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# A NEW ANALOGUE CHAOTIC SYSTEM FOR NOISE GENERATION

BY

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**Abstract.** The present paper introduces a novel analogue nonlinear circuit that exhibit rich dynamics. The proposed fourth order circuit is based on a piecewise linear algebraic nonlinear function, with discontinuity in the origin. The dynamic part of the system is developed by closing two feedback loops around a differential amplifier, which drives the nonlinear circuit. For a wide range of parameter values, the proposed system behaves chaotically, exhibiting dissipative property, sensitivity to the initial condition, unpredictable time evolution and ergodicity. These properties are fit for designing generators of analogue noise with flat power spectrum in a defined frequency band. The paper presents both system analysis properties of the proposed nonlinear system and simulation results, confirming the presented theoretical aspects.

**Key words:** chaotic dynamics; nonlinear circuits; noise generators.

## 1. Introduction

The field of nonlinear chaotic circuits has exerted increasing attention to the research community in the last decades, due to both interesting theoretical

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results and possible engineering applications (Kennedy, 1994; Sparrow, 1983; Ueta & Chen, 2000; Aguirre & Bilings, 1994; Grigoraş & Grigoraş, 2005a, 2005b). One application of interest is analogue noise generation, due to the possibility of generating true random signals with ergodic properties for test and measurement equipment, communication security and spread spectrum clock generation. Many research results reported are based on analog implementations (Stojanovsky & Kocarev, 2001; Udaltsov *et al*., 2002; Yalcin *et al*., 2004) or fixed point digital systems (Leon *et al*., 2001; Yang *et al*., 2004).

In the present paper we propose an alternative fourth order analogue nonlinear system, exhibiting rich dynamics, based on two feedback loops around a differential amplifier and a piecewise nonlinear function, aiming at noise generation applications. In the following we will review some system analysis properties of the proposed circuit, present simulation results and highlight conclusions on possible application areas and future research.

## 2. System Analysis of the Nonlinear Circuit

The proposed system, aiming at noise generation applications, is based on a double feedback loop circuit, around a differential amplifier, which drives a nonlinear circuit, described by the algebraic function

$$f(x) = -x + \mathrm{sign}(x), \tag{1}$$

depicted in Fig. 1, where the sign($x$) function, denotes the '***signum***' function, implementable using a single comparator:

$$\mathrm{sign}(x) = \begin{cases} -1 \text{ for } x < 0, \\ 0 \text{ for } x = 0, \\ 1 \text{ for } x > 0. \end{cases} \tag{2}$$

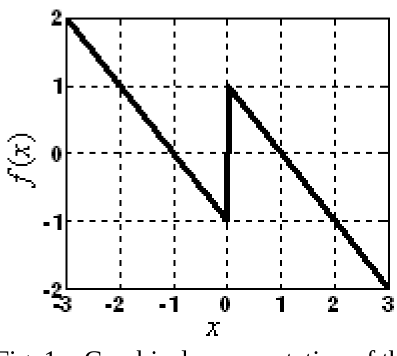


Fig. 1 – Graphical representation of the algebraic nonlinear function.

The positive feedback loop around the algebraic part of the circuit is implemented using a first order high pass filter, described by the transfer function

$$H(s) = \frac{s}{s+\alpha}. \tag{3}$$

The negative feedback counterpart contains a second order high pass filter

$$H(s) = \frac{s^2}{s^2 + 2\beta s + \omega_0^2}. \tag{4}$$

The differential amplifier has a gain, $G$, a fixed pole, normalized at $s_0 = -0.1$ and an output characteristic with limitation at the unitary value of the saturation voltage.

Taking into account the presented system structure, the state eqs.

$$\begin{cases} x_1' = -0.1x_1 - \alpha x_2 + 2\beta x_3 + \omega_0^2 x_4, \\ x_2' = -0.1Gx_1 + \mathrm{sign}(x_1) - \alpha x_2, \\ x_3' = -0.1Gx_1 + \mathrm{sign}(x_1) - 2\beta x_3 - \omega_0^2 x_4, \\ x_4' = x_3, \end{cases} \tag{5}$$

can be deduced.

The equilibrium points of the resulting system, (5), can be obtained from the condition

$$\mathbf{x}'(t) = 0, \tag{6}$$

where

$$\mathbf{x} = [x_1 \quad x_2 \quad x_3 \quad x_4]^T. \tag{7}$$

is the state variable vector, with $[\ \dots\ ]^T$ denoting the transposed vector-matrix.

This leads to the algebraic system of eqs.

$$\begin{cases} -0.1x_1 - \alpha x_2 + 2\beta x_3 + \omega_0^2 x_4 = 0, \\ -0.1Gx_1 + \mathrm{sign}(x_1) - \alpha x_2 = 0, \\ -0.1Gx_1 + \mathrm{sign}(x_1) - 2\beta x_3 - \omega_0^2 x_4 = 0, \\ x_3 = 0. \end{cases} \tag{8}$$

The system (8) yields the unique null solution, $\mathbf{x} = 0$. The simulation results in the following section show that the unique null equilibrium point is unstable for the parameter sets of interest.

Further analysing the resulting state eqs., we can easily deduce that the nonlinear system is dissipative, for positive values of the coefficients,

$$\nabla f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} + \frac{\partial f_4}{\partial x_4} = -0.1 - \alpha - 2\beta. \tag{9}$$

Actually, the positive condition can be relaxed to

$$0.1 + \alpha + 2\beta > 0, \tag{10}$$

although, from the implementation point of view, negative coefficients are not practical due to the stability condition imposed to the two linear filters.

The other two conditions for a nonlinear system to behave chaotically, namely ergodicity and sensitivity to initial conditions, are not deducible from the state eqs. and will be estimated in the following section, by numerical simulation.

## 3. Simulation Results

In order to identify the types of nonlinear dynamics of the proposed system and choose the right parameter values for the coefficients of the eqs. in (5), for the desired chaotic behavior, bifurcation diagrams were plotted. In Fig. 2 we give examples of the bifurcation diagram, performed at the variation

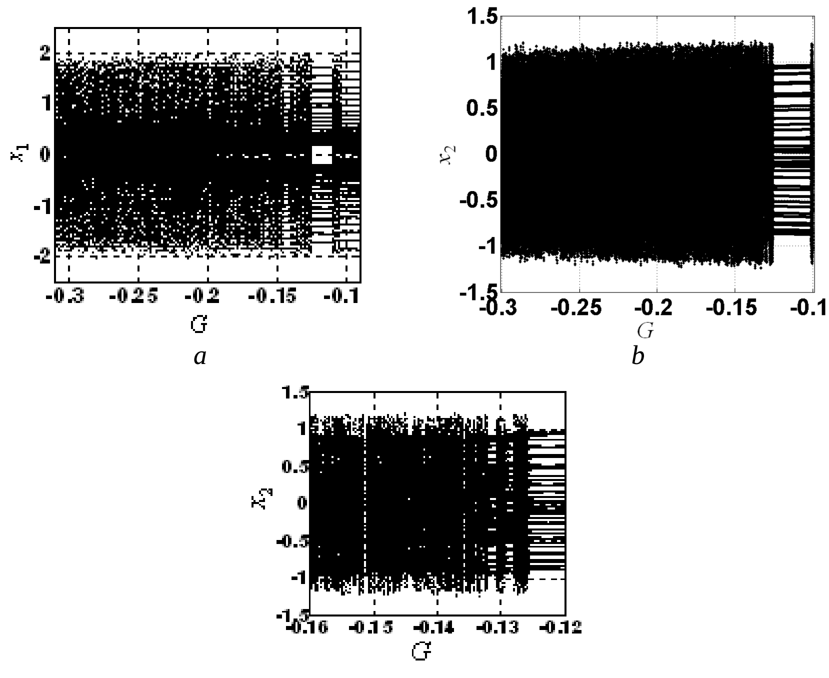


Fig. 2 – Bifurcation diagrams for the first state variable (*a*) and second one (*b*), with a detail on a reduced gain variation interval (*c*).

of the gain of the differential amplifier, $G$. The first graph (Fig. 2 *a*), shows the $x_1$ state variable *versus* the gain parameter, highlighting periodic dynamics for small values of the gain, alternated with chaotic regions, which become larger for increasing gain values. Similar behavior is obtained by sampling the second state variable, $x_2$, as suggested in the second graph (Fig. 2 *b*), and the detail in the third one (Fig. 2 *c*). For gain values reaching unity, only chaotic behavior can be observed, all further simulations being made for this value.

For normalized parameter values ensuring chaotic behavior

$$\alpha = 0.2,\ \beta = 0.01,\ \omega_0^2 = 0.1,\ G = 1, \tag{11}$$

we present examples of the time evolution for the system state variables in Fig. 3. The noise-like of the resulting waveforms suggest the chaotic behavior, for the parameter set chosen from the bifurcation diagrams.

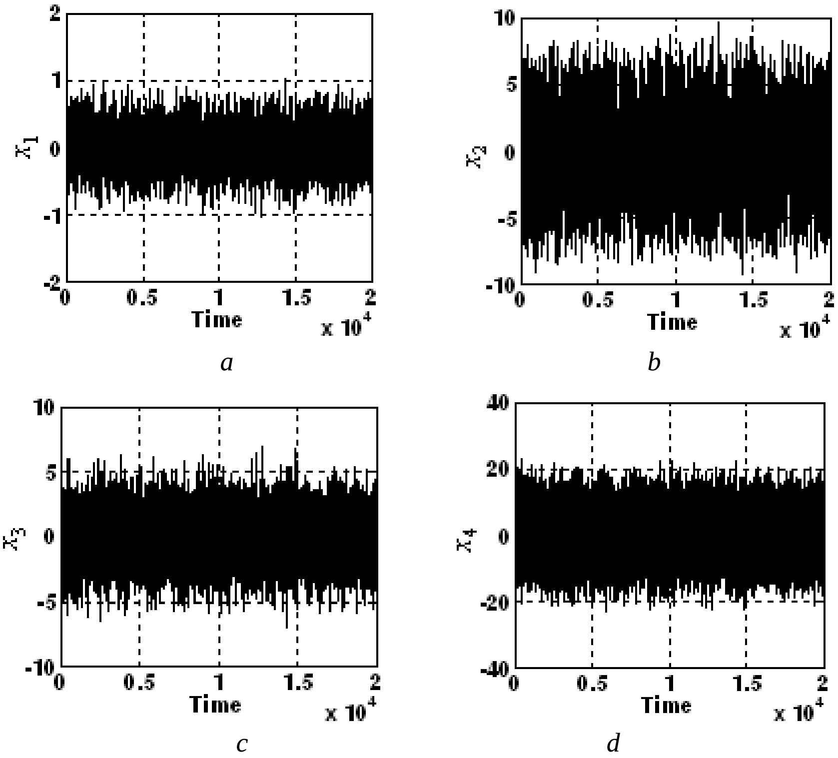


Fig. 3 – Time evolution of state variables $x_1$ (*a*), $x_2$ (*b*), $x_3$ (*c*) and $x_4$ (*d*).

By composing the state variables waveforms, we obtained 3-D projections of the system state space, such as the one in Fig. 4, consistent with the chaotic dynamics of the system.

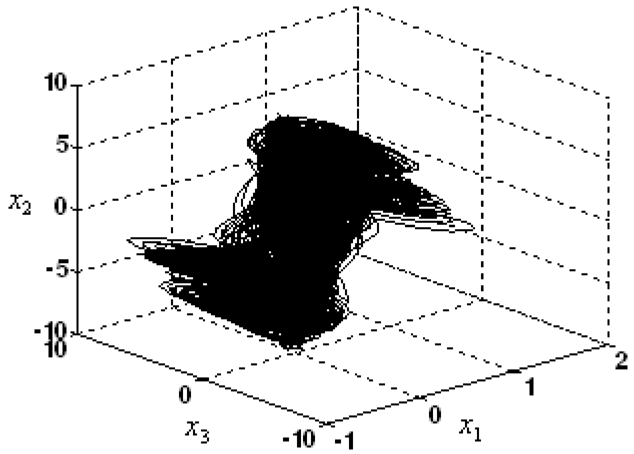


Fig. 4 – 3-D projection of the state portrait for the proposed system.

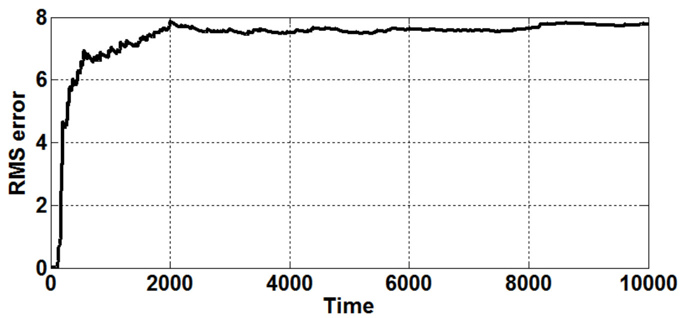


Fig. 5 – RMS error of the state vectors of two instances of the chaotic system.

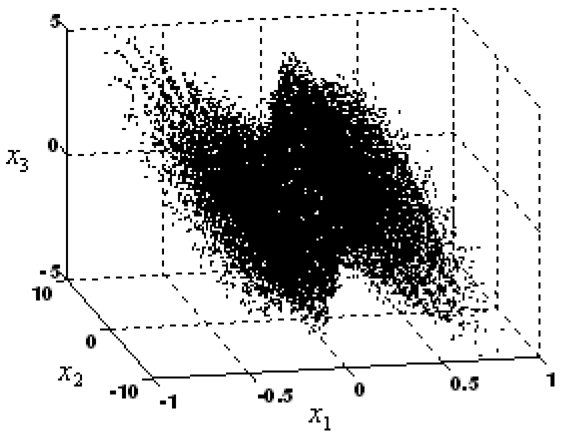


Fig. 6 – Poincaré section for the proposed nonlinear system.

To confirm the chaotic behavior of the proposed system, for the set of parameters in eq. (8), we checked for the property of initial condition sensitivity for two identical systems, starting from initial conditions differing with $\varepsilon = 0.001$. The simulations were performed by calculating the running RMS value of the Euclidean norm of the error vector. The obtained results are graphically depicted in Fig. 5, highlighting the exponential divergence of the systems trajectories at initial time interval, followed by a region of smaller increase and final saturation at a larger value than the RMS of the state vector.

For a better understanding of the nonlinear dynamics of the proposed system, Poincaré sections were obtained. The results, depicted in Fig. 6, confirm the ergodic property, suggested by the phase portrait in Fig. 4. The dense distribution of the samples in the discrete state space of the Poincaré section also suggests that statistical properties of the generated output signals may lead to the possibility of applying the proposed system for noise generation.

To estimate the statistical properties of the generated signals, we performed power spectrum estimations, based on the periodogram method. For instance, in Fig. 7, we present the result of such an estimation for the power spectrum density, $S_{33}(\omega)$, of the third state variable, $x_3(t)$, performed on a 32,768 samples period, by performing the average of four, 8,192 long, FFT instances. The obtained results highlight a reasonably flat spectrum in the targeted frequency band, limited by the cutoff frequencies of the filters in the feedback loops.

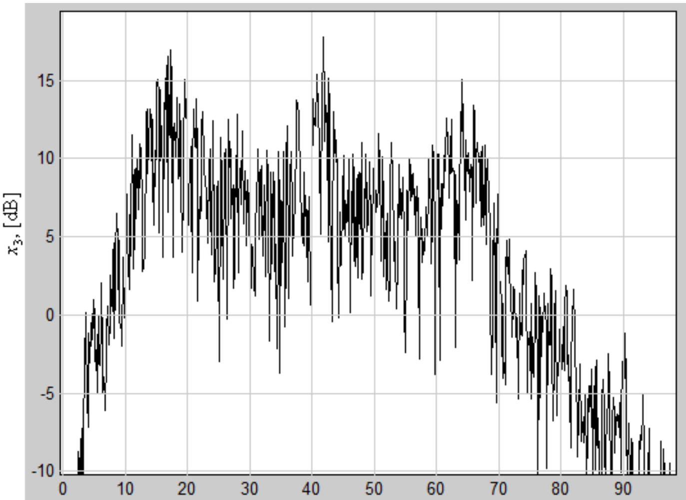

Fig. 7 – Logarithmic power spectrum of the output signal.

## 4. Conclusion

In the present paper we propose a new fourth order, analogue nonlinear circuit, exhibiting periodic and chaotic dynamics. The system level analysis performed on the circuit includes the deduction of its state eqs. and the demonstration of its dissipative property. The simulations included in this paper concentrate on the chaotic behavior of the system, aiming at analogue noise generation. Time domain evolution of the state variables, phase portrait, Poincaré section and power spectral density results confirm the complex dynamics of the proposed system. The presented properties open the way to further research aiming at IC implementation of the analysed nonlinear circuit.

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## UN NOU SISTEM ANALOGIC HAOTIC PENTRU GENERAREA DE ZGOMOT

(Rezumat)

Se introduce un nou circuit neliniar analogic, care evidenţiază comportări dinamice bogate. Circuitul de ordinul patru propus este bazat pe o funcţie neliniară algebrică liniară pe porţiuni, cu discontinuitate în origine. Partea dinamică a circuitului este realizată prin închiderea a două bucle de reacție în jurul unui amplificator diferențial, care comandă circuitul neliniar. Pentru o gamă largă de valori ale parametrilor, sistemul propus se comportă haotic, fiind disipativ, senzitiv la condițiile inițiale, având o evoluție în timp nepredictibilă şi o comportare ergodică. Aceste proprietăți sunt potrivite pentru a proiecta generatoare analogice de zgomot, având o densitate spectrală de putere constantă, într-o bandă de frecvențe bine definită. Sunt prezentate atât rezultate de analiză la nivel de sistem, cât şi rezultate de simulare, care confirmă aspectele teoretice prezentate.