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RESONANCES AT THE ACCESS GATES OF A LINEAR, NON-AUTONOMOUS AND RECIPROCAL GENERAL TWO-PORT, WORKING AT EMPTY LOAD OR IN SHORT-CIRCUIT, IN HARMONIC STEADY-STATE

BY

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Abstract. The necessary and sufficient conditions to realize the resonance at the input gate (*1*), (*1′*) of a linear, non-autonomous and reciprocal two-port, are established when either the gate (*2*), (*2′*) or the gate (*3*), (*3′*) is open or in short-circuit. The two-port is considered as working in harmonic steady-state.

The conditions which assure the simultaneous realization of a resonance regime at the input gate (*1*), (*1′*) and (*3*), (*3′*) gate when the gate (*2*), (*2′*) is open or in short-circuit or the simultaneous realization of a resonance regime at the input gate (*1*), (*1′*) and (*3*), (*3′*) gate when the gate (*2*), (*2′*) is open or in short-circuit are determined too.

Key words: linear, non-autonomous and reciprocal general two-port; working at empty load or in short-circuit; resonance at the input gate (*1*), (*1′*); double resonance at (*1*), (*1′*) and (*2*), (*2′*) or (*1*), (*1′*) and (*3*), (*3′*) gates.

1. Introduction

In several previous papers, firstly (Rosman, 1989) in a more accessible form and then (Rosman, 2004), in a much simpler form, the necessary and sufficient conditions which assure the realization of resonance at the input gate (*1*), (*1′*) , of a linear, non-autonomous and reciprocal general two-port, working

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in harmonic steady-state, were determined. It is necessary to underline that while in the first cited paper were studied the general two-ports in “hard” sense (Rosman, 2008), only the general two-ports in “soft” sense (Rosman, 2008) were considered in the second paper.

The aim of this paper is to propose some specifications concerning the resonance at the input gate of a linear, non-autonomous and reciprocal general two-port, working either at empty load or in short-circuit. In what follows such a two-port is considered, represented in Fig. 1 and abbreviatedly named LNRGT, having in harmonic steady-state, the equations (Sigorsky, 1962):

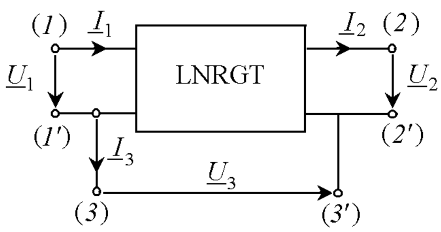


Fig. 1

$$\begin{bmatrix} \underline{U}_1 \\ \underline{I}_1 \\ \underline{U}_3 \end{bmatrix} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} & \underline{A}_{13} \\ \underline{A}_{21} & \underline{A}_{22} & \underline{A}_{23} \\ \underline{A}_{31} & \underline{A}_{32} & \underline{A}_{33} \end{bmatrix} \begin{bmatrix} \underline{U}_2 \\ \underline{I}_2 \\ \underline{I}_3 \end{bmatrix}, \tag{1}$$

where $\underline{A}_{ij}$, (i, j = 1, 2, 3), represent the fundamental parameters of the considered LNRGT, which satisfy the reciprocity relations (Sigorsky, 1962)

$$\left|\underline{A}\right| = \det\left[\underline{A}_{ij}\right] = \underline{A}_{33},\ \underline{A}_{12}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{21} = \underline{A}_{32},\ \underline{A}_{11}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31} = \underline{A}_{13}. \tag{2}$$

At the same time

$$\underline{Z}_2 = R_2 + \mathrm{j}X_2,\ \ \underline{Z}_3 = R_3 + \mathrm{j}X_3 \tag{3}$$

represent the receivers complex impedance considered passive ($R_2 \geq 0$, $R_3 \geq 0$), at the gates (*2*), (*2′*) and, respectively, (*3*), (*3′*), while

$$\underline{Z}_{e1} = R_{e1} + \mathrm{j}X_{e1} = \frac{\underline{U}_1}{\underline{I}_1} \tag{4}$$

is the complex equivalent impedance at the LNRGT’s input gate (*1*), (*1′*). It is evident that if the LNRGT is also passive (as the complex impedances $\underline{Z}_2$ and $\underline{Z}_3$), then $R_{e1} \geq 0$.

If signals $\underline{U}_2$, $\underline{I}_2$, $\underline{U}_3$, $\underline{I}_3$ are eliminated between eqs. (1), (3), (4) the equivalent complex impedance’s expression at the LNRGT’s input gate (*1*), (*1′*) is obtained namely

$$\underline{Z}_{e1} = \frac{\left(\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{12}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{22}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}, \tag{5}$$

which may be considered as function of complex impedances $\underline{Z}_2$, $\underline{Z}_3$ and of

LNRGT's fundamental parameters, $\underline{A}_{ij}$, (i,j = 1, 2, 3).

In the studied case, considering that is a matter of an LNRGT in "hard" sense, it is possible to conceive two distinct working regimes: at empty load when either $\underline{Z}_2 \rightarrow \infty$ or $\underline{Z}_3 \rightarrow \infty$, characterized by the equivalent complex impedances at the input gate

$$\underline{Z}_{e10}^{(2)} = \frac{\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}}, \tag{6}$$

respectively,

$$\underline{Z}_{e10}^{(3)} = \frac{\underline{A}_{11}\underline{Z}_2 + \underline{A}_{12}}{\underline{A}_{21}\underline{Z}_2 + \underline{A}_{22}}, \tag{7}$$

as well as two distinct working regimes in short-circuit when either $\underline{Z}_2$ =0 or $\underline{Z}_3$ = 0, characterized by the equivalent complex impedances at the input gate

$$\underline{Z}_{e1\text{sc}}^{(2)} = \frac{\underline{A}_{11}\underline{Z}_3 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_3 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{21}\underline{A}_{33}} \tag{8}$$

respectively,

$$\underline{Z}_{e1\text{sc}}^{(3)} = \frac{\left(\underline{A}_{13}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{21}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_2 + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}. \tag{9}$$

It is necessary to observe that $\underline{Z}_{e10}^{(3)}$ represents, in the same time, the expression of the equivalent complex impedance at the input gate of an in restricted sense two-port working at empty load. Also, it is possible to ascertain that expressions (6),…,(9) represents *conformal transformations* (Stoilov, 1964) of half-planes $R_2 \geq 0$, respectively $R_3 \geq 0$ (having in view that the complex impedances $\underline{Z}_2$, $\underline{Z}_3$ are supposed to be passive) in circles situated in complex plane (R_{e1} , jX_{e1}) (strictly speaking $\left(R_{e10}^{(2)},\ \text{j}X_{e10}^{(2)}\right)$, $\left(R_{e1\text{sc}}^{(2)},\ \text{j}X_{e1\text{sc}}^{(2)}\right)$, respectively $\left(R_{e10}^{(3)},\ \text{j}X_{e10}^{(3)}\right)$, $\left(R_{e1\text{sc}}^{(3)},\ \text{j}X_{e1\text{sc}}^{(3)}\right)$). The first two circle's eqs., established by Rosman & Belaus, (2000), are

$$a'\left(R_{e10}^{(2)2} + X_{e10}^{(2)2}\right) + b'R_{e10}^{(2)} + c'X_{e10}^{(2)} + d' = 0, \tag{10}$$

$$a''\left(R_{e1\text{sc}}^{(2)2} + X_{e1\text{sc}}^{(2)2}\right) + b''R_{e1\text{sc}}^{(2)} + c''X_{e1\text{sc}}^{(2)} + d'' = 0, \tag{11}$$

while the ones of the two other circles, established also by Rosman & Belaus, (2000), are

$$e'\left(R_{e10}^{(3)2}+X_{e10}^{(3)2}\right)+f'R_{e10}^{(3)}+g'X_{e10}^{(3)}+h'=0, \tag{12}$$

$$e''\left(R_{e1sc}^{(3)2}+X_{e1sc}^{(3)2}\right)+f''R_{e1sc}^{(3)}+g''X_{e1sc}^{(3)}+h''=0. \tag{13}$$

Coefficients a', b', c', d', a'', b'', c'', d'', e', f', g', h', e'', f'', g'', h'' expressions, which are functions only of LNRGT's fundamental parameters, $\underline{A}_{ij}$, $(i, j = 1, 2, 3)$, are given in an other previous paper (Rosman, 2013).

Having in view that the considered LNRGT is supposed as being passive it results that necessarily the circles (10),…, (13) are situated in half-plane $R_{e10} \geq 0$ so that the theirs center abscissae are positive and consequently

$$\text{sign } a' = -\text{sign } b',\ \text{sign } a'' = -\text{sign } b'',\ \text{sign } c' = -\text{sign } d',\ \text{sign } c'' = -\text{sign } d''. \tag{14}$$

In the same time the circles (10,…,(13) not intersect the axis $X_{e10}^{(2)}=0$, $X_{e1sc}^{(2)}=0$, $X_{e10}^{(3)}=0$, $X_{e1sc}^{(3)}=0$, so that

$$c'^2 < 4a'd',\ \ c''^2 < 4a''d'',\ \ g'^2 < 4e'f',\ \ g''^2 < 4e''f''. \tag{15}$$

2. Resonance at the Input Gate

At the input gate of an LNRGT may be distinguished more possibilities to realize the resonance when this one works either at empty load or in short-circuit. A first possibility may be render evident when the gate (*2*), (*2′*) of the LNRGT's is open. In this case eq. (10) defines the frontier of equivalent complex impedance $\underline{Z}_{e10}^{(2)}$ existence domain. In view to realize the resonance at the input gate; in this case, it is necessary that the circle (10) intersect the $X_{e10} = 0$ axis, or, at most, be tangent to this one. This condition is realized when parameters a', b', d' satisfy the inequality

$$b'^2 \geq a'd'. \tag{16}$$

Having in view the inequality (15_1), this one, combined with (16) may lead to the double inequality

$$c'^2 < 4a'd' \leq b'^2. \tag{17}$$

The eq.'s $a'R_{e10}^2 + c'R_{e10} + d' = 0$ roots represent the equivalent resistances at the input gate in the regimes which realize the resonance at this gate. Taking into account relation (14_1) these resistances are situated in the range

$$R_{e10}^{(2)} \in \left[\frac{-b' + \sqrt{b'^2 - 4a'd'}}{2a'},\ \frac{-b' - \sqrt{b'^2 - 4a'd'}}{2a'} \right], \tag{18}$$

when $a' < 0$, $b' > 0$; a similar relation may be established when $a' > 0$, $b' < 0$. The limits of this range correspond to points M, respectively N (Fig. 2).

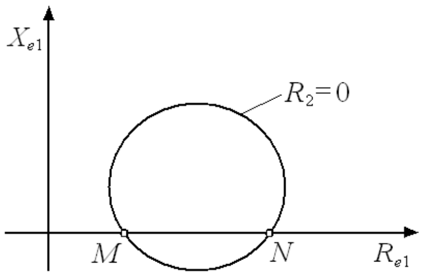


Fig. 2

It is obvious that the realization of resonance at the input gate of a LNRGT working with the gate (*2*), (*2′*) open implies the necessity that the double inequality (17) be satisfied. Since the coefficients a', b', c', d' are functions of fundamental parameters, $\underline{A}_{ij}$, $(i,j = 1, 2, 3)$, only, and these ones are, at their turn, functions of the harmonic tension frequency applied at the LNRGT's input gate it results that the considered resonance regime may be realized only in a certain frequency range.

In a similar way it can be studied the possibilities to realize the resonance at the input gate of an LNRGT when: a) the (*2*), (*2′*) gate is in short-circuit; b) the (*3*), (*3′*) gate is open; c) the gate (*3*), (*3′*) is in short-circuit.

In the first case the necessary and sufficient condition to realize the resonance at the LNRGT's input gate is that the double inequality

$$a''^2 < 4a''d'' \le b''^2 \tag{19}$$

be satisfied; if $a'' < 0$, $b'' > 0$ the resistances at the two-port's input gate are situated, at resonance, in the range

$$R_{e1sc}^{(2)} \in \left[\frac{-b'' + \sqrt{b''^2 - 4a''d''}}{2a''},\ \frac{-b'' - \sqrt{b''^2 - 4a''d''}}{2a''} \right]. \tag{20}$$

In the other two cases, using a similar proceeding, it results that the double inequalities

$$g'^2 < 4e'h' \le f'^2, \tag{21}$$

or, respectively,

$$g''^2 < 4e''h'' \le f''^2, \tag{22}$$

must be satisfied, the ranges in which the equivalent resistance at the LNRGT's input gate exists being

$$R_{e10}^{(3)} \in \left[\frac{-f'+\sqrt{f'^2-4e'h'}}{2e'},\ \frac{-f'-\sqrt{f'^2-4e'h'}}{2e'}\right], \tag{23}$$

when $e' < 0$, $f' > 0$, respectively

$$R_{e1sc}^{(3)} \in \left[\frac{-f''+\sqrt{f''^2-4e''h''}}{2e''},\ \frac{-f''-\sqrt{f''^2-4e''h''}}{2e''}\right]. \tag{24}$$

if $e'' < 0$, $f'' > 0$.

3. Double Resonance at the Input Gate and One of the Output Gates

In view to establish the conditions in which the resonance is eventually realized, simultaneously both at the input gate and at the other gate than the one which works either at empty load or in short-circuit, it is necessary to determine, beforehand, in the (R_{e1} , jX_{e1}) plane, the curves corresponding either to case when $X_3 = 0$ (the (*2*), (*2′*) gate is open or works in short-circuit), or to case when $X_2 = 0$ (the (*3*), (*3′*) gate is open or works in short-circuit). In the first two cases relations (6) and (8) are utilized and in the last two cases, relations (7) and (9) are used. In the above situations the complex impedances $\underline{Z}_2$, respectively $\underline{Z}_3$, are explicited, imposing the condition that theirs imaginary part (X_2 , respectively X_3) be null. In this case are taken into account too that relations (6),…,(9) represent conformal transformations which transform circles from (R_3 , jX_3) plane in circles situated in (R_1 , jX_1) plane (Stoilov, 1964). Applying this proceeding, details being indicated in the Appendix, the following eqs.

$$\alpha'\left(R_{e10}^{(2)2} + X_{e10}^{(2)2}\right) + \beta' R_{e10}^{(2)} + \gamma' X_{e10}^{(2)} + \delta' = 0, \tag{25}$$

$$\alpha''\left(R_{e1sc}^{(2)2} + X_{e1sc}^{(2)2}\right) + \beta'' R_{e1sc}^{(2)} + \gamma'' X_{e1sc}^{(2)} + \delta'' = 0, \tag{26}$$

respectively

$$\varepsilon'\left(R_{e10}^{(3)2}+X_{e10}^{(3)2}\right)+\varphi' R_{e10}^{(3)}+\lambda' X_{e10}^{(3)}+\mu'=0, \tag{27}$$

$$\varepsilon''\left(R_{e1sc}^{(3)2}+X_{e1sc}^{(3)2}\right)+\varphi'' R_{e1sc}^{(3)}+\lambda'' X_{e1sc}^{(3)}+\mu''=0, \tag{28}$$

are obtained. The expressions of coefficients α', β', γ', δ', α'', β'', γ'', δ'', ε', φ', λ', μ', ε'', φ'', λ'', μ'' are given in the Appendix.

The considered two-port as the complex impedances $\underline{Z}_2$, $\underline{Z}_3$ being passive, the circles (24),…,(27) are situated entirely as the half-plane $R_{e1} \geq 0$ and consequently

$$\alpha'^2 > 4\alpha'\delta', \;\; \alpha''^2 > 4\alpha''\delta'', \;\; \lambda'^2 > 4\varepsilon'\mu', \;\; \lambda''^2 > 4\varepsilon''\mu'', \tag{29}$$

when the resonance is realized at the (*3*), (*3′*) gate. Considering, for instance, the case characterized by eq. (24) any point situated on this circle's contour corresponds to a resonance regime realized at the (*3*), (*3′*) gate.

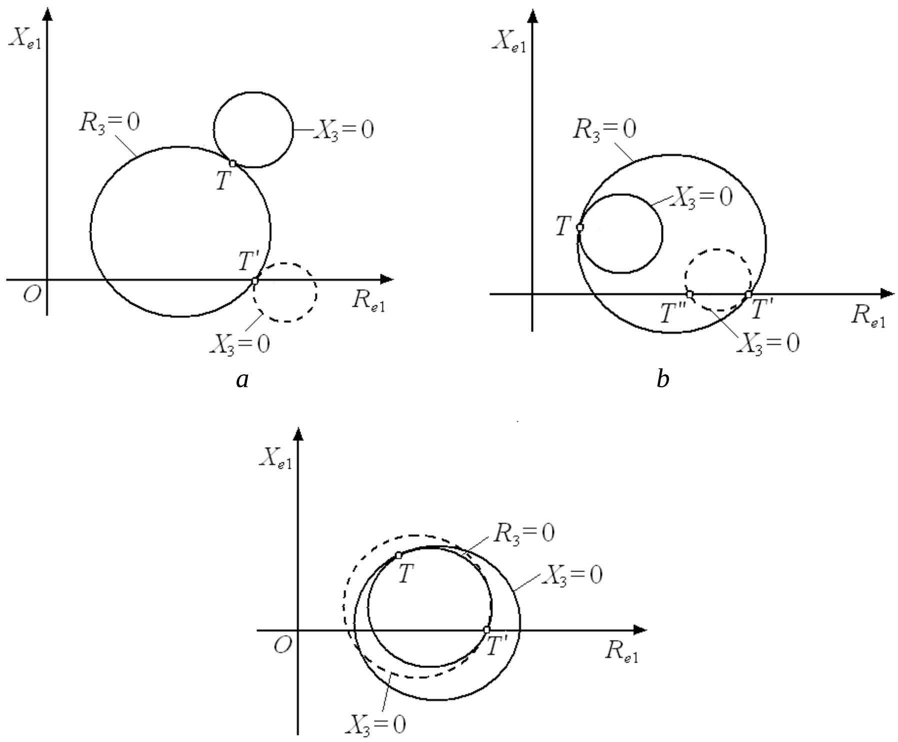
If is considered, for instance, the case when the LNRGT works at empty load with the (*2*), (*2′*) gate open ($\underline{I}_2 = 0$), the existence domain of the equivalent complex impedance at the LNRGT's input gate (*1*), (*1′*) is situated inside and of the frontier of circle (10) (circle $R_3 = 0$ – Fig. 3 *a*); it is important to establish the circle's (24) ($X_3 = 0$) position in the complex plane (R_{e1} , jX_{e1}). Having in view relation (6) it results

$$\underline{Z}_{e10}^{(2)}=\underline{\tilde{Z}}_{e10}^{(2)}=\frac{\underline{A}_{13}\underline{A}_{21}-\underline{A}_{11}\underline{A}_{33}}{\underline{A}_{23}\underline{A}_{31}-\underline{A}_{21}\underline{A}_{33}}, \tag{30}$$

when simultaneously $R_3 = 0$, $X_3 = 0$. The obtained result point out that circles $R_3 = 0$ (s. rel. (10)) and $X_3 = 0$ (s. rel. (24)) are *tangent*. These ones may be tangent *exterior* (Fig. 3 *a*) or *interior* (Fig. 3 *b*), *T* being the tangency point. The fact that these circles are tangent exterior or interior depends on certain metric relations between the circles (10) and (24) centers coordinates and theirs radiuses. In the particular case when LNRGT's fundamental parameters satisfy relation $\Im m\left(\underline{\tilde{Z}}_{e10}^{(2)}\right)=0$, that is

$$A_{31}^2\Im m\left(\underline{A}_{13}\underline{A}_{23}^*\right)+A_{32}^2\Im m\left(\underline{A}_{11}\underline{A}_{21}^*\right)=\Im m\left[\underline{A}_{21}\underline{A}_{33}^*\left(\underline{A}_{13}\underline{A}_{31}^*-\underline{A}_{11}\underline{A}_{33}^*\right)\right], \tag{31}$$

where relation (29) was taken into account, the tangency point of circles (10) and (24) is situated on the $X_{e1} = 0$ axis, corresponding to a double resonance, simultaneously realized at the input gate (*1*), (*1′*) and at the gate (*3*), (*3′*), the gate (*2*), (*2′*) being open (working at empty load). A second point, *T″*, in which the circle $X_3 = 0$ intersects, in plane (R_{e1}, X_{e1}), the $X_{e1} = 0$ axis is possible to be obtained only in the case represented in Fig. 3 *b*.



c

Fig. 3

Similarly may be studied the case when an LNRGT works with the gate (*2*), (*2′*) in short-circuit. In this situation the complex impedance which corresponds to circles (11) and (25) tangency point is

$$\underline{Z}^{(2)}_{e1\mathrm{sc}} = \underline{\tilde{Z}}^{(2)}_{e1\mathrm{sc}} = \frac{\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{23}}{\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}, \tag{32}$$

where relations (8) and $\underline{Z}_3 = 0$ were taked into account. In this case, the tangency point, T'', is situated on the $X_{e1} = 0$ axis corresponding to a double resonance both at the LNRGT's input gate, when this one works in short-circuit at the (*2*), (*2′*) gate and the (*3*), (*3′*) gate if relation $\Im m\left(\underline{\tilde{Z}}^{(2)}_{e1\mathrm{sc}}\right) = 0$ is satisfied namely

$$A_{32}^2 \Im m\left(\underline{A}_{13}\underline{A}_{23}^*\right) + A_{33}^2 \Im m\left(\underline{A}_{12}\underline{A}_{22}^*\right) = \Im m\left[\underline{A}_{32}\underline{A}_{33}^*\left(\underline{A}_{13}\underline{A}_{22}^* + \underline{A}_{12}\underline{A}_{23}^*\right)\right]. \tag{33}$$

Analogous results may be obtained when the LNRGT works with the (*3*), (*3′*) gate either open, or in short-circuit. It is useful to mention that instead of circles $R_3 = 0$ (eq. (10)) it is necessary to consider, in plane (R_{e1}, jX_{e1}), the circles $R_2 = 0$ (eq. (11)) and $X_2 = 0$ (eq. (25)). The graphical aspect is similarly to that represented in Fig. 3, the complex impedances which correspond to the tangency point (T) of circles (11) and (25) being

$$\underline{Z}_{e10}^{(3)} = \underline{\tilde{Z}}_{e10}^{(3)} = \frac{\underline{A}_{12}}{\underline{A}_{22}}, \tag{34}$$

respectively

$$\underline{Z}_{e1\text{sc}}^{(3)} = \underline{\tilde{Z}}_{e1\text{sc}}^{(3)} = \frac{\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\underline{A}_{23}\underline{A}_{32} - \underline{A}_{22}\underline{A}_{33}}. \tag{35}$$

It is easy to observe that

$$\underline{Z}_{e1\text{sc}}^{(2)} = \underline{\tilde{Z}}_{e1\text{sc}}^{(3)}. \tag{36}$$

The circles (11) and (25) tangency point is situated on the $X_{e1} = 0$ axis if

$$\Im m\left(\underline{A}_{12}\underline{A}_{22}^{*}\right) = 0, \tag{37}$$

respectively if relation (32) is satisfied rendering evident the existence of a double resonance.

In this case too it is possible to observe the existence of a second double resonance regime, simultaneously, at the (*1*), (*1′*) gate and at the (*3*), (*3′*) gate when the (*2*), (*2′*) gate is either open or in short-circuit.

5. Conclusions

Utilizing some conformal transformations the existence domain of the equivalent complex impedances at the input gate of a linear, non-autonomous, reciprocal, general two-port are determined when either the (*2*), (*2′*) gate or the (*3*), (*3′*) gate is open or in short-circuit. If the curves which mark these domains are known the necessary and sufficient conditions which assure the realization of resonance at the input gate of such a two-port, in the mentioned working regimes, are determined.

The conditions which assure the realization, simultaneously, of resonance either at the gates (*1*), (*1′*) and (*3*), (*3′*) when (*2*), (*2′*) gate is either open or in short-circuit, or at the gates (*1*), (*1′*) and (*2*), (*2′*) when (*3*), (*3′*) gate is either open or in short-circuit are determined too.

Appendix

Annulling the imaginary part of complex impedance

$$\underline{Z}_3 = \frac{\left(\underline{A}_{21}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{31}\right)\underline{Z}_{e10}^{(2)} + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{11}\underline{A}_{33}}{\underline{A}_{21}\underline{Z}_{e10}^{(2)} - \underline{A}_{12}}, \quad \left(\underline{Z}_{e10} \neq \frac{\underline{A}_{11}}{\underline{A}_{21}}\right), \tag{A.1}$$

deduced from relation (6), eq. (24) is obtained, where

$$\begin{aligned}
\alpha' &= \Im m\left[\underline{A}_{21}^{*}\left(\underline{A}_{21}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{31}\right)\right],\\
\beta' &= -2\Im m\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) + \Im m\left[\underline{A}_{31}\left(\underline{A}_{11}^{*}\underline{A}_{23} + \underline{A}_{13}\underline{A}_{21}^{*}\right)\right],\\
\gamma' &= 2\Im m\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) + \Re e\left[\underline{A}_{21}\left(\underline{A}_{11}^{*}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{21}^{*}\right)\right],\\
\delta' &= -\Im m\left[\underline{A}_{11}^{*}\left(\underline{A}_{12}\underline{A}_{31} - \underline{A}_{11}\underline{A}_{33}\right)\right].
\end{aligned} \tag{A.2}$$

If the imaginary part of complex impedance

$$\underline{Z}_3 = \frac{\left(\underline{A}_{22}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{32}\right)\underline{Z}_{e1sc}^{(2)} + \underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31}}{\underline{A}_{22}\underline{Z}_{e1sc}^{(2)} - \underline{A}_{12}}, \quad \left(\underline{Z}_{e1sc}^{(2)} \neq \frac{\underline{A}_{12}}{\underline{A}_{22}}\right), \tag{A.3}$$

obtained from expression (8) is annulled, it results eq. (23) with

$$\begin{aligned}
\alpha'' &= \Im m\left[\underline{A}_{22}^{*}\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\right],\\
\beta'' &= -2\Im m\left(\underline{A}_{33}\right)\Re e\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) + \Im m\left[\underline{A}_{32}\left(\underline{A}_{12}^{*}\underline{A}_{23} + \underline{A}_{13}\underline{A}_{32}^{*}\right)\right],\\
\gamma'' &= -2\Im m\left(\underline{A}_{33}\right)\Im m\left(\underline{A}_{11}\underline{A}_{21}^{*}\right) + \Re e\left[\underline{A}_{32}\left(-\underline{A}_{12}^{*}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{32}^{*}\right)\right],\\
\delta'' &= -\Im m\left[\underline{A}_{12}^{*}\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}\right)\right].
\end{aligned} \tag{A.4}$$

Relation (7) permits to obtain the expression

$$\underline{Z}_2 = -\frac{\underline{A}_{12} - \underline{A}_{22}\underline{Z}_{e10}^{(3)}}{\underline{A}_{21}\underline{Z}_{e10}^{(3)} - \underline{A}_{11}}, \quad \left(\underline{Z}_{e10} \neq \frac{\underline{A}_{11}}{\underline{A}_{21}}\right), \tag{A.5}$$

which leads to eq. (26) if her imaginary part is annulled, the coefficients of this eq. having the expressions

$$\begin{aligned}
\varepsilon' &= \Im m\left(\underline{A}_{21}\underline{A}_{22}^{*}\right), \quad \varphi' = \Im m\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right),\\
\lambda' &= \Re e\left(\underline{A}_{11}\underline{A}_{22}^{*} - \underline{A}_{12}\underline{A}_{21}^{*}\right), \quad \mu' = \Im m\left(\underline{A}_{11}\underline{A}_{12}^{*}\right).
\end{aligned} \tag{A.6}$$

Using relation (8) the expression of complex impedance

$$\underline{Z}_2 = \frac{\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\underline{Z}_{e1sc}^{(3)} + \underline{A}_{23}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{33}}{\left(\underline{A}_{23}\underline{A}_{31} - \underline{A}_{21}\underline{A}_{33}\right)\underline{Z}_{e1sc}^{(3)} + \underline{A}_{11}\underline{A}_{33} - \underline{A}_{13}\underline{A}_{31}} \quad \text{(A.7)}$$

may be obtained; annulling her imaginary part; it results eq. (27) where

$$\begin{aligned}
\varepsilon" &= \Im m\left[\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\left(\underline{A}^*_{23}\underline{A}^*_{31} - \underline{A}^*_{21}\underline{A}^*_{33}\right)\right],\\
\varphi" &= \Im m\left[\left(\underline{A}_{22}\underline{A}_{33} - \underline{A}_{23}\underline{A}_{32}\right)\left(\underline{A}^*_{11}\underline{A}^*_{33} - \underline{A}^*_{13}\underline{A}^*_{31}\right) + \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31}\right)\left(\underline{A}^*_{23}\underline{A}^*_{31} - \underline{A}^*_{21}\underline{A}^*_{33}\right)\right],\\
\lambda" &= \Re e\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31}\right)\left(\underline{A}^*_{21}\underline{A}^*_{31} - \underline{A}^*_{21}\underline{A}^*_{33}\right) - \left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31}\right)\left(\underline{A}^*_{11}\underline{A}^*_{33} - \underline{A}^*_{13}\underline{A}^*_{31}\right)\right],\\
\mu" &= \Im m\left[\left(\underline{A}_{13}\underline{A}_{32} - \underline{A}_{12}\underline{A}_{31}\right)\left(\underline{A}^*_{11}\underline{A}^*_{33} - \underline{A}^*_{13}\underline{A}^*_{31}\right)\right].
\end{aligned} \quad \text{(A.8)}$$

REFERENCES

Rosman H., Belaus D., *Sur les domaines de fonctionnement à vide et en court-circuit des quadripôles généraux linéaires, non-autonomes, en régime permanent harnonique*. Bul. Inst. Politehnic, Iaşi, **XLVI (L)**, *3-4*, s. Electrot., Energ., Electron., 13-22 (2000).

Rosman H., *La résonance à la porte d'entrée d'un quadripôle général linéaire et passif en régime permanent harnonique*. Bul. Inst. Politehnic, Iaşi, **XXXV (XXXIX)**, *3-4*, s. Electrot., Energ., Electron., 1-11 (1989).

Rosman H., *On the Working Conditions at Empty Load and Short-Circuit of a Linear, Non-Autonomous and Reciprocal General Two-Port in Harmonic Steady-State*. Bul. Inst. Politehnic, Iaşi, **LIX (LXIII)**, *2*, s. Electrot., Energ., Electron., 55-64 (2013).

Rosman H., *Sur les conditions de réalisation de la résonance à la porte d'entrée d'un quadripôle général linéaire non-autonome, alimentant un récepteur passif, en régime permanent harnonique*. Bul. Inst. Politehnic, Iaşi, **XLIX (LIII)**, *1-2*, s. Electrot., Energ., Electron., 31-38 (2004).

Rosman H., *About the Equivalence between a General and a Restricted Sense Two-Ports, both Linear and Non-Autonomous.*Rev. Roum. Sci. Techn., s. Electrot. et Energ., **LII**, *1*, 3-10 (2008).

Sigorsky V.P., *General Theory of Two-Ports* (in Russian). Ed. Acad. of Sci., Ukraine, Kiev, 1962.

Stoilov S., *Teoria funcţiilor de variabilă complexă*. Edit. Acad., Bucureşti, 1964.

REZONANŢE LA PORŢILE DE ACCES ALE UNUI CUADRIPOL GENERAL LINIAR, NEAUTONOM ŞI RECIPROC, FUNCŢIONÂND LA GOL SAU ÎN SCURTCIRCUIT, ÎN REGIM PERMANENT ARMONIC

(Rezumat)

Se stabilisc condiţiile necesare şi suficiente de realizare a rezonanţelor la poarta de intrare, (*1*), (*1'*), a unui cuadripol general liniar, neautonom şi reciproc având fie

poarta (*2*), (*2′*), fie poarta (*3*), (*3′*), la gol sau în scurtcircuit, în cazul în care cuadripolul funcționează în regim permanent armonic.

Se precizează condițiile în care un astfel de coadripol realizează, simultan, rezonanța, la perțile (*1*), (*1′*) și (*2*), (*2′*) (sau (*3*), (*3′*)) atunci când poarta (*3*), (*3′*) (sau (*2*), (*2′*)) este la gol sau în scurtcircuit.