boldsymbol{L_{\min}} = \infty$.
   - **2.3.** Set the iteration counter: ***counter*** = 1.
   - **2.4.** The pheromone concentrations initialization on the edges and distance computations:

```
for i = 1 to n do
    for k = 1 to n do
        if i ≠ k then τ_ik = c
        if i < k then d_ik = sqrt[(x_i – x_k)^2 + (y_i – y_k)^2]
            else if i > k then : d_ki = d_ik
```

3. **Select the location nodes for each ant, using a random function**:

```
for j = 1 to na do
    o = (j – 1) div m; Nod_j = o + 1
```

4. **Insert the first node (current node) in tabu list of each ant**:

```
for j= 1 to na do
    Tabu_j(s) = Nod_j
```

5. **Set the current node position in tabu list**: ***s=1***.
6. **Establish routes (completing tabu lists)**:

```
Repeat
    s = s + 1
  for j =1 to na do
      i = Nod_j ; Sum = 0
  for k = 1 to n do
      if k ∉ Tabu_j then
       Sum = Sum + (τ_ik)^α(1 / d_ik)^β
  for k = 1 to n do
      if k ∉ Tabu_j then
       P_ik = (τ_ik)^α·(1 / d_ik)^β/Sum
    else P_ik = 0
       P_max = 0; k* = 0
  for k = 1 to n do
    if k ∉ Tabu_j then
      if P_ik > P_max then
        P_max = P_ik; k* = k ; Oras_j = k* ; Tabu_j(s) = k*
 Until tabu list is full (s = n)
```

7. **Total length computation of each ant route:**

```
for j = 1 to na do          L_k = 0
  for p = 1 to s do
     i = Tabu_j(p); k = Tabu_j(p+1) ; L_k = L_k + d_ik
```

8. **Select the minimum length route:**

```
for j = 1 to na do
         if L_k < L_min then L_min = L_j.
```

9. **Update pheromone concentrations on the graph edges**:

```
for k = 1 to na do
  for p = 1 to s do
      i = Tabu_j(p); k = Tabu_j (p+1); Δτ_ik = Δτ_ik +Q / L_j
  for i = 1 to n do
   for k =1 to n do
      if i ≠ k then
        τ_ik = ρ· τ_ik + Δτ_ik
```

10. **Stopping criterion:**
    - **10.1** Ending the initial tree calculation: **counter** = $\boldsymbol{T_{max}}$.
    - **10.2** Empty the tabu lists:

```
for j = 1 to na do
   for p = 1 to s+1 do
     Tabu_j(p) = 0 and return at five step.
```

11. **Build the minimum tree from previously obtained trees by each ant**. For a ant j:
    - **11.1** Find the lowest edge distance from the tree $\boldsymbol{T^j}$.
    - **11.2** If the edge does not already exist, add to the list.
12. **The length computation of all obtained edges in the previous step list**.
13. **Displays the total minimum length tree**.
14. **End algorithm.**

Fig. 1 – The ACO pseudo-code adapted for determining the optimal route of an electric energy distribution network.

algorithm shown in Fig. 1 the route lengths for all the ants must be calculated and will store the minimum length, which coincides with the final iteration. Before switching to another step, the pheromone concentration must be updated on each graph edge by using the following relationship:

$$\tau_{ik} = \rho\tau_{ik} + \Delta\tau_{ik} , \tag{2}$$

where ρ is a subunit coefficient, from which it results the pheromone evaporation rate on the established routes $(1 - \rho)$. Coefficient ρ always is choosen subunit (ρ = 0.1), because should be avoided unlimited accumulation of pheromones on the graph edges. The $\Delta\tau_{ik}$ term represents the pheromone concentration correction on the edge (i,k) determined by the total ant number who move from the node i to node k, using the relation

$$\Delta\tau_{ik} = \sum_{j=1}^{n_a} \Delta\tau_{ik}^{j} , \tag{3}$$

where $\Delta\tau_{ik}^{j}$ represents the deposited pheromone quantity on edge (i, k) by ant j, determined as follows:

$$\Delta\tau_{ik}^{j} = \begin{cases} \dfrac{Q}{L_j} \text{ if } i,k \in \text{Tabu}_j \text{ and } i = \text{Tabu}_j(p); k = \text{Tabu}_j(p+1), \\ 0, \text{ otherwise.} \end{cases} \tag{4}$$

Stopping criterion (step 10 from Fig.1) coincides with the maximum number of iterations ($T_{\max}$). While $t < T_{\max}$, reset tabu lists of the ants and the procedure is restarted by resetting first element of every tabu list with current node number where each ant is located. ACO algorithm is (Gavrilaş, 2008):

a) *Natural* – based on the ants behavior looking for the shortest route between nest and food source.

b) *Parallel* and *Distributed* – treating an ants population moving simultaneously and independently in the absence of the external control.

c) *Cooperative* – each ant chooses the route based on information added by the other ants (pheromones) that previously had chosen the same route.

d) *Versatile* – easy to apply similar versions to the same problem.

e) *Robust* – can be applied to a variety of combinatorial optimization problems carrying out some minimal changes.

2.2. Distribution Network Route Reconfiguration Using Steiner Method

In the literature, the minimum network length determined through union of the system formed by initial nodes (generally known) and an arbitrary number of nodes newly introduced is known as the generalized Steiner problem.

Considering these aspects, the minimum length tree from all trees with additional nodes is called *minimal Steiner tree*, which results at the searching process among the all trees that can be obtained based on all possible combinations of initial and additional nodes. Of course, these trees types are numerous, their number is $(n + l)^{n+l+2}$, where n represent the initial number nodes and l is the additional number nodes. Thus, for the minimum network length search the mathematical methods based on analysed variables number reduction, are used (Neagu *et al.*, 2013).

For example in a power network, if the power supply node is known, the proposed algorithm is carried out starting from this one. Fig. 2 presents an example of five minimum tree network configurations, and can easily observe that a new additional branching node allows a tree configuration with a total length less than aforementioned. For example, Fig. 3 illustrates a tree formed by three nodes, arranged in an equilateral triangle vertex.

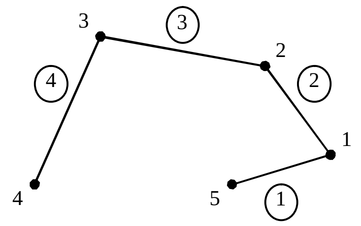


Fig. 2 – Example for minimum length network.

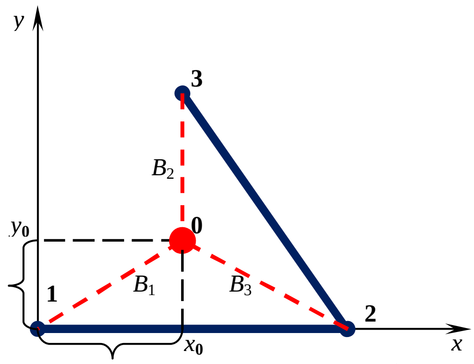


Fig. 3 – Steiner tree construction for three nodes.

The minimum network length construction methods, using the Steiner points, form a whole class of methods based on the following properties of Steiner trees, namely

i) The branches that connect the initial nodes with the Steiner points are arranged at 120 degrees angles.

ii) One Steiner point corresponds to three vertices (nodes); theoretically, the number of Steiner points is unlimited ($0 \leq k \leq n - 2$).

iii) Best solution is for a network with least three nodes and one Steiner point.

Hereinafter the Steiner tree construction using the so-called Euclidean constructions is presented (Fig. 4). For the network with initial nodes, a_1, a_2, a_3, is necessary an additional point, b_1. This point will coincide with one of the given points if any defined angle by the nodes is greater or equal with 120º (if the angle $a_2a_1a_3 \geq 120^{\circ}$, then b_1 coincides with a_1); if all angles are less than 120º, then b_1 is found within the triangle formed by a_1, a_2, a_3 vertices.

The Euclidean construction is obtained as follows: by using one of the branches, for example a_2a_3 , an equilateral triangle is formed and the peak S is situated in opposition with a_1 node in the a_2a_3 edge. Point b_1 will be situated on the circumscribed circle of the triangle. In this case, the Steiner point is situated at the intersection of the circle with the right point Sa_1 called the *equivalent of the* a_2 *and* a_3 *nodes*. The main steps of the minimum length network construction (the optimal route from the economic point of view) in all Steiner method variants are the following:

i) The initial set of nodes is decomposed into subsets which allow Steiner tree construction (for problem size reduction).

ii) For each subset of nodes, using the described procedure a Steiner tree topology is obtained.

iii) Minimum tree length is obtained by aggregating the separate subsets.

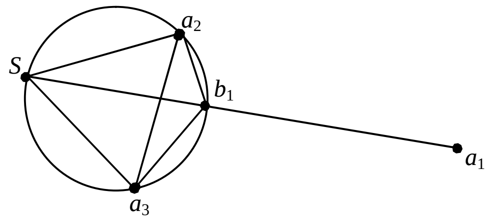


Fig. 4 – Euclidian construction of the Steiner tree.

3. A New Approach for Distribution Network Route Optimization

The choice of economic indicators for various variants estimation of networks routes depends of the nature problem to be solved and the particular characteristics of each design level. Here can also be included the values of total updated expenses, total investment, energy losses, operating costs voltage drops, damages caused by the unpowered consumers, etc.

Radial network synthesis problem is divided in two stages: one uses Prim algorithm building the minimum length network and the second improves the network by adding supplementary branch nodes. The diagram or route network improvement can be achieved by shifting the source nodes to the ends and *vice versa*, with the particularity that the latter would be preferable because the ends power flows are known. By using the mathematical model above described, an arborescence distribution networks route optimization application was developed. The application uses a combination of ACO and Steiner algorithms and the goal function is minimum network length with radial structure restrictions. Fig. 5 shows the flowchart that contains following steps:

a) Input data: general data (source and PQ nodes); consumers data (Cartesian coordinates, input nodes name); the source node is always node 1.

b) Minimum length tree construction with ACO algorithm.

c) Steiner algorithm application for network built in the previous step, by additional branch nodes introduction, called *Steiner points*.

d) Display the partial results (total network length) for the current version, to select the optimal variant. Then, display the network topology and the global minimum network length.

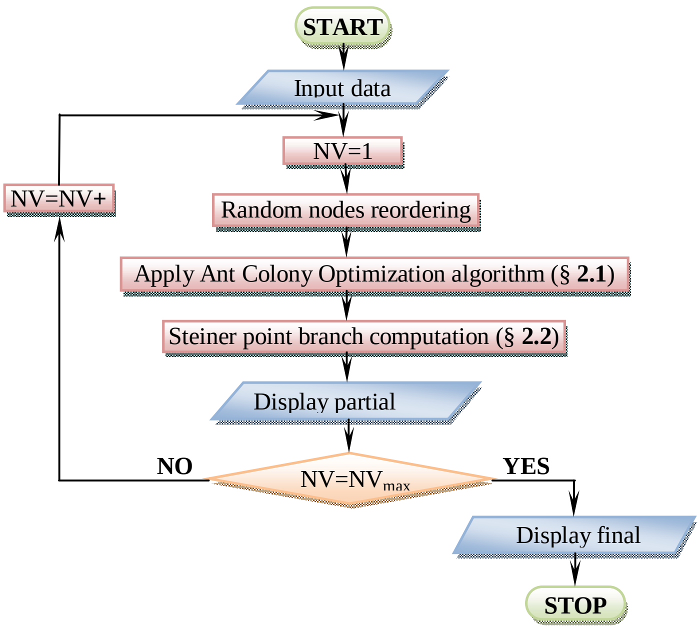


Fig. 5 – The arborescence route network optimization flowchart.

3. Case Example

To highlighten the utility of the algorithm proposed in this paper, in order to optimal route determination of arborescence distribution networks, several medium voltage distribution networks located in Iaşi County were analysed. In this context, Table 1 presents the input data of a real test distribution network with 108 nodes (consumer points). The developed application input data were the nodes defined by consumers (name and Cartesian coordinates of the node).

Based on presented methodology through application of first step with a successive search technique (ACO), the network length results of 5.338 km. By optimizing minimum graph length obtained using ACO, in a second step, to allow new route construction, are needed 36 additional branch nodes (Steiner points), and minimum length resulted is of 4.735 km. The additional Steiner points are summarized in Table 2 such as: added node number, the Cartesian coordinates and the three nodes which could form the Steiner branch point.

Table 1
Cartesian Coordinates of the Test Distribution Network

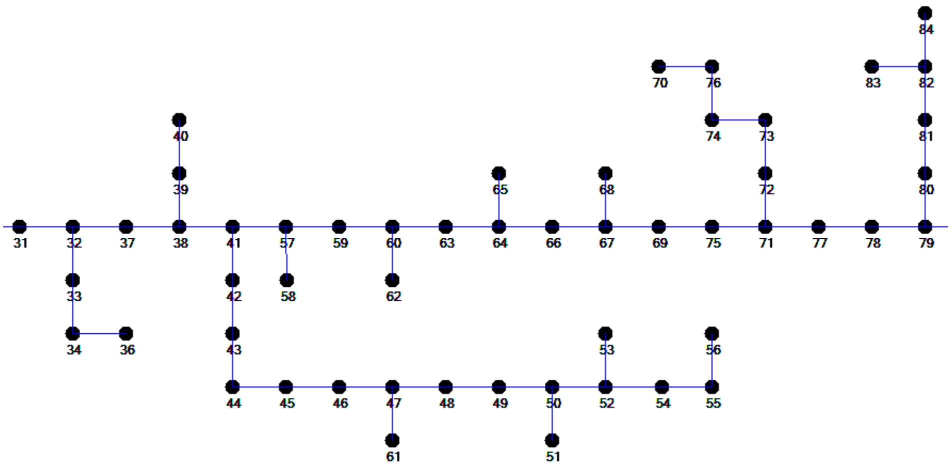
Node	*x*	*y*	Node	*x*	*y*	Node	*x*	*y*	Node	*x*	*y*
1	50	250	28	500	250	55	1,250	400	82	1,450	100
2	100	250	29	550	250	56	1,250	350	83	1,400	100
3	150	250	30	550	200	57	850	250	84	1,450	50
4	150	200	31	600	250	58	851	300	85	1,500	250
5	150	150	32	650	250	59	900	250	86	1,550	250
6	150	100	33	650	300	60	950	250	87	1,550	200
7	200	100	34	650	350	61	950	450	88	1,600	250
8	250	100	35	550	351	62	950	300	89	1,600	300
9	250	150	36	700	350	63	1,000	250	90	1,650	250
10	250	200	37	700	250	64	1,050	250	91	1,700	250
11	250	250	38	750	250	65	1,050	200	92	1,700	200
12	250	300	39	750	200	66	1,100	250	93	1,700	150
13	250	350	40	750	150	67	1,150	250	94	1,700	100
14	300	250	41	800	250	68	1,150	200	95	1,700	50
15	350	250	42	800	300	69	1,200	250	96	1,750	150
16	350	299	43	800	350	70	1,200	100	97	1,800	150
17	350	350	44	800	400	71	1,300	250	98	1,750	50
18	350	400	45	850	400	72	1,300	200	99	1,800	50
19	350	450	46	900	400	73	1,300	150	100	1,850	10
20	300	400	47	950	400	74	1,250	150	101	1,850	50
21	300	451	48	1,000	400	75	1,250	250	102	1,900	50
22	400	250	49	1,050	400	76	1,250	100	103	1,950	50
23	450	250	50	1,100	400	77	1,350	250	104	2,000	50
24	402	300	51	1,100	450	78	1,400	250	105	2,000	100
25	402	348	52	1,150	400	79	1,450	250	106	2,000	150
26	500	300	53	1,150	350	80	1,450	200	107	2,050	150
27	500	350	54	1,203	400	81	1,450	150	108	2,050	200

Table 2
Additional Branch Points (Steiner Points) Necessary in the Analysed Network

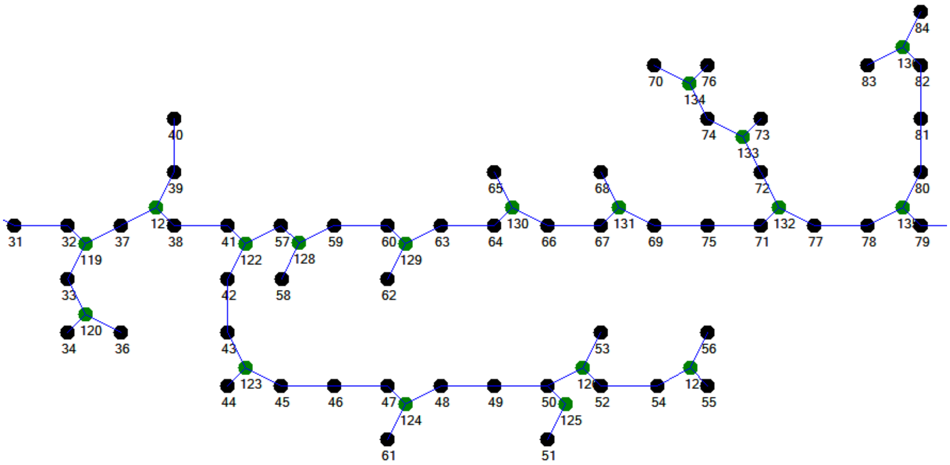
No of Steiner point	Steiner point coordinates		Nodes which form the Steiner point			No of Steiner point	Steiner point coordinates		Nodes which form the Steiner point		
	x	*y*					*x*	*y*			
109	142	241	2	3	4	127	1,241	390	54	55	56
110	174	123	5	6	7	128	874	274	57	58	59
111	239	123	7	8	9	129	975	274	60	62	63
112	272	273	11	12	14	130	1,075	242	64	65	66
113	340	272	14	15	16	131	1,174	242	67	68	69
114	341	423	18	19	20	132	1,324	242	71	72	77
115	342	441	18	19	21	133	1,291	174	72	73	74
116	425	274	22	23	24	134	1,242	123	70	74	76
117	524	341	26	27	35	135	1,441	241	78	79	80
118	573	241	29	30	31	136	1,441	90	82	83	84
119	674	274	32	33	37	137	1,574	242	86	87	88
120	673	341	33	34	36	138	1,624	274	88	89	90
121	740	240	37	38	39	139	1,692	240	90	91	92
122	824	274	41	42	57	140	1,724	174	92	93	94
123	824	391	43	44	45	141	1,724	74	94	95	98
124	973	424	47	48	61	142	1,874	44	100	101	102
125	1,123	424	50	51	52	143	1,990	74	103	104	105
126	1,141	390	50	52	53	144	2,024	142	105	106	107

The software developed has a friendly interface and allows user to upload data both the keyboard and from a .txt file, and display the network from graphical point of view. Fig. 6 shows a network zone (network route between 31 and 84 nodes), in order to view the two steps for route optimization of the electric energy distribution networks, in a radial or arborescence configuration.

Following the real distribution networks analysis it is found that through the proposed methodology usage, the route of the analysed network is reduced to 603 m. Taking into account such considerations, it should be noted that for rural distribution electrical networks from our country, which may have long lengths (tens of kilometers), the algorithm proposed in this paper proves to be an effective tool in design process, leading to a minimization of route length.



a



b

Fig. 6 – Arborescence test distribution network route optimization: *a* – ACO algorithm (step 1); *b* – Steiner branch nodes introduction (step 2).

4. Conclusions

The proposed reconfiguration algorithm showed to be capable of finding all optimal tree configurations with a low computational effort. The use of a met-heuristic algorithm (first step) for the tree configuration allowed the reduction of the search space, making the application of the algorithm possible for large distribution systems, with a less computational effort.

In order to optimize the tree networks routes, the mathematical model proposed in this paper includes two steps, namely: a first step consists in determining the minimum length complete graph using all network nodes (a single source node and the all consuming nodes), and a second step corresponds to a length graph optimization (power network route) resulting minimum from the first step by introducing an arbitrary number of additional nodes or Steiner points. The proposed mathematical model, algorithm and software application represents an effective tool in power networks design for optimal route determination of the medium voltage distribution networks.

Also, the proposed algorithms must be completed by various technical restrictions: network routes problems, geographical and ecological barriers, proximity to low current lines, etc.

REFERENCES

Avella P., Villacci D., Sforza A., *A Steiner Arborescence Model for the Feeder Reconfiguration in Electric Distribution Networks*. Eur. J. Oper. Res., **164**, 505–509 (2005).

Eremia M. *et al*., *Electric Power Systems. Electric Networks*. Edit. Acad. Rom., Bucureşti, 2006.

Gavrilaş M., *Inteligenţă artificială şi aplicaţii în energetică*. Edit. „Gh. Asachi”, Iaşi, 2002.

Georgescu Gh., Neagu B., *Proiectarea şi exploatarea asistată de calculator a sistemelor publice de repartiţie şi distribuţie a energiei electrice*. Vol. **2**, Edit. PIM, Iaşi, 2012.

Greenberg H.. *Greedy Algorithms for Minimum Spanning Tree*. Univ. of Colorado at Denver, www.cudenver.edu/hgreenbe/, March 28, 1998.

Hamdaoui H., Hadjeri S., Zeblah A., *A New Constructive Method for Electric Power System Reconfiguration Using Ant Colony*. Leonardo Electronic J., Practices and Technol., **12**, 49-60 (2008).

Neagu B., Georgescu Gh., *Optimization Possibilities for Radial Electric Energy Distribution Network Routes*. Bul. Inst. Politehnic, Iaşi, **LIX (LXIII)**, *1*, s. Electrot., Energ., Electron. , Iaşi, 133-142 (2013).

Neagu B., Georgescu G., Elges A., *Monitoring System of Electric Energy Consumption to Users*. Internat. Conf. of Electr. a. Power Engng., Iaşi, 2012, 265-270.

Sudhakar T.D. *et al.*, *A Graph Theory – Based Distribution Feeder Reconfiguration for Service Restoration*. Internat. J. of Power a. Engng. Syst., **30**, *3*, 161-168 (2010).

Zhou G. *et al.*, *A Genetic Algorithm Approach on Capacitated Minimum Spanning Tree Problem*. Internat. Conf. on Comput. Intell. a. Secur., 2006, 215–218.

O NOUĂ ABORDARE PRIVIND OPTIMIZAREA TRASEELOR REȚELELOR ELECTRICE DE DISTRIBUȚIE

(Rezumat)

Stabilirea configurațiilor optime ale rețelelor de distribuție reprezintă una din cele mai complexe etape în procesul de proiectare şi exploatare al acestora. Evoluția tehnicii de calcul în direcția unor performanțe remarcabile a influențat direct perfecționarea modelelor şi metodelor matematice şi a algoritmilor de calcul privind proiectarea şi exploatarea sistemelor de distribuție a energiei electrice.

Ținând seama de structura rețelelor de distribuție, se propune o metodologie de optimizare a configurației rețelelor de distribuție radiale. În acest sens se propune un algoritm care, plecând de la mulțimea nodurilor definite de consumatori, permite construirea traseului optim, pe baza unei metode euristice (algoritmul furnicii) completat de o metodă clasică (metoda Steiner) care reoptimizează traseul prin introducerea unor noduri suplimentare de ramificație. În finalul lucrării, pentru a evidenția utilitatea algoritmului propus, s-a analizat o rețea reală de distribuție de medie tensiune.