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**THE ENERGY REGIME OF A LINEAR, NON-AUTONOMOUS
AND PASSIVE GENERAL TWO-PORT, SUPPLIED, IN
HARMONIC STEADY-STATE, SIMULTANEOUSLY AT THE
GATES (1), (1') and (2), (2'), HAVING BETWEEN THESE GATES
A NON-LINEAR INERTIAL COUPLING COMPLEX
IMPEDANCE (II)**

BY

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Abstract. The conditions assuring the realization of an extreme value of the active power dissipated in the coupling branch relying the gates (1), (1') and (2), (2') of a general, linear, non-autonomous and passive two-port, supplied, in harmonic steady-state, simultaneously, at these gates, are established when this branch is realized by series connection of a resistor, a coil and a condenser, all the three elements being non-linear inertial. In the same working conditions a similar problem concerning the efficiency of the dissipated active power transfer in the coupling branch is outlined too.

Key words: linear, non-autonomous and passive two-port; simultaneous supply at the ports (1), (1') and (2), (2'); harmonic steady-state; extreme values of the active power and of the efficiency.

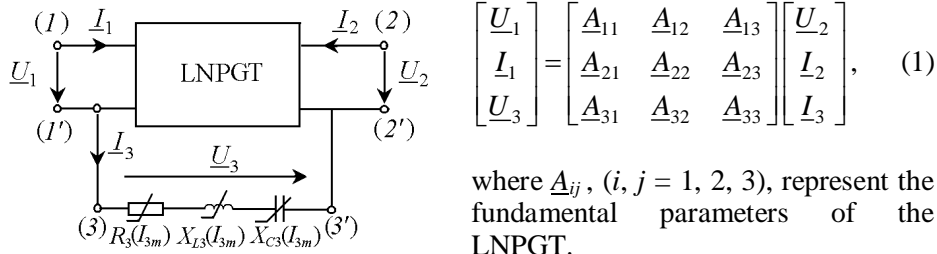
1. Introduction

In a previous papers (Rosman, 2011) the problem concerning the determination of conditions when either the active power, P_3 , dissipated in the

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coupling branch between the gate (1), (1') and (2), (2') of a linear, non-autonomous and passive general two-port (LNPGT) supplied, in harmonic steady-state, simultaneously, at these gates, or the efficiency of this dissipated active power transfer, has extreme values, are approached. The coupling branch is considered as being non-linear inertial, having an arbitrary structure, supposing that both the equivalent resistance and the equivalent reactance of the branch are functions of the amplitude of a same arbitrary harmonic current.

In what follows similar problems are studied, in the particular case when the coupling branch between the gates (1), (1') and (2), (2') is realized by the series connection of a resistor, a coil and a condenser, all the three elements being non-linear inertial (Fig. 1). The eqs. of such a two-port are (Sigorsky, 1956):



where \underline{A}_{ij} , ($i, j = 1, 2, 3$), represent the fundamental parameters of the LNPGT.

Fig. 1

As regards the coupling branch complex impedance connected at gate (3), (3'), this is given by relation

$$\underline{Z}_3(I_{3m}) = R_3(I_{3m}) + jX_3(I_{3m}), \quad (2)$$

having in view that in this case it is possible to utilize the symbolic complex method of harmonic signals representation because the input-output characteristic of a non-linear inertial element is linear in instantaneous values (Philippow, 1963) and consequently the steady-state of LNPGT represented in Fig. 1 is harmonic if the supplying voltages, u_1 , u_2 , are harmonic, having the same frequency.

The parameters $R_3(I_{3m})$, $X_{L3}(I_{3m})$, $X_{C3}(I_{3m})$ are given by (Savin & Rosman, 1973)

$$\begin{cases} R_3(I_{3m}) = \alpha_1 + \frac{3}{4}\alpha_3 I_{3m}^2, & (\alpha_1 > 0, \alpha_3 \leq 0), \\ X_{L3}(I_{3m}) = \beta_1 \omega - \frac{3}{4}\beta_3 \omega I_{3m}^2, & (\beta_1 > 0, \beta_3 \leq 0), \\ X_{C3}(I_{3m}) = \frac{\gamma_1}{\omega} + \frac{3}{4} \cdot \frac{\gamma_3}{\omega^3} I_{3m}^2, & (\gamma_1 > 0, \gamma_3 \leq 0), \end{cases} \quad (3)$$

so that

$$X_3(I_{3m}) = X_{L_3}(I_{3m}) - X_{C_3}(I_{3m}) = \beta_1 \omega - \frac{\gamma_1}{\omega} - \frac{3}{4} \left(\beta_3 \omega + \frac{\gamma_3}{\omega^3} \right) I_{3m}^2 \leq 0. \quad (4)$$

2. Active Power Dissipated in the Coupling Branch

The expression of the active power dissipated in the coupling branch between the gates (1), (1') and (2), (2') of the LNPGT represented in Fig. 1 was established in a previous paper (Rosman, 2004) namely

$$P_3 = \frac{R_3 |\underline{A}_{32} \underline{U}_1 + \underline{A}_{13} \underline{U}_2|^2}{e(R_3^2 + X_3^2) + fR_3 + gX_3 + h}, \quad (5)$$

where the coefficients e, f, g, h are given in Appendix 1, being functions only of LNPGT's characteristic parameters, \underline{A}_{ij} , ($i, j = 1, 2, 3$).

Referring to the coupling branch represented in Fig. 1, the R_3 and X_3 parameters from relation (5) must be substituted with (3₁) and (4), obtaining

$$P_3 = |\underline{A}_{32} \underline{U}_1 + \underline{A}_{13} \underline{U}_2|^2 \frac{A + BI_{3m}^2}{CI_{3m}^4 + DI_{3m}^2 + E}, \quad (6)$$

which may be studied with the usual proceeding regarding the functions of a single independent variable. Coefficients A, B, C, D, E expressions are given in Appendix 2.

Firstly the derivative

$$\frac{dP_3}{dI_m} = -2 |\underline{A}_{32} \underline{U}_1 + \underline{A}_{13} \underline{U}_2|^2 \frac{BCI_{3m}^4 + 2DCI_{3m}^2 + AD - BE}{(CI_{3m}^4 + DI_{3m}^2 + E)^2} I_{3m} \quad (7)$$

is performed. Annulling this derivative, an algebraic biquadratic eq. of fourth degree, in I_{3m} , is obtained namely

$$BCI_{3m}^4 + 2ACI_{3m}^2 + AD - BE = 0, \quad (8)$$

having the roots

$$I_{3m, 1,2,3,4} = \pm \sqrt{\frac{-AC \pm \sqrt{A^2 C^2 - BC(AD - BE)}}{BC}}, \quad (9)$$

at which must be added the root $I_{3m_5} = 0$, unacceptable from physical point of view. Also, the roots having the sign “-” in front of the external radical

(I_{3m_3}, I_{3m_4}) are unacceptable from same point of view. As regards the other two roots (I_{3m_1}, I_{3m_2}) , these can be acceptable only if their expressions are real and positive. A first condition which must be satisfied is referred to the expression situated under the inner radical: this one must be non-negative. Since $C > 0$ (s. Appendix 2) the above condition becomes

$$B(AD - BE) \leq A^2C. \quad (10)$$

Having in view that $C > 0$ too (s. Appendix 2) it results that the roots I_{3m_1}, I_{3m_2} can be real and positive if

$$\text{sign}B = -\text{sign}(AD - BE). \quad (11)$$

The following possible cases must be taken into account:

a) If $B > 0$ and consequently $AD - BE < 0$, only the root

$$I_{3m_1} = \sqrt{\frac{-AC + \sqrt{A^2C^2 - BC(AD - BE)}}{BC}} \quad (12)$$

is real and positive.

b) If $B < 0$ and consequently $AD - BE > 0$, on the contrary, only the root

$$I_{3m_2} = \sqrt{\frac{-AC - \sqrt{A^2C^2 - BC(AD - BE)}}{BC}} \quad (13)$$

is real and positive.

c) If $B(AD - BE) = A^2C$ the derivative's (8) root which can be real and positive is

$$\tilde{I}_{3m} = \sqrt{-\frac{A}{B}}, \quad (14)$$

which is acceptable only if $B < 0$.

In view to establish if the determined roots of eq. (8) correspond to an extreme value of function $P_3(I_{3m})$, the second derivative of this function with respect to I_{3m} is performed namely

$$\frac{d^2P_3}{dI_{3m}^2} = -2|A_{32}U_1 + A_{13}U_2|^2 \frac{5BCI_{3m}^4 + 6ACI_{3m}^2 - AD + BE}{(CI_{2m}^4 + DI_{2m}^2 + E)^2}. \quad (15)$$

It is necessary to determine the sign of this derivative when $I_{3m} = I_{3m_1}$ or $I_{3m} = I_{3m_2}$. Having in view that $2|A_{32}U_1 + A_{13}U_2|^2 / (CI_{2m}^4 + DI_{2m}^2 + E) > 0$ for any value of I_{3m} it is sufficient to determine only the sign of the numerator of the fraction from relation (15) for these two values of I_{3m} . Performing the calculus it results

$$\begin{aligned} & 5BCI_{3m_1}^4 + 4ACI_{3m_1}^2 - AD + BE = \\ & = \frac{4\sqrt{A^2C^2 - BC(AD - BE)} \left[\sqrt{A^2C^2 - BC(AD - BE)} - AC \right]}{BC}, \end{aligned} \quad (16)$$

respectively

$$\begin{aligned} & 5BCI_{3m_1}^4 + 4ACI_{3m_1}^2 - AD + BE = \\ & = \frac{4\sqrt{A^2C^2 - BC(AD - BE)} \left[\sqrt{A^2C^2 - BC(AD - BE)} + AC \right]}{BC}. \end{aligned} \quad (17)$$

Consequently

- a) if $B > 0$, $d^2P_3/dI_{3m}^2|_{I_{3m}=I_{3m_1}} > 0$ and $d^2P_3/dI_{3m}^2|_{I_{3m}=I_{3m_2}} < 0$;
- b) if $B < 0$, $d^2P_3/dI_{3m}^2|_{I_{3m}=I_{3m_1}} < 0$ and $d^2P_3/dI_{3m}^2|_{I_{3m}=I_{3m_2}} < 0$.

It is possible to conclude that the function $P_3(I_{3m})$ is: a) *maximum* if $I_{3m} = I_{3m_2}$ and $B > 0$ or $I_{3m} = I_{3m_1}$ and $B < 0$; b) *minimum* if $I_{3m} = I_{3m_1}$ and $B < 0$ or $I_{3m} = I_{3m_2}$ and $B > 0$.

If $AD = BE$, $d^2I_m/dI_3^2 = 0$ for $I_{3m} = \tilde{I}_{3m}$ (s. rel. (14)), a supplementary study being necessary in this case.

Substituting in relation (6) expressions (12) and (13) of I_{3m_1} , respectively I_{3m_2} , it may be obtained the maximum, or, respectively, the minimum value of P_3 . Performing the calculus it results

$$P_{3\text{extr}} = \frac{\pm B^2 |A_{32}U_1 + A_{13}U_2|^2}{2\sqrt{A^2C^2 - BC(AD - BE)} \mp 2AC \mp BD}, \quad (18)$$

where the superior sign corresponds to I_{3m_1} and the inferior one, to I_{3m_2} .

3. Efficiency, η , of Dissipated Active Power Transferred to the Coupling Branch

The expression of the efficiency characterizing the active power's transfer from gates (1), (1') and (2), (2') to the coupling branch between these gates, in the case of a LNPGT supplied at the specified gates with harmonic voltages having the same frequency, was established in a previous paper (Rosman, 2006) namely

$$\eta = \frac{R_3 |\underline{A}_{23} \underline{U}_1 + \underline{A}_{13} \underline{U}_2|^2}{a(R_3^2 + X_3^2) + bR_3 + cX_3 + d}, \quad (19)$$

where the expressions of a, b, c, d are given in Appendix 3.

It is useful to observe that relations (5) and (19) have a similar structure so that the established conclusions concerning the active power, P_3 , dissipated in the coupling branch (with complex impedance \underline{Z}_3) between the gates (1), (1') and (2), (2') of the LNPGT supplied, simultaneously, at these gates, may be easily adapted to the study of the efficiency, η , of the transfer of this power. With this end in view is sufficiently to substitute coefficients e, f, g, h (s. Appendix 1) with, respectively, a, b, c, d (s. Appendix 3). The explicit formulation of the conclusions regarding this last case has a high degree of difficulty on account of a, b, c, d expressions complexity. In any case is possible to affirm that because R_3, X_3 parameters are functions of the current amplitude, I_{3m} , which flows through the coupling branch relying the gates (1), (1') and (2), (2') (s. rel. (3) and (4)), the efficiency is also a function of the same current amplitude, $\eta(I_{3m})$. A detailed study of this function, analogous to that of function $P_3(I_{3m})$, performed in §2 of this paper, leads to the conclusion that the efficiency can be extreme for certain values of I_{3m} . In the same time it is possible to determine these values when existing.

4. Conclusions

1. The dissipated active power, P_3 , in the coupling branch between the gates (1), (1') and (2), (2') of a linear, non-autonomous and general two-port, is determined when this one is supplied at the above mentioned gates with harmonic voltages having the same frequency. The coupling branch is considered as a series connexion of a resistor, a coil and a condenser, all three non-linear inertial.

2. The efficiency, η , of active power's, P_3 , transfer through the considered two-port to the non-linear inertial branch is determined too.

3. The conditions in which either the active power, P_3 , or the efficiency, η , have extreme values are determined.

Appendix 1

In a previous paper (Rosman, 2004) the active power, P_3 , dissipated in the coupling branch between the gates (1), (1') and (2), (2'), having the complex impedance $\underline{Z}_3 = R_3 + jX_3$, in case of a LNPGT supplied at these gates, simultaneously, by two harmonic voltages having the same frequency, was determined namely

$$P_3 = R_3 \frac{|\underline{A}_{32}\underline{U}_1 + \underline{A}_{13}\underline{U}_2|}{|\underline{A}_{12}(\underline{Z}_3 - \underline{A}_{33}) + \underline{A}_{13}\underline{A}_{32}|}. \quad (\text{A.1})$$

Performing the calculus it results relation (5) where

$$\begin{aligned} e &= A_{12}^2 > 0, \quad f = 2\Re\left[\underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{12}\underline{A}_{33}^*\right)\right], \\ g &= -2\Im\left[\underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{12}\underline{A}_{33}^*\right)\right], \quad h = \left|\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right|^2 > 0. \end{aligned} \quad (\text{A.2})$$

Appendix 2

If in relation (5) are substituted, resistance R_3 with expression (3₁) and reactance X_3 with (4) it results relation (6) where

$$\begin{aligned} A &= \alpha_1 > 0, \quad B = \frac{3}{4}\alpha_3, \quad C = \frac{9A_{12}^2}{16}\left[\alpha_3^2 + \left(\beta_3\omega + \frac{\gamma_3}{\omega^3}\right)^2\right] > 0, \\ D &= \frac{3A_{12}}{2}\left[\alpha_1\alpha_3 - \left(\beta_1\omega - \frac{\gamma_1}{\omega}\right)\left(\beta_3\omega + \frac{\gamma_3}{\omega^3}\right)\right] + \frac{3}{2}\left\{\alpha_3\Re\left[\underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{12}\underline{A}_{33}^*\right) - \right. \right. \\ &\quad \left. \left. - \left(\beta_3\omega + \frac{\gamma_3}{\omega^3}\right)\Im\left[\underline{A}_{12}\left(\underline{A}_{22}\underline{A}_{31}^* - \underline{A}_{21}\underline{A}_{32}^*\right)\right]\right\}, \quad (\text{A.3}) \\ E &= A_{12}^2\left[\alpha_1^2 + \left(\beta_1\omega - \frac{\gamma_1}{\omega}\right)^2\right] + 2\left\{\alpha_1\Re\left[\underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{12}\underline{A}_{33}^*\right) - \right. \right. \\ &\quad \left. \left. - \left(\beta_1\omega - \frac{\gamma_1}{\omega}\right)\Im\left[\underline{A}_{12}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{12}\underline{A}_{33}^*\right)\right]\right\}. \end{aligned}$$

Appendix 3

Coefficients a , b , c , d expressions were established in a previous paper (Rosman, 2006) namely

$$\begin{aligned}
a &= \Re(\underline{A}_{12}\underline{A}_{22}^*)U_1^2 + \Re(\underline{A}_{11}\underline{A}_{12})U_2^2 - \Re\left[\underline{A}_{12}\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*\right) + 1\right]\Re(\underline{U}_1\underline{U}_2^*) + \\
&\quad + \Im\left[\underline{A}_{12}\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^*\right) - 1\right]\Im(\underline{U}_1\underline{U}_2^*), \\
b &= \Re\left[\underline{A}_{22}^*\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right) + \underline{A}_{12}\left(\underline{A}_{23}\underline{A}_{32}^* - \underline{A}_{22}\underline{A}_{33}^*\right)\right]U_1^2 + \\
&\quad + \Re\left[\underline{A}_{11}\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right) + \underline{A}_{32}\left(\underline{A}_{13}\underline{A}_{32}^* - \underline{A}_{11}\underline{A}_{33}^*\right)\right]U_2^2 - \\
&\quad - \Re\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^* + 1\right) + 2\underline{A}_{12}\underline{A}_{33}^*\right]\Re(\underline{U}_1\underline{U}_2^*) + \\
&\quad + \Im\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^* - 1\right)\right]\Im(\underline{U}_1\underline{U}_2^*), \\
c &= \Im\left[\underline{A}_{22}^*\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right) - \underline{A}_{12}\left(\underline{A}_{23}\underline{A}_{32}^* - \underline{A}_{22}\underline{A}_{33}^*\right)\right]U_1^2 + \\
&\quad + \Im\left[\underline{A}_{11}\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right) - \underline{A}_{12}\left(\underline{A}_{23}\underline{A}_{31}^* - \underline{A}_{11}\underline{A}_{33}^*\right)\right]U_2^2 - \\
&\quad - \Im\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^* - 1\right) + 2\underline{A}_{12}\underline{A}_{33}^*\right]\Re(\underline{U}_1\underline{U}_2^*) + \\
&\quad + \Re\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\left(\underline{A}_{11}\underline{A}_{22}^* - \underline{A}_{12}\underline{A}_{21}^* + 1\right)\right]\Im(\underline{U}_1\underline{U}_2^*), \\
d &= \Re\left[\left(\underline{A}_{13}\underline{A}_{31}^* - \underline{A}_{22}\underline{A}_{33}^*\right)\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right]U_1^2 + \Re\left[\left(\underline{A}_{13}\underline{A}_{31}^* - \underline{A}_{11}\underline{A}_{33}^*\right) \times \right. \\
&\quad \left. \times \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right]U_2^2 + 2\Re\left[\underline{A}_{33}^*\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right]\Re(\underline{U}_1\underline{U}_2^*).
\end{aligned} \tag{A.4}$$

It is necessary to observe that while coefficients e, f, g, h (s. rel (A.1)) are functions only of LNGPT's fundamental parameters, \underline{A}_{ij} , ($i, j = 1, 2, 3$), coefficients a, b, c, d are, in addition, functions of the voltages values, $\underline{U}_1, \underline{U}_2$, applied at the gates (1), (1') and, respectively, (2), (2') of the two-port too.

REFERENCES

- Philippow E., *Nichtlineare Elektrotechnik*. Akad. Verlagsges. Geest u. Portig., K.G., Leipzig, 1963.
- Rosman H., *La transmission de la puissance active par un quadripôle général linéaire, non-autonome, réciproque, en régime permanent harmonique, alimenté simultanément aux portes (1), (1') et (2), (2')*. Bul. Inst. Politehnic, Iași, **L (LIV)**, 3-4, s. Electrot., Energ., Electron., 23-28 (2004).
- Rosman H., *Le rendement de la transmission de la puissance active par un quadripôle général linéaire non-autonome, réciproque, alimenté, en régime permanent harmonique, simultanément aux portes (1), (1') et (2), (2')*. Bul. Inst. Politehnic, Iași, **LII (LVI)**, 1-2, s. Electrot., Energ., Electron., 23-29 (2006).
- Rosman H., *The Energy Regime of a Linear, Non-Autonomous and Passive General Two-Port, Supplied, in Harmonic Steady-State, Simultaneously, at the Gates (1), (1') and (2), (2'), Having between these Gates a Non-Linear Inertial Coupling Complex Impedance (I)*. Bul. Inst. Politehnic, Iași, **LVI (LX)**, 4, s. Electrot., Energ., Electron., 19-28 (2010).

Savin Gh., Rosman H., *Circuite electrice neliniare și parametrice*. Edit. Tehnică, București, 1973.

Sigorsky V.P., *General Theory of Two-Ports* (in Russian). Ed. Acad. of Sci., Ukraine, Kiev, 1962.

REGIMUL ENERGETIC AL UNUI CUADRIPOLE GENERAL LINIAR,
NEAUTONOM ȘI PASIV, ÎN REGIM PERMANENT ARMONIC,
ALIMENTAT SIMULTAN LA PORȚILE (1), (1') ȘI (2), (2'), AVÂND
ÎNTRE ACESTE PORȚI O IMPEDANȚĂ COMPLEXĂ DE CUPLAJ
NELINIARĂ INERȚIALĂ (II)

(Rezumat)

Se determină condițiile în care puterea activă, P_3 , disipată în impedanța de cuplaj dintre porțile (1), (1') și (2), (2') ale unui cuadripol general liniar, neautonom și pasiv, în regim permanent armonic, alimentat simultan pe la aceste porți, are valori extreme atunci când impedanța de cuplaj dintre porțile respective este constituită din gruparea în serie a unui rezistor, unei bobine și unui condensator, toate trei neliniare inerțiale.

În aceleași condiții se abordează și problema determinării randamentului cu care este transferată impedanței de cuplaj puterea activă, P_3 , disipată în aceasta.

