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# ON FINITE REPRESENTATION IMPLEMENTATION OF GABOR FILTERS

BY

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**Abstract.** Gabor filters have been widely used in signal processing and pattern recognition applications. In this paper the implementation of recursive filters which approximate Gabor filters frequency response using fixed-point number representation has been analysed. The minimum bits number, necessary to avoid overflows has been determined.

Key words: Gabor filters; recursive filters; fixed-point arithmetic.

# 1. Introduction

Gabor filters have been proposed by Denis Gabor in 1946 and they are used in image processing and pattern recognition applications: human face recognition (Wiscott *et al.*,1995; Choi *et al*, 2008; Lei *et al.*, 2011) textures recognition (Manjunath *et al.*, 2000; David & Ungureanu, 2007; Han *et al.*, 2007), edge detection (Kovesi, 1999), audio signal recognition (Wu *et al.*, 2011), quality image improvement (Yang & Li, 2005) or, more recently, in new signal processing research fields like compressed sensing (Cho & Jay Kuo, 2011). Gabor filters have been also chosen as descriptors for textures in the

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MPEG-7 video compression standard. The motivation for using Gabor filters is related to the maximum information processing in both time domain and frequency domain. Another advantage is that the magnitude and phase filter responses can be analysed separately. In many papers, the observation made by Daugman, (1985), that the time response of mammals primary visual cortex cells resembles the Gabor filter response is mentioned.

The main disadvantage of the Gabor filters signal processing systems is that it usually uses a bank of 30...40 digital filters. This fact affects the digital systems computational needs. In order to reduce the multiplications and additions number, of the filtering can be made in the frequency domain using FFT algorithm. The signal dimension should be power of 2 or algorithms as overlap-save, overlap-save should be used. Another solution, less computationally intensive, is to use infinite impulse response filters which approximate as precisely as possible Gabor filters frequency response or time response. In this case the computational complexity is O(2M(N + 1)), where M represents the signal dimension and 2N represents the IIR filter order. One solution has been proposed by Young et al., (2002), where a rational approximation of Gaussian function has been used, the Laplace transform to obtain a H(s) transfer function and a change of variables  $(s = (1 - z^{-1})/T))$  in order to obtain H(z) transfer function. Another solution has been proposed by David & Ungureanu, (2005), where the Padé and respective Padé-Chebyshev approximation have been used in order to approximate the frequency characteristics of Gabor filters. Using Padé approximation have been obtained approximately the same errors as those presented by Young et al., (2002), but the advantage is that analytically coefficients of discrete filter can be obtained. The method can also be used in order to obtain discrete filters with orders higher then 6. Using Padé-Chebyshev approximation, have been obtained maximum absolute errors with approximately one order of magnitude lower than using Padé approximation or the method proposed by Young et al., (2002).

### 2. 1-D and 2-D Gabor Filters

In a typical image processing/analysis system that uses Gabor filters, the signal is filtered using each of the filters of the bank, the norm of each resulted signal is calculated, and a feature vector results. The feature vectors are used in the classification process.

An 1-D Gabor filter is described by a Gaussian function modulated in amplitude

$$g(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-x^2/2\sigma^2} e^{j\Omega_0 x}.$$
 (1)

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 $\sqrt{2}^{i}$ , where i = 1,...,N - 1. Correspondingly, the frequency characteristic of such a filter will be a Gaussian function defined in the frequency domain shifted to the central frequency,  $\Omega_0$ 

In a typical pattern recognition system, the standard deviation is  $2^{i}$  or

$$G(\omega) = \sqrt{\sigma} e^{-(\omega - \Omega_0)^2 \sigma^2/2}.$$
 (2)

Many applications, use filters which have the following wavelet property,  $\Omega_0 \sigma = k$ . Thus the Gabor filters responses are self-similar (scaled and dilated versions of a mother function).

For image processing applications, 2-D Gabor filters are used. In this case  $\sigma_x$  and  $\sigma_y$  represent standard deviations along x and y axis, respectively

$$g(x, y) = g(x)g(y) = \frac{1}{2\pi\sqrt{\sigma_x \sigma_y}} e^{-\frac{x^2}{2\sigma_x^2} \cdot \frac{y^2}{2\sigma_y^2}} e^{j(\Omega_x x \Omega_y y)}.$$
 (3)

The frequency characteristic of such a 2-D Gabor filter is a 2-D Gaussian function shifted to the frequency  $(\Omega_x, \Omega_y)$ 

$$G(\omega_x, \omega_y) = \sqrt{\sigma_x \sigma_y} \exp\left[-\frac{(\omega_x - \Omega_x)^2 \sigma_x^2}{2} - \frac{(\omega_y - \Omega_y)^2 \sigma_y^2}{2}\right].$$
 (4)

Between  $\sigma_x$ ,  $\sigma_y$ ,  $\Omega_x$ ,  $\Omega_y$  the following relations can exist:

$$\Omega_x = \Omega_v \cos \varphi_u, \ \Omega_y = \Omega_v \sin \varphi_u, \ \Omega_v = 2^{-2} \pi, \ \varphi_u = \frac{u\pi}{N};$$

$$(v = 1, ..., M; \ u = 0, ..., N - 1).$$
(5)

# 3. Rational Approximations of Gabor Filters Frequency Characteristics

In order to approximate the Gabor filters frequency characteristic (David & Ungureanu, 2005), the Gaussian function was approximated using a Padé or a Padé-Chebyshev approximation. If the Padé approximation has been used the following frequency characteristics for N=1, 2 and 3 has been obtained:

$$H_{1}(\omega) = \sqrt{\sigma} \left(1 + \sigma^{2} - \sigma^{2} \cos \omega\right)^{-1},$$

$$H_{2}(\omega) = 12\sqrt{\sigma} \left[12 + 15\sigma^{2} + 9\sigma^{4} - \left(16\sigma^{2} + 12\sigma^{4}\right) + \left(\sigma^{2} + 3\sigma^{4}\right)\cos 2\omega\right]^{-1},$$

$$H_{3}(\omega) = 360\sqrt{\sigma} \left[360 + 490\sigma^{2} + 420\sigma^{4} + 150\sigma^{6} - \left(540\sigma^{2} + 585\sigma^{4} + 225\sigma^{6}\right)\cos \omega + ...\right]$$

$$\dots + \left(54\sigma^{2} + 180\sigma^{4} + 90\sigma^{6}\right)\cos 2\omega - \left(4\sigma^{2} + 15\sigma^{4} + 15\sigma^{6}\right)\cos 3\omega\right]^{-1}.$$
(6)

In order to obtain Gabor filters frequency characteristic, the rational approximation has been shifted to the frequency  $\Omega_0$ .

Using the change of variable  $z = e^{i\omega}$ , the discrete transfer function of the Gaussian filter has been obtained. Using the change of variable  $z \rightarrow z e^{i\omega}$ , the transfer function of a Gabor filter has been obtained. In the eq.

$$H\left(z\mathrm{e}^{-\mathrm{j}\Omega_{0}}\right) = \sqrt{\sigma} \frac{z^{N}\mathrm{e}^{-\mathrm{j}\Omega_{0}}}{c\prod_{k=1}^{N}\left(z\mathrm{e}^{-\mathrm{j}\Omega_{0}} - p_{k}\right)\left(z\mathrm{e}^{-\mathrm{j}\Omega_{0}} - \frac{1}{p_{k}}\right)};\tag{7}$$

 $p_k$  and  $1/p_k$  represent H(z) poles.

The transfer function (7) is the product of a stable transfer function (which has the poles  $p_k$ ), and an unstable transfer function (which has the poles  $1/p_k$ ).

For N = 1, the Gaussian filter has the following transfer function:

$$H(z) = \frac{\sqrt{\sigma}}{-\frac{\sigma^2}{2}z^{-1} + (1+\sigma^2) - \frac{\sigma^2}{2}z} = -\frac{2}{\sigma\sqrt{\sigma}} \cdot \frac{z}{(z-p)\left(z-\frac{1}{p}\right)},$$
(8)

where the pole *p* has the value (|p| < 1):

$$p = \frac{(1+\sigma^2) - \sqrt{1+2\sigma^2}}{\sigma^2}.$$
 (2)

The Gaussian filter transfer function can be written as a product between a stable transfer function  $(H_{-}(z))$  and an unstable transfer function  $(H_{+}(z))$ 

$$H(z) = \left(\frac{\sqrt{2p}}{\sigma} \cdot \frac{z}{z-p}\right) \left(\frac{\sqrt{2p}}{\sqrt{\sigma}} \cdot \frac{-1}{zp-1}\right) = H_{-}(z)H_{+}(z).$$
(3)

The stable and unstable Gabor filters transfer functions are

$$H_{-}(ze^{-j\Omega_{0}}) = \frac{\sqrt{2p}}{\sigma} \cdot \frac{1}{1 - pe^{j\Omega_{0}}z^{-1}},$$
(4)

respectively

$$H_{+}(ze^{-j\Omega_{0}}) = \frac{\sqrt{2p}}{\sqrt{\sigma}} \cdot \frac{1}{ze^{-j\Omega_{0}}p - 1}.$$
(5)

For the implementation of the unstable transfer function, a method proposed by Mitra, (1998), and Young *et al.*, (2002), has been used. The main idea is that a stable filter has the same amplitude frequency characteristic as an unstable one. A discrete filter which has a pole outside of unit circle becomes stable if the change of variable  $z \rightarrow z^{-1}$  is used. In order to obtain an unchanged spectrum of the output filter, the input signal of the unstable filter is flipped and the output signal is flipped again. Gabor filter implementation using this method is presented in Fig. 1.

$$e[n] \longrightarrow H_{-}(ze^{-j\Omega_{0}}) \longrightarrow H_{+}(z^{-1}e^{-j\Omega_{0}}) \longrightarrow I$$

Fig. 1 – Gabor filter implementation.

Thus, although the filter  $H(ze^{-j\Omega_0})$  is unstable, the signal y[n] from Fig. 1, has the same spectrum as filter  $H(ze^{-j\Omega_0})$  output.

The expression of  $H_+(z^{-1}e^{-j\Omega_0})$  transfer function is

$$H_{+}(z^{-1}e^{-j\Omega_{0}}) = \frac{\sqrt{2p}}{\sqrt{\sigma}} \cdot \frac{1}{1 - pe^{-j\Omega_{0}}z^{-1}}.$$
 (6)

The notations from Fig. 1 are

$$H_{-}(ze^{-j\Omega_{0}}) = \frac{\tilde{W}(z)}{E(z)} \text{ and } H_{+}(z^{-1}e^{-j\Omega_{0}}) = \frac{\tilde{Y}(z)}{W(z)},$$

$$e[n] = e_{\text{real}}[n] + je_{\text{imag}}[n],$$

$$\tilde{w}[n] = \tilde{w}_{\text{real}}[n] + j\tilde{W}_{\text{imag}}[n],$$

$$y[n] = y_{\text{real}}[n] + j y_{\text{imag}}[n].$$
(7)

# 4. The Influence of Bits Number on Filter Response

The implementation of the stable Gabor filter using an IIR approximation is presented in Fig. 2. For the representation of input signal, L bits are used. The multiplicators have L bits inputs and 2L bits output. The

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adders have 2L bits inputs and 2L bits output. A quantizer that reduces the number of bits from 2L bits to L bits has been used.



Fig. 2 – Gabor filter implementation for N = 1.

In order to avoid signal overflows it is necessary to have an estimation for  $|u_{\text{1real}}[n]|$ ,  $|u_{\text{1imag}}[n]|$ ,  $|u_{2\text{real}}[n]|$ ,  $|u_{1\text{imag}}[n]|$  maximum values. The relations (8), (10) give  $|u_1[n]|$  and  $|u_2[n]|$  maximum values. It can be considered, also, that these relations give estimations for maximum absolute value of the real and imaginary parts.

$$\frac{U_{1}(z)}{E(z)} = \frac{1}{1 - p e^{j\Omega_{0}} z^{-1}} \Longrightarrow u_{1}[n] = \sum_{m=0}^{\infty} e[n - m](p e^{j\Omega_{0}})^{m},$$

$$|u_{1}[n]| = \left|\sum_{m=0}^{\infty} e[n - m](p e^{j\Omega_{0}})^{m}\right| < \sum_{m=0}^{\infty} \left|(p e^{j\Omega_{0}})^{m}\right| = \sum_{m=0}^{\infty} |p|^{m} = \frac{1}{1 - |p|},$$

$$|\tilde{w}[n]| = \left|\frac{\sqrt{2p}}{\sigma} u[n]\right| < \frac{\sqrt{2p}}{\sigma} \frac{1}{1 - |p|} = 1.$$
(9)

Then

$$\left| u_{2}[n] \right| < \left| \sum_{m=0}^{\infty} \tilde{w}[n-m](pe^{-j\Omega_{0}})^{m} \right| < \left| \sum_{m=0}^{\infty} (pe^{-j\Omega_{0}})^{m} \right| = \sum_{m=0}^{\infty} \left| p \right|^{m} = \frac{1}{1-\left| p \right|}, \quad (10)$$

$$|y[n]| < \sum_{m=0}^{\infty} |p|^m = \frac{\sqrt{2p}}{\sqrt{\sigma}} \frac{1}{1-|p|} = \sqrt{\sigma}.$$
 (11)

Fig. 3 presents the estimated maximum absolute values for  $u_{\text{lreal}}[n]$ ,  $u_{\text{limag}}[n]$ ,  $u_{2\text{real}}[n]$ , and  $u_{2\text{imag}}[n]$  at  $\sigma$  variation. In Fig. 3 there are presented the variations with  $\sigma$  of maximum absolute values for a random signal with uniform distribution, a random signal with normal distribution and an 1-D signal obtained by concatenation of Lena image rows.

From Fig. 3 it results that in order to avoid overflows, for the maximum value of  $\sigma = 32$ , at least L = 6 bits are needed if the estimated values are used, and at least L = 5 bits are needed if the values obtained by simulation are used.

In Fig. 5 the filtered Lena images for  $\sigma = 4$  and  $\sigma = 8$  using L = 5 bits, 8 bits and, respectively, 12 bits are presented. The filtered images have been compared with the filtered images obtained using L = 64 bits.



Fig. 3. – The absolute maximum values obtained though simulation and the estimation given by relations (9) and (10).

The coefficient  $2p/\sigma\sqrt{\sigma}$  has been spllited in two multipliers  $\sqrt{2p}/\sigma$ and  $\sqrt{2p}/\sqrt{\sigma}$ . This two coefficients appears in the (11) and (12) transfer functions, respectively. The advantage is that the maximum value of  $|\tilde{w}[n]|$  is limitated at 1 (eq. (9)) and both  $|u_1[n]|$  and  $|u_2[n]|$  are limited at the same value, 1/1-|p|. If the multiplier only in (12) whould be used, the  $|u_1[n]|$  and  $|u_2[n]|$ dynamics would be small in comparison with y[n] dinamics and would require a higher bits number in order to represent both small amplitudes and the big ones.

In Figs. 4 *a* and 4 *b* the real part, imaginary part and the absolute values of the impulse response for Gabor filters with  $\sigma = 8$ , respectively, are presented.

Image filtered with Gabor filters with  $\sigma = 4$  and  $\sigma = 8$  and for L = 5, 8, 12 bits implementation are represented in Fig. 5.



Fig. 4 *a* – Absolute value, real part and imaginary part of Gabor filters impulse response with  $\sigma = 8$  for L = 5 bits implementation.



Fig. 4 *b* – Absolute value, real part and imaginary part of Gabor filters impulse response with  $\sigma = 8$  for L = 8 bits implementation.

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Fig. 5 – Image filtered with Gabor filters with  $\sigma = 4$  and  $\sigma = 8$  and for L = 5, 8, 12 bits implementation.

# **5.** Conclusions

In this paper the implementation of Gabor filters using IIR filter implementation is presented. Examples of signal response (filter impulse responses and Lena image) for different number of bits representation have been analysed. From simulations can be observed that there are not visible differences between responses for representations using more then 8 bits. The estimation of the minimum number of bits necessary in order to avoid overflows has been also determined. Using random signals and rows of Lena image, the estimation has been confirmed though simulations. Future work will analyse the performances of a system classification using Gabor filters for representations using different number of bits.

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## ANALIZA IMPLEMENTĂRII FILTRELOR GABOR FOLOSIND UN NUMĂR FINIT DE BIȚI

### (Rezumat)

Se prezintă rezultatele obținute de autor în ceea ce privește implementarea filtrelor Gabor folosind filtre cu răspuns infinit la impuls. S-au analizat, de asemenea, limitările impuse de folosirea unui număr finit de biți în ceea ce privește depășirea de format, specifică implementărilor în virgulă fixă și s-a comparat răspunsul la impuls al filtrelor implementate folosind 64 de biți și cel al răspunsurilor folosind 5, 8 și 12 biți. Rezultatele obținute arată că există posibilitatea să se implementeze filtrele de tip Gabor folosind 5 biți pentru o valoare maximă a deviației standard egală cu 32. În cercetările viitoare se va analiza măsura în care numărul de biți utilizați în implementarea filtrelor de tip IIR, afectează performanțele sistemului de clasificare a texturilor.