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SIMPLIFIED ORTHOGONAL MODEL IN TERMS OF FLUXES FOR AN INDUCTION MACHINE USING PSPICE

BY

VOINEA-RADU COCIU* and LIVIA COCIU

“Gheorghe Asachi” Technical University of Iaşi
Faculty of Electrical Engineering

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Abstract. Starting from the PSpice implementation of the orthogonal model of the induction machine in terms of fluxes, the paper aims to simplify the equivalent electrical schematic in order to increase simulation convergence safety and to decrease simulation time. Through adequate transformations based on equality between a stator voltage and a rotor voltage, a simplified equivalent schematic is attained. The main idea is to find the relationship between the two voltages in different circumstances and evaluate the error introduced by the assumption of them being equal. Conclusions are drawn regarding computing speed, size of output data file, precision and convergence.

Key words: induction machine; two-phase model; equivalent circuit; PSpice

1. Introduction

To describe the behavior of the induction machines, mathematical models with distributed or concentrated parameters have been suggested. The two-phase (orthogonal) model is a natural model where single-phase and two-phase induction machines are involved. For three-phase induction machines the natural model in phase coordinates, the three-phase model, is better suited. Most

* Corresponding author: *e-mail*: cociuvr@ee.tuiasi.ro

times however, the orthogonal (two-phase) model is used in the study of the three-phase induction machine operating as a motor or generator.

Different forms of mathematical models were considered, using only currents or only fluxes. The first case is the most common in literature (Ong, 1997), while the second models that retain the total fluxes, is newer (Simion, 2008). In each case, an equivalent circuit has been established and different implementations of the model have been suggested in order to simulate them with the aid of different programs (Justus,1993).

The study presented by Cociu & Cociu, (2012), highlights the differences between the two types of modelling, concerning model parameters and computing speed. When the second model is used, a new unusual parameters must additionally be calculated by the program to solve the system of eqs.

Comparing the voltage eqs. in classical form, in terms of currents, and in the new approach, in terms of fluxes, a similar format can be seen, except the two derivatives in the classical form *versus* only one derivative when total fluxes are used. The first step in implementing the system of eqs. in PSpice is establishing the equivalent electrical circuits in both cases (Cociu & Cociu, 1997, 2012). The two circuits give similar but not identical topologies. The model using only fluxes has an extra controlled-voltage source and an extra node. That is why, in this case, the simulation is a little slower and the output data file a little larger. These results are peculiarities only of PSpice implementation.

2. New Approach of Two-Phase (Orthogonal) Model dq-dq

To allow for a more compact form of the eqs., the following will use spatial phasors to represent the quantities of interest of the machine. Using a stationary reference frame, the eqs. describing the behavior of an induction machine, in their classical form, are the following:

a) voltage eqs. for stator and rotor

$$\begin{cases} \underline{v}_s = R_s \underline{i}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t}, \\ \underline{v}_r' = R_r' \underline{i}_r' + \dfrac{\mathrm{d}\underline{\psi}_r'}{\mathrm{d}t} - \mathrm{j}\omega_r \underline{\psi}_r'; \end{cases} \tag{1}$$

b) stator and rotor flux eqs.

$$\begin{cases} \underline{\psi}_s = L_s \underline{i}_s + L_m \underline{i}_r'; \qquad L_s = L_{\sigma s} + L_m, \\ \underline{\psi}_r' = L_s \underline{i}_r' + L_m \underline{i}_s; \qquad L_r' = L_{\sigma r}' + L_m; \end{cases} \tag{2}$$

c) electromagnetic torque expression (in terms of fluxes)

$$t_e = \frac{p}{\Delta_L} p \left| \underline{\psi}_s \times \underline{\psi}_r^{'} \right| ; \tag{3}$$

d) motion eq.

$$t_e - t_l = \frac{J}{p} \cdot \frac{\mathrm{d}\omega_r}{\mathrm{d}t} + F_\alpha \frac{\omega_r}{p} . \tag{4}$$

All the rotor quantities are transformed to have the same frequency as the stator quantities, using well-known relations.

In order to establish the system of eqs. in terms of total fluxes and to eliminate the current, the following notation is used:

$$\Delta_L = L_s L_r^{'} - L_m^2 , \tag{5}$$

and we can determine the current expressions with respect to the fluxes

$$\underline{i}_s = \frac{L_r^{'}}{\Delta_L} \underline{\psi}_s - \frac{L_m}{\Delta_L} \underline{\psi}_r^{'} ,\ \underline{i}_r^{'} = \frac{L_s}{\Delta_L} \underline{\psi}_r^{'} - \frac{L_m}{\Delta_L} \underline{\psi}_s . \tag{6}$$

Replacing these current expressions in (1) and using the notations

$$\begin{aligned} v_{sr} &= \frac{R_s L_r^{'}}{\Delta_L},\ v_{sm} = \frac{R_s L_m}{\Delta_L}, \\ v_{rs} &= \frac{R_r^{'} L_s}{\Delta_L},\ v_{sm} = \frac{R_r^{'} L_m}{\Delta_L}, \end{aligned} \tag{7}$$

we get a new form of the voltage eqs., in terms of fluxes

$$\begin{cases} \underline{v}_s = v_{sr} \underline{\psi}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} - v_{sm} \underline{\psi}_r^{'} , \\ \underline{v}_r^{'} = v_{rs} \underline{\psi}_r^{'} + \dfrac{\mathrm{d}\underline{\psi}_r^{'}}{\mathrm{d}t} - v_{rm} \underline{\psi}_s - \mathrm{j}\omega_r \underline{\psi}_r^{'} . \end{cases} \tag{8}$$

Using the notations

$$\underline{e}_s = v_{sm} \underline{\psi}_r^{'} ,\ \underline{e}_r = v_{rm} \underline{\psi}_s + \mathrm{j}\omega_r \underline{\psi}_r^{'} , \tag{9}$$

is obtained the final form of the voltage eqs.

$$\begin{cases} \underline{v}_s = \nu_{sr}\underline{\psi}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} - \underline{e}_s, \\ \underline{v}_r^{'} = \nu_{rs}\underline{\psi}_r^{'} + \dfrac{\mathrm{d}\underline{\psi}_r^{'}}{\mathrm{d}t} - \underline{e}_r. \end{cases} \tag{10}$$

Using this voltage eqs. system, the new equivalent circuit can be established like in Fig. 1 (Cociu & Cociu, 2012).

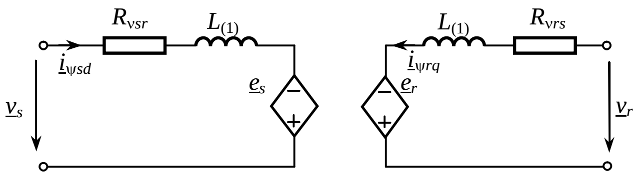

Fig. 1 – Equivalent circuits of the two-phase model of the induction machine in terms of fluxes.

Starting from the observation that the parameters ν_{sr}, ν_{sm} and ν_{rs}, ν_{rm} have common parts, the voltage eqs. (8) can be further processed by associating terms number one and three

$$\begin{aligned} &\nu_{sr}\underline{\psi}_s - \nu_{sm}\underline{\psi}_r^{'} = \\ &= \frac{R_s L_r^{'}}{\Delta_L}\underline{\psi}_s - \frac{R_s L_m}{\Delta_L}\underline{\psi}_r^{'} = \frac{R_s L_{\sigma r}^{'}}{\Delta_L}\underline{\psi}_s + \frac{R_s L_m}{\Delta_L}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right) = \\ &= \nu_{s\sigma r}\underline{\psi}_s + \nu_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \end{aligned} \tag{11}$$

$$\begin{aligned} &\nu_{rs}\underline{\psi}_r^{'} - \nu_{rm}\underline{\psi}_s = \\ &= \frac{R_r^{'} L_s}{\Delta_L}\underline{\psi}_r - \frac{R_r^{'} L_m}{\Delta_L}\underline{\psi}_s = \frac{R_r^{'} L_{\sigma s}}{\Delta_L}\underline{\psi}_r + \frac{R_r^{'} L_m}{\Delta_L}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right) = \\ &= \nu_{r\sigma s}\underline{\psi}_r^{'} + \nu_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right). \end{aligned} \tag{12}$$

New notations are introduced

$$\nu_{s\sigma r} = \frac{R_s L_{\sigma r}^{'}}{\Delta_L}, \quad \nu_{r\sigma s} = \frac{R_r^{'} L_{\sigma s}}{\Delta_L}. \tag{13}$$

Since $L_{\sigma s} << L_s$ and $L_{\sigma r}^{'} << L_r^{'}$, it follows that $\nu_{s\sigma r} << \nu_{sr}$ and $\nu_{r\sigma s} << \nu_{rs}$. The final form of the voltage eqs. becomes:

$$\begin{cases} \underline{v}_s = v_{s\sigma r}\,\underline{\psi}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} + v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \\ \underline{v}_r^{'} = v_{r\sigma s}\,\underline{\psi}_r^{'} + \dfrac{\mathrm{d}\underline{\psi}_r^{'}}{\mathrm{d}t} + v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right) - \mathrm{j}\omega_r\,\underline{\psi}_r^{'}. \end{cases} \tag{14}$$

An analysis of the three terms of the voltages eqs. reveals that, compared to (8) and (10), the first term is noticeably smaller, the second is the same, and the third is quite different. By separating the term corresponding to the rotation-induced voltage, the following notations can be adopted:

$$\begin{aligned} \underline{e}_s^{*} &= v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right), \\ \underline{e}_r^{*} &= v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right), \\ \underline{e}_{r\omega}^{*} &= \mathrm{j}\omega_r\,\underline{\psi}_r^{'} . \end{aligned} \tag{15}$$

The results of the calculations carried out so far lead to the equivalent electrical schematic shown in Fig. 2, which corresponds to the voltages eqs. (14).

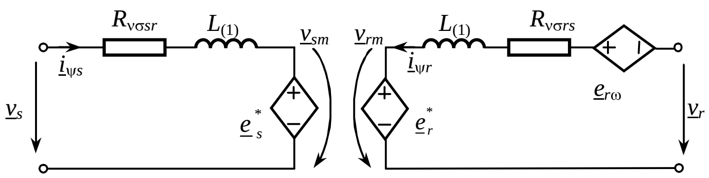

Fig. 2 – New equivalent circuits.

The main result that is of practical interest is that the two voltages, $\underline{v}_{sm}$ and $\underline{v}_{rm}$, are similar in structure and that their values might be close

$$\underline{v}_{sm} = v_{sm}\left(\underline{\psi}_s - \underline{\psi}_r^{'}\right),\ \underline{v}_{rm} = v_{rm}\left(\underline{\psi}_r^{'} - \underline{\psi}_s\right). \tag{16}$$

When the values of the two voltages are close enough, they can be considered equal, leading to a simplified equivalent electrical schematic. By removing a node, the complexity of the circuit drops, with positive effects on the simulation convergence and the simulation time when PSpice is used. The following aims to find the relationship between the two voltages in different situations in which the induction machine may operate. This is used to evaluate the degree to which simplifying the equivalent electrical schematic is allowed, when the errors introduced are to be kept in check.

3. Evaluating and Comparing Voltages $\underline{v}_{sm}$ and $\underline{v}_{rm}$

For a quantitative evaluation of the total fluxes, ψ_s and ψ_r , and of the two new voltages, $\underline{v}_{sm}$ and $\underline{v}_{rm}$ (the third term of the voltages eqs.), numerical values are required for the machine parameters.

An orthogonal model of an induction machine has been performed, rated as follows:

$P_n = 5$ kW; $R_s = 1\Omega$; $L_{\sigma s} = 8$ mH; $J = 30$ g.m^2;
$U_{11n} = 380$ V; $R_r^{'} = 1\,\Omega$; $L_{\sigma r}^{'} = 8$ mH; $F_\alpha = 30$ N.m.s/rad;
$f_1 = 50$ Hz; Y connection; $L_m = 120$ mH; $p = 1$.

The first goal was to determine the way in which the machine operating mode influences the values of total fluxes and their difference. To this end, the values of fluxes for slips corresponding to angular speed values of 0…350 rad/s, have been evaluated. This range covers most of the operating states that can be found in practice, like start-up, no load or load in motor or generator operating. The results yielded by simulation are shown in Fig. 3.

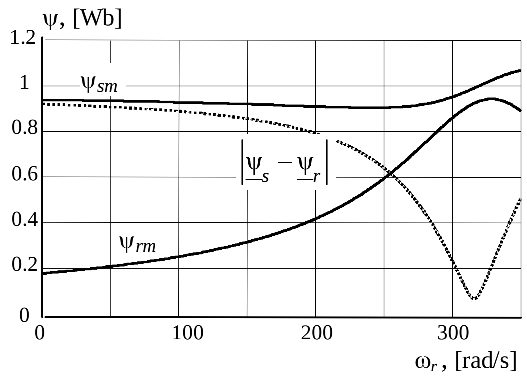

Fig. 3 – Total fluxes variation *versus* angular rotor speed.

The value of the total stator flux is approximately constant. It is mostly dependent on the value of the supply voltage, while the slip has little influence on it. The total rotor flux behaves differently, increasing with the rotor angular speed. Consequently, the difference between the two spatial phasors of total fluxes is strongly dependent on the slip. Around synchronism the difference between fluxes is low in motor and generator operating as well.

This leads to voltages like, $\underline{v}_{sm}$ and $\underline{v}_{rm}$, that have low values in the usual operating range. Fig. 4 gives the variation of the $V = V_{sm} = V_{rm}$ voltage when

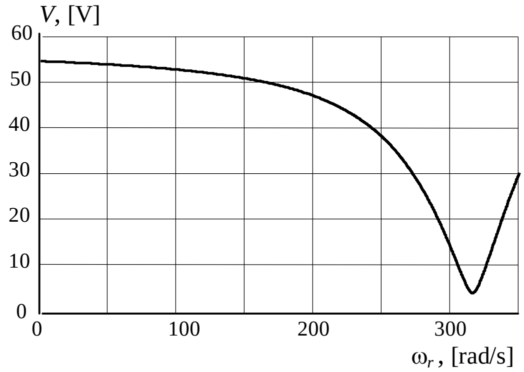

Fig. 4 – Voltage *V* variation *versus* angular rotor speed.

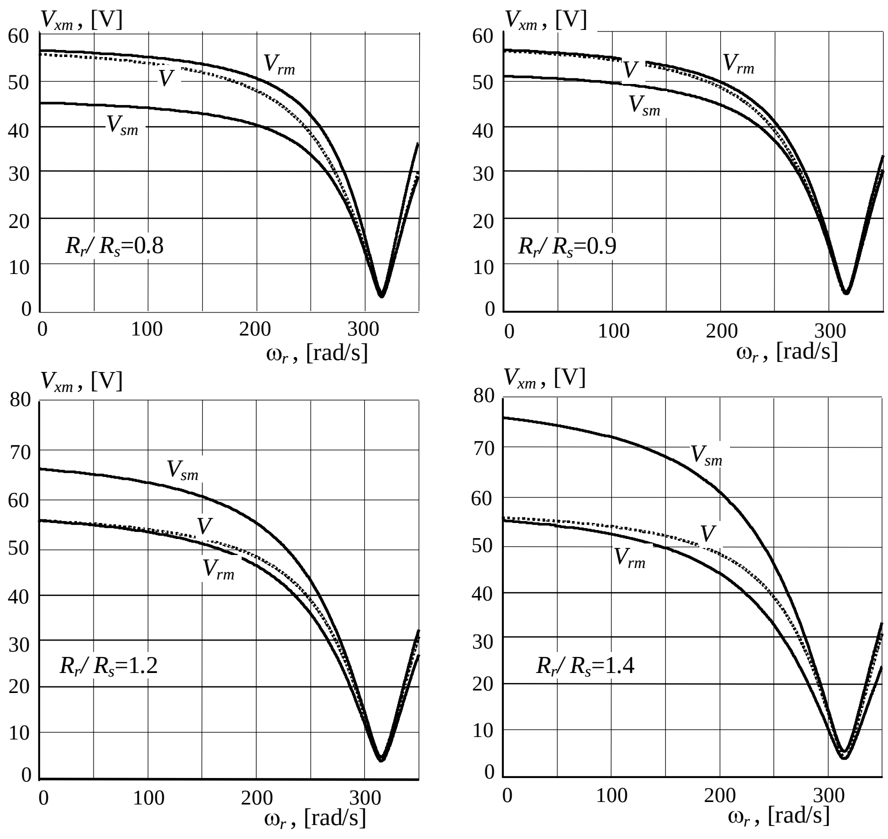

Fig. 5 – Voltage variations *versus* angular rotor speed.

$R_s = R_r^{'}$. Taking into account that the machine is supplied at 380 V, values of 5…20 V of the regular operating range can be considered secondary in the economy of the voltages of the voltages eqs. Even deviations of a few volts may be considered negligible in a first approximation.

The two voltages, $\underline{v}_{sm}$ and $\underline{v}_{rm}$, are similar in structure and close in values. Their mathematical expressions differ only by R_s that shows up in $\underline{v}_{sm}$ and $R_r^{'}$ that shows up in $\underline{v}_{rm.}$ In what follows, the difference between the two voltages is evaluated when the stator and rotor resistances are different, $R_s \neq R_r^{'}$. Fig. 5 illustrates more cases in which the ratio of the two resistances is varied in the range of $R_r^{'} / R_s = 0.8...1.4$. In all cases the sum of the two resistances has been kept at a constant, $R_s + R_r^{'} = \text{const.}$ The dotted line shows the voltage reference value, *V*, obtained when $R_s = R_r^{'} = R$.

As expected, the more the ratio of resistances grows further away from one, the more the voltages of interest, V_s and V_r , deviate from the common value of the ideal case, *V*. In all cases when $R_r^{'} < R < R_s$, a similar relationship between voltages has been found, $V_r < V < V_s$, and the other way around. Around synchronism, in the usual operating range, in both motor and generator operating, there were minor differences between voltages, on the order of a few volts.

4. Simplifying the Equivalent Electrical Schematic

Accepting the $V_r \cong V_s \cong V$ approximation even when $R_s \neq R_r^{'}$, is mathematically equivalent to:

$$R_s = R'_r = R \rightarrow v_{sm} = v_{rm} = v_m = \frac{RL_m}{\Delta_L}, \tag{17}$$

$$\begin{cases} \underline{v}_s = v_{s\sigma r}\underline{\psi}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} + \underbrace{v_m\left(\underline{\psi}_s - \underline{\psi}_r\right)}_{\underline{v}_m}, \\ \underline{v}_r = v_{r\sigma s}\underline{\psi}_r + \dfrac{\mathrm{d}\underline{\psi}_r}{\mathrm{d}t} - \underbrace{v_m\left(\underline{\psi}_s - \underline{\psi}_r\right)}_{\underline{v}_m} - \mathrm{j}\omega_r\underline{\psi}_r, \end{cases} \tag{18}$$

$$\underline{v}_m = v_m\left(\underline{\psi}_s - \underline{\psi}_r\right). \tag{19}$$

In the equivalent electrical schematic, the two stator and rotor circuits, which so far have been separated, can be coupled on the common part, consisting of the R_{vm} resistor through which a current corresponding to the difference of total fluxes passes. To preserve the signs specific to the rotor voltage eq., it was necessary to reverse the direction of the current, $\underline{i}_{\psi r}$, and of the optional rotor supply voltage, $\underline{v}_r^{'}$. Thus the two total fluxes pass through the resistor in opposite directions, and, consequently, is necessary their subtraction.

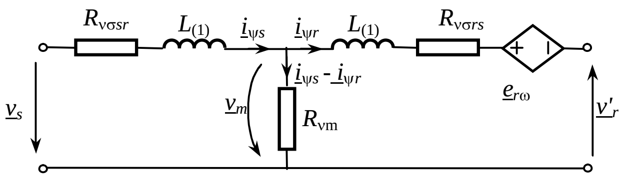

Fig. 6 – Simplified equivalent circuits of the two-phase model of the induction machine using fluxes.

When implementing in PSpice, sources and the generic integrator need to be added to the equivalent circuit in order to complete the orthogonal model and to calculate the quantities of interest (see Cociu & Cociu, 2012).

In this manner, a new PSpice implementation of the orthogonal model expressed only in terms of fluxes has been obtained. The main advantage of this implementation comes from simplifying the equivalent electrical schematic. By removing a node, the complexity of the circuit drops, with positive effects on the simulation convergence and on the simulation speed.

5. Simulation Results

Simplifying the equivalent electrical schematic definitely leads to errors in the final results of the simulation. If the values of the two resistances are close, and the induction machine operates in the regular operating range around the synchronism speed, it is very likely that the magnitude of errors is kept in check. Right now the paper does not aim to evaluate the errors, but to analyse the possible advantages that using the simplified equivalent electrical schematic can bring.

To evaluate the new orthogonal model implementations in PSpice, the simulation of the behavior of an induction machine has been performed, using the PSpice implementation of three different orthogonal models: orthogonal model in terms of currents (classical model), orthogonal model in terms of fluxes and the new simplified orthogonal model in terms of fluxes.

The analysis has been made using the following computer configuration: Gigabyte 945GCMX-S2 MB, Intel Pentium D 925 2.4 GHz CPU, 2048 MB DDR2 RAM, Samsung SP0812N HDD. For a nominal supply of the machine, two different operating states have been considered:

A. Start-up operation of an induction machine during $t = 0\ldots2$ s.

B. No load steady-state operation during $t = 0\ldots1$ s followed by the transient due to the sudden full load at $t = 1$ s till $t = 2$ s.

The analysis time interval has been in every case 2 s and the integration step time was 0.1 ms and 0.5 ms. Simulations took into account two different approaches:

a) only the output mechanical quantities, torque and angular speed have been considered;

b) the output mechanical quantities torque and angular speed together with internal quantities, fluxes in the currents-only model case and currents in the fluxes-only model case, have been considered.

The results yielded by simulation have been organized and presented in Table 1. For each simulation task, five different simulations have been carried out and the average value is what has been taken into account.

Table 1

Simulation characteristics			Currents only model		Fluxes only model		Simplified fluxes only model	
	Step time ms		Simulation time, [s]	File size kB	Simulation time, [s]	File size kB	Simulation time, [s]	File size kB
A	0.1	a	3.47	4,666	3.72	5,292	3.12	4,666
		b	4.91	7,482	5.01	8,108	4.48	7,482
	0.5	a	0.70	938	0.75	1,064	0.63	938
		b	0.99	1,504	1.01	1,603	0.92	1,504
B	0.1	a	3.51	4,823	3.70	5,446	3.14	4,823
		b	4.98	7,638	5.07	8,264	4.49	7,638
	0.5	a	0.71	970	0.76	1,095	0.65	970
		b	1.00	1,535	1.02	1,661	0.89	1,535

All the simulation results are practically identical when using the three orthogonal models (in the third case for $R_s = R_r^{'}$). This is a normal outcome, since they refer to the same system of eqs. processed in three different ways. The comparison between the first and the second model was made by Cociu & Cociu, (2012). The classical type of orthogonal model, using only currents, proved to be a little faster than the fluxes-only model. In all simulations, the computing time was smaller in the first case (see columns 4 and 6).

The results obtained using the fluxes-only simplified model turned out as expected. In all situations, the simulation time was lower by 10% (see columns 6 and 8), and the size of the output file was identical to the one when simulating based on the classical model (see columns 5 and 9). The simplified model performed the best out of the three models under comparison.

The topology of the new equivalent circuit (Fig. 6) is identical to the one from the classical case of the currents-only model, with the same number of nodes and controlled sources. For this reason, the size of the output file is identical. Regarding the increased simulation speed, the authors have yet to find a plausible explanation.

5. Conclusions

If the purpose of machine analyse is to determine the mechanical output quantities, it is natural to eliminate the currents or the fluxes from the voltage eqs. in order to simplify the mathematical calculation. The usual method when using PSpice is based on equivalent circuit implementation in order to solve the voltage eqs. and generic integrators utilization for mechanical eqs.

The classical type of orthogonal model, using currents, has proved to be a little faster than the fluxes model. In all situations, the computing time was smaller in the first case. But the new approach presented in this paper changes the situation. If the two stator and rotor resistances are equal the resulting equivalent circuit can be simplified.

The two stator and rotor circuits, which so far have been separated, can be coupled on the common part, consisting of the R_{vm} resistor through which a current corresponding to the difference of total fluxes flows .

The topology of the new equivalent circuit is identical to the one from the classical case of the currents-only model. It has the same number of nodes and controlled sources. For this reason, the size of the output file is identical. But the main result is the increase of integration speed in all cases. This implementation of the fluxes-only model proves to be faster than the classical currents-only one.

To conclude, the simplified equivalent circuit brings clear advantages in machine behaviour simulation in terms of integration speed and size of output data file. On the other hand, it can not be applied without introducing errors than if the two stator and rotor resistance are closed.

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MODEL IN FLUXURI SIMPLIFICAT AL MAŞINII DE INDUCȚIE UTILIZÂND PSpice

(Rezumat)

Pornind de la modul specific de implementare în PSpice a modelului ortogonal al maşinii de inducție utilizând doar fluxuri totale, lucrarea îşi propune simplificarea schemei electrice echivalente în scopul creşterii siguranței convergenței simulării şi a reducerii timpului de simulare. Prin prelucrări adecvate se ajunge la o schemă echivalentă simplificată pe baza considerării egalității a două tensiuni, una statorică şi alta rotorică. Se urmăreşte determinarea relației existente între cele două tensiuni în diferite situații şi se evaluează eroarea introdusă în cazul considerării egalității celor două tensiuni. Se trag concluzii privind simularea, mărimea fişierului de ieşire, precizia şi convergența.