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MANLEY-ROWE TYPE RELATIONS CONCERNING THE ACTIVE PSEUDOPOWERS

BY

HUGO ROSMAN*

“Gheorghe Asachi” Technical University of Iaşi
Faculty of Electrical Engineering

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Abstract. It is proved that the active pseudopowers, intervening in Tellegen’s theorem, satisfy Manley-Rowe type relations when the voltages and the currents are double periodical functions of time, having the same frequencies, $f_1 = \omega_1/2\pi$, $f_2 = \omega_2/2\pi$.

Key words: pseudopowers; active pseudopowers; Manley-Rowe relations; generalized Manley-Rowe relations.

1. Introduction

In 1952 Tellegen has established an important theorem, called in specialty literature as *Tellegen’s theorem*. This one has determined the introduction, in the electrical network theory, of the *pseudopower* concept; for instance such a power was utilized among the first researchers, by Penfield, Spence and Dunker in their book published in 1970.

A generalization of Tellegen’s theorem in the electromagnetic field and, implicitly, the defining of pseudopowers in such a case, was proposed by Andrei Ţugulea, (1986), named by the author *electromagnetic pseudopower*.

It is necessary to underline that in all these cases it is a matter of instantaneous powers.

In view to define the pseudopowers in an electrical network it is

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* *e-mail*: adi_rotaru2005@yahoo.com

necessary, beforehand to formulate Tellegen's theorem in case of an electrical network. Let be two insulated active or passive, linear, non-linear or parametric electrical networks, (I) and (II), having identical oriented topological graphs, and let be $u_k^{(\mathrm{I})}(t')$ the voltage between the nodes incident to branch k in network (I) at moment t' and $i_k^{(\mathrm{II})}(t'')$ – the current which flows through branch k of network (II) at moment t'', with $k = 1,2,\ldots,B$, (B number of branches). The two networks' working regime is considered quasistationary, of anelectrical type. Relation

$$\sum_{k=1}^{B} u_k^{(\mathrm{I})}(t')\, i_k^{(\mathrm{II})}(t'') = 0 \tag{1}$$

is satisfied, representing Tellegen's theorem (in case of electrical networks). Expression

$$p_k^{(\mathrm{I}),(\mathrm{II})}(t',t'') = u_k^{(\mathrm{I})}(t')\, i_k^{(\mathrm{II})}(t'') \tag{2}$$

represents the instantaneous pseudopower in networks (I) and (II) k branch. In the particular case when $t' = t'' = t$ the previous relation becomes

$$p_k^{(\mathrm{I}),(\mathrm{II})} = u_k^{(\mathrm{I})}(t)\, i_k^{(\mathrm{II})}(t). \tag{3}$$

If signals $u_k^{(\mathrm{I})}(t)$, $i_k^{(\mathrm{II})}(t)$ are periodic, having a harmonic variation *vs*. time with the same frequency $f = 1/T$, it is possible to define: a) an *active pseudopower* in networks' (I) and (II) k branch

$$P_k^{(\mathrm{I}),(\mathrm{II})} = \frac{1}{T}\int_0^T p_k^{(\mathrm{I}),(\mathrm{II})}(t)\,\mathrm{d}t, \tag{4}$$

which admits the well known symbolical representation

$$P_k^{(\mathrm{I}),(\mathrm{II})} = \Re e\left(\underline{U}_k^{(\mathrm{I})}\, \underline{I}_k^{(\mathrm{II})*}\right) \tag{5}$$

and b) a *reactive pseudopower*

$$Q_k^{(\mathrm{I}),(\mathrm{II})} = \Im m\left(\underline{U}_k^{(\mathrm{I})}\, \underline{I}_k^{(\mathrm{II})*}\right) \tag{6}$$

in k branch of the same networks.

2. Manley-Rowe Type Relations Concerning the Active Pseudopowers

In view to establish such relations some results obtained in a previous work (Rosman, 1972), were taken into account. Namely, considering two scalar functions, $m(t)$, $n(t)$, double periodic with respect the time, which admit the developments in complex double Fourier series (Tolstov, 1956)

$$\begin{cases} m(t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} M_{p,q} \mathrm{e}^{\mathrm{j}(p\omega_1 + q\omega_2)t}, \\ n(t) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} N_{p,q} \mathrm{e}^{\mathrm{j}(p\omega_1 + q\omega_2)t}, \end{cases} \tag{7}$$

where $T_1 = 2\pi/\omega_1$, $T_2 = 2\pi/\omega_2$ represent the two double periodic functions periods and

$$\begin{cases} \underline{M}_{p,q} = \dfrac{1}{4\pi^2} \int_0^{2\pi} \mathrm{d}(\omega_2 t) \int_0^{2\pi} m(t) \mathrm{e}^{\mathrm{j}(p\omega_1 + q\omega_2)t} \mathrm{d}(\omega_1 t), \\ \underline{N}_{p,q} = \dfrac{1}{4\pi^2} \int_0^{2\pi} \mathrm{d}(\omega_2 t) \int_0^{2\pi} n(t) \mathrm{e}^{\mathrm{j}(p\omega_1 + q\omega_2)t} \mathrm{d}(\omega_1 t), \end{cases} \tag{8}$$

represent the complex double Fourier series (7) inverse transfoms, take place the following Manley-Rowe (M.-R.) type relations (Rosman, 2004)

$$\sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{p \Re e\left(\underline{M}_{p,q} \underline{N}^*_{p,q}\right)}{p\omega_1 + q\omega_2} = 0, \quad \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \frac{q \Re e\left(\underline{M}_{p,q} \underline{N}^*_{p,q}\right)}{p\omega_1 + q\omega_2} = 0 \tag{9}$$

available only if the function $m(n)$ satisfies the unicity condition

$$\oint m(n) \mathrm{d}n = 0. \tag{10}$$

Considering that

$$m = u_k^{(\mathrm{I})}, \quad n = u_k^{(\mathrm{II})}, \quad (k = 1, 2, ..., B) \tag{11}$$

the networks (I) and (II) having B branches each one, taking in account expressions (8) it is possible to establish the following relations

$$\begin{cases} \underline{U}^{(\mathrm{I})}_{k;p,q} = \dfrac{1}{4\pi^2} \int_0^{2\pi} \mathrm{d}(\omega_2 t') \int_0^{2\pi} u_k^{(\mathrm{I})}(t') \mathrm{e}^{-\mathrm{j}(p\omega_1 + q\omega_2)t'} \mathrm{d}(\omega_1 t'), \\ \underline{I}^{(\mathrm{II})}_{k;p,q} = \dfrac{1}{4\pi^2} \int_0^{2\pi} \mathrm{d}(\omega_2 t'') \int_0^{2\pi} I_k^{(\mathrm{II})}(t'') \mathrm{e}^{-\mathrm{j}(p\omega_1 + q\omega_2)t} \mathrm{d}(\omega_2 t''). \end{cases} \tag{12}$$

Using the same proceeding as in the previous section and supposing that

$$\oint u_k \mathrm{d} i_k = 0 \tag{13}$$

relations

$$\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{p\Re e\left(\underline{U}^{(\mathrm{I})}_{k;p,q}\underline{I}^{(\mathrm{II})*}_{k;p,q}\right)}{p\omega_1+q\omega_2}=0,\quad \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{q\Re e\left(\underline{U}^{(\mathrm{I})}_{k;p,q}\underline{I}^{(\mathrm{II})*}_{k;p,q}\right)}{p\omega_1+q\omega_2}=0 \tag{14}$$

are obtained analogous to (9). Having in view relation (5), expressions (14) may be written

$$\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{pP^{(\mathrm{I}),(\mathrm{II})}_{k;p,q}}{p\omega_1+q\omega_2}=0,\quad \sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{qP^{(\mathrm{I}),(\mathrm{II})}_{k;p,q}}{p\omega_1+q\omega_2}=0, \tag{15}$$

where

$$P^{(\mathrm{I}),(\mathrm{II})}_{k;p,q}=\Re e\left(U^{(\mathrm{I})}_{k;p,q}I^{(\mathrm{II})*}_{k;p,q}\right) \tag{16}$$

represents the active pseudopower changed by harmonics of rank *p* and *q* between the networks' (I) and (II) *k* branches.

Relations (15) may be considered as representing M.-R. type relations regarding the active pseudopowers where *k* branches of networks (I) and (II). In the particular case when networks (I) and (II) coincide the proper M.-R. relations concerning the *k* branch of respective network are obtained.

3. Generalized Manley-Rowe Type Relations Concerning the Active Pseudopowers

In a recent paper (Rosman, 2004) was proven that considering two scalar functions, *m*(*t*), *n*(*t*), double periodic with respect to time, which admit the developments in complex double Fourier series (7), generalized M.-R. type relations are satisfied, named by us Manley-Rowe-Kontorovich (M.-R.-K.) relations (8), having the following expressions

$$\begin{cases}\displaystyle\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{\psi(p,q)\Re e\left(\underline{M}_{p+\mu,q+\nu}\underline{N}^{*}_{p+\sigma,q+\tau}\right)}{p\omega_1+q\omega_2}=0,\\ \displaystyle\sum_{p=-\infty}^{\infty}\sum_{q=-\infty}^{\infty}\frac{\varphi(p,q)\Im m\left(\underline{M}_{p+\mu,q+\nu}\underline{N}^{*}_{p+\sigma,q+\tau}\right)}{p\omega_1+q\omega_2}=0,\end{cases} \tag{17}$$

where $\underline{M}_{p+\mu,q+\nu}$, $\underline{N}_{p+\sigma,q+\tau}$ are given by relations analogous to (8), and

$$\psi(p,q) = -\psi(-p,-q), \quad \varphi(p,q) = \varphi(-p,-q) \tag{18}$$

are two arbitrary functions, the first, odd and the second, even, with respect to p and q, and μ, ν, σ, $\tau \in \mathbb{N}$.

Using an analogous procedure to that utilized in section **2** the relations

$$\begin{cases} \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \dfrac{\psi(p,q)\Re e\left(\underline{U}_{k;p+\mu,q+\nu}^{(\mathrm{I})} \underline{I}_{k;p+\sigma,q+\tau}^{(\mathrm{II})*}\right)}{p\omega_1 + q\omega_2} = 0, \\ \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \dfrac{\varphi(p,q)\Im m\left(\underline{U}_{k;p+\mu,q+\nu}^{(\mathrm{I})} \underline{N}_{k;p+\sigma,q+\tau}^{(\mathrm{II})*}\right)}{p\omega_1 + q\omega_2} = 0, \end{cases} \tag{19}$$

are obtained. These relations may be considered as representing generalized M.-R.-K. relations concerning the active pseudopowers.

It is necessary to underline that the expressions $\Re e\left(\underline{U}_{k;p+\mu,q+\nu}^{(\mathrm{I})} \underline{I}_{k;p+\sigma,q+\tau}^{(\mathrm{II})*}\right)$, $\Im m\left(\underline{U}_{k;p+\mu,q+\nu}^{(\mathrm{I})} \underline{N}_{k;p+\sigma,q+\tau}^{(\mathrm{II})*}\right)$ represent pseudoactive, respectively pseudoreactive powers, in the sense used by us.

4. Conclusions

Considering two linear, non-linear or parametric insulated electric networks, (I) and (II), having identical oriented topological graphs and the branchs voltages and currents double periodical functions with respect to time, denoting with $\underline{U}_{k;p,q}^{(\mathrm{I})}$ and $\underline{I}_{k;p,q}^{(\mathrm{II})}$ the complex double Fourier series inverse transformations of the voltage between the terminals of network's (I) branch and, respectively, of the current which flows through network's (II) k branch, (k = 1, 2,…, B) the two signals referring to same harmonics at moment t', respectively t''. Manley-Rowe type relations concerning the active pseudopowers in k-branch of the two networks, (I) and (II), are established

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RELAŢII DE TIP MANLEY-ROWE RELATIVE LA PSEUDOPUTERILE ACTIVE ŞI REACTIVE

(Rezumat)

Se stabilisc relaţii de tip Manley-Rowe relative la pseudoputerile active şi reactive, definite de către Penfield, Spence şi Duinker în legătură cu teorema lui Tellegen. Se propune şi o generalizare a acestor relaţii.