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CONSIDERATIONS ON THE TRANSIENT OF THE SERIES DC MOTOR

BY

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Abstract. The paper presents some considerations on the transient response of a series dc machine. Starting from the idea that the machine operation is described by a nonlinear equation system, the linearization around an operating point of the steady state method was considered. Small variations of the supply voltage and torque resistant were considered as input quantities. The transfer functions of both rotor angular speed and current and the characteristic parameters of the second order system were established under these conditions. Analytical results have been validated by computer simulation using PSpice program.

Key words: induction machine; two-phase model; equivalent circuit; PSpice.

1. Introduction

The series dc motor is very used in traction applications due to the large starting torque and the capability to be operated at low speed.

This motor has the field winding connected in series with the rotor winding so a single voltage supply u is required. The armature and field currents become identical $i_A = i_E = i$. The two voltage equations become only one, and adding the torque expression and the motion equation we have the whole set of equations describing the motor behavior (Ong, 1997; Lyshevsky, 1999).

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Using the notations of Fig. 1 it results:

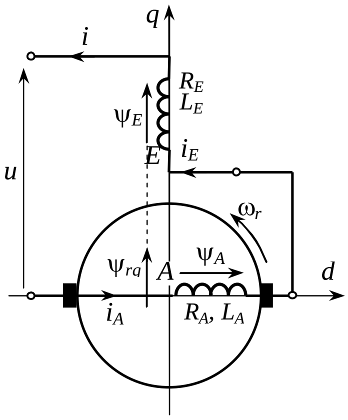

Fig. 1 – Orthogonal model of the series dc motor.

$$\begin{aligned} &u = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \omega_r L_{mq} i, \\ &m_e = \psi_{rq} i = L_{mq} i^2, \\ &\frac{\mathrm{d}\omega_r}{\mathrm{d}t} = \frac{1}{J}(t_e - t_l), \end{aligned} \tag{1}$$

where: $R = (R_A + R_E)$ and $L = (L_A + L_E)$. The system of three equations with three unknown quantities i, m_e and ω_r is uniquely determined. Solving the system leads to the transient of the motor. Although relatively simple, the system is nonlinear due to the multiplying of variable quantities $i(t)\omega_r(t)$ and $i^2(t)$ that hampers an analytical study.

2. The Transient at Small Variations of Input Quatities

Even if the system of equations describing the series dc motor behavior is nonlinear, the linearization around an operating point can be applied to partial solve the problem (Covacs,1980; Das,2010). In this case we may consider small variations of the input quantities Δu and Δt_l.

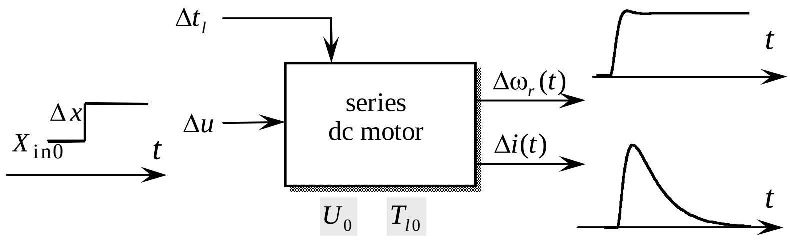

Fig. 2 – Small variations of input quatities.

Eliminating the electromagnetic torque and considering steady state values U_0 and T_{l0}, from eq. (1) it results:

$$\begin{aligned} &U_0 = RI_0 + \omega_0 L_{mq} I_0, \\ &L_{mq} I_0^2 = T_{l0}. \end{aligned} \tag{2}$$

In the presence of small variations of the input quantities is obtained:

$$u \rightarrow U_0 + \Delta u\,;\; i \rightarrow I_0 + \Delta i;$$
$$t_e \rightarrow T_{e0} + \Delta t_e\,;\; T_l \rightarrow T_{L0} + \Delta t_l\,;\; \omega_r \rightarrow \omega_{r0} + \Delta\omega. \tag{3}$$

Replacing in eq. (1) gives:

$$U_0 + \Delta u = R(I_0 + \Delta i) + L\frac{\mathrm{d}(I_0 + \Delta i)}{\mathrm{d}t} + (\omega_{r0} + \Delta\omega)L_{mq}(I_0 + \Delta i),$$
$$L_{mq}(I_0 + \Delta i)^2 = J\frac{\mathrm{d}(\omega_{r0} + \Delta\omega)}{\mathrm{d}t} + T_{l0} + \Delta t_l. \tag{4}$$

Separating the small variations and neglecting the small variations products it results:

$$\Delta u = R\Delta i + L\frac{\mathrm{d}\Delta i}{\mathrm{d}t} + \omega_{r0}L_{mq}\Delta i + L_{mq}I_0\Delta\omega,$$
$$2L_{mq}I_0\Delta i = J\frac{\mathrm{d}\Delta\omega}{\mathrm{d}t} + \Delta t_l. \tag{5}$$

Using the Laplace transform gives:

$$\Delta U = R\Delta I + sL\Delta I + \omega_{r0}L_{mq}\Delta I + L_{mq}I_0\Delta\Omega,$$
$$2L_{mq}I_0\Delta I = J\frac{\mathrm{d}\Delta\Omega}{\mathrm{d}t} + \Delta T_l, \tag{6}$$

$$\left(R + \omega_{r0}L_{mq} + sL\right)\Delta I + L_{mq}I_0\Delta\Omega = \Delta U(s),$$
$$2L_{mq}I_0\Delta I - sJ\Delta\Omega = \Delta T_l(s). \tag{7}$$

The determinant of the matrix coefficients results as:

$$\Delta = -JL\left(s^2 + s\frac{R + \omega_{r0}L_{mq}}{L} + \frac{2L_{mq}^2 I_0^2}{JL}\right). \tag{8}$$

The resulting second order system is characterized by the natural frequency and damping coefficient:

$$\omega_n = L_{mq}I_0\sqrt{\frac{2}{JL}}\;;\quad \zeta = \frac{R + \omega_{r0}L_{mq}}{2\sqrt{2}L_{mq}I_0}\sqrt{\frac{J}{L}}\,. \tag{9}$$

As can be seen, $\omega_n = f(I_0)$ and $\zeta = f(\omega_{r0}, I_0)$. In the case of separately excited or shunt dc motors, the characteristic values depend only on the dc machine constructive parameters. In our case, this dependence is extended also to the operating point. From eq. (2) it can be deduced:

$$I_0 = \sqrt{\frac{T_{l0}}{L_{mq}}}; \quad (R + \omega_{r0} L_{mq}) = \frac{U_0}{I_0} = U_0 \sqrt{\frac{L_{mq}}{T_{l0}}}. \tag{10}$$

Substituting in eq. (9) we obtain:

$$\omega_n = \sqrt{\frac{2L_{mq}}{JL}} \sqrt{T_{l0}}\ ; \quad \zeta = \frac{1}{2\sqrt{2}} \sqrt{\frac{J}{L}} \frac{U_0}{T_{l0}} \tag{11}$$

In order to validate the analytical results, a simulation of the behavior of the dc machine has been performed in PSpice (Justus, 1993; Cociu & Cociu1997), rated as follows:

$P_n = 12$ kW; $U_n = 220$ V; $I_n = 70$ A; $n_n = 1{,}500$ rot/min;
$R_A = 0.32\ \Omega$; $R_E = 0.215\ \Omega$;
$L_A = 3.5$ mH; $L_E = 64.5$ mH; $L_{mq} = 15.6$mH;
$J = 0.1\ \text{kg}\cdot\text{m}^2$; $F_\alpha = 10^{-3}$ Nms/rad.

Fig. 3 reveals the influence of the steady state operation quantities U_0 and T_{l0} on the specific parameters that define the characteristic response of the second order system.

Three cases of different value but constant ratio for supply voltage and load torque were considered. As shown in eq. (11), the constant ratio of voltage and load torque leads to the same value of damping coefficient in all three cases. Obviously the damping coefficient changes with the moment of inertia; but the change has the same value in all three cases. The other parameter, the natural angular frequency depends only on the load torque. At low load it result low natural angular frequency and vice versa.

Since the coefficient matrix determinant Δ is the same in all of interest transfer functions, this behavior is expected to be present both in the case of angular speed and current.

3. The Transient at Small Variations of the Supply Voltage

Consider a constant non zero value load torque. The machine works in steady state supplied by U_0 voltage. Supposing the voltage rises with a low value Δu. From eq. (7) considering $\Delta t_l = 0$ and using eq. (8) it follows:

$$\Delta_\Omega = -2L_{mq} I_0 \Delta U(s) \tag{12}$$

$$\Delta\Omega(s) = \frac{\Delta_\Omega}{\Delta} = \frac{2L_{mq} I_0}{JL(s^2 + 2\zeta\omega_n + \omega_n^2)} \frac{\Delta U}{s} \tag{13}$$

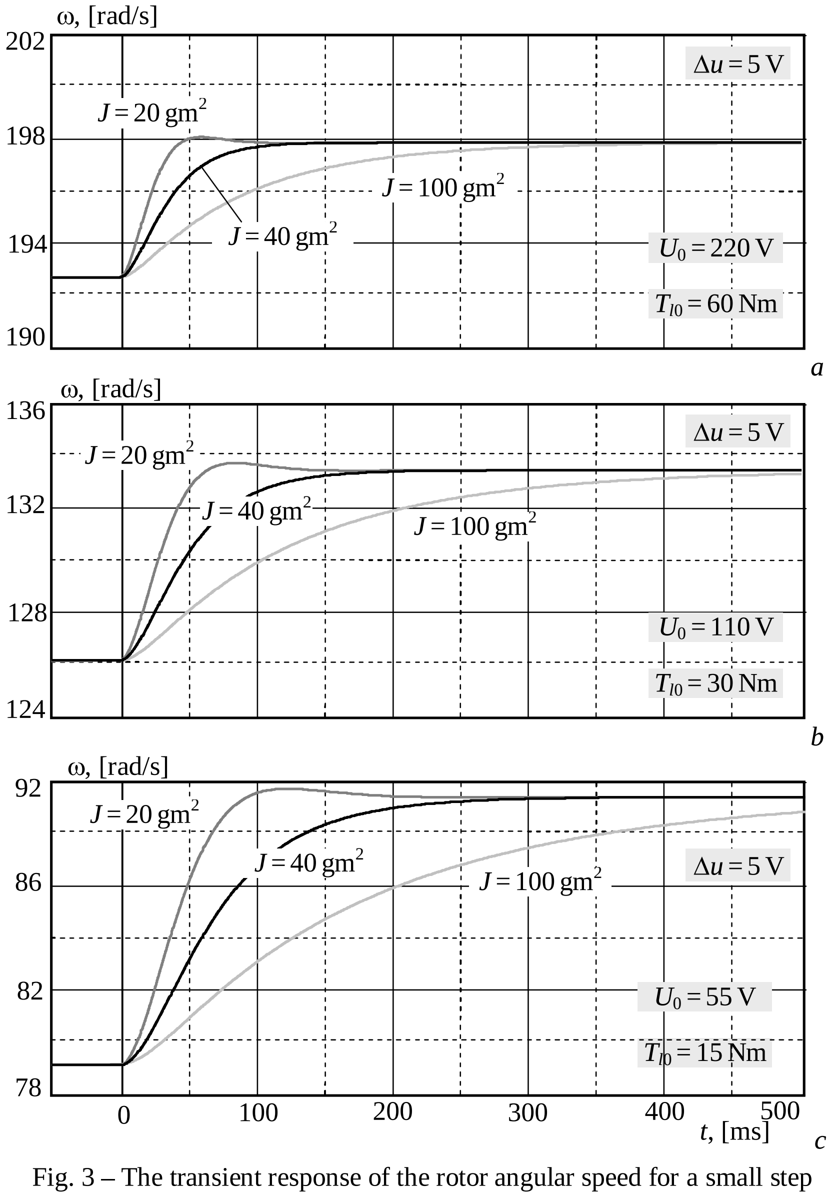

Fig. 3 – The transient response of the rotor angular speed for a small step of the supply voltage: *a* – $U_0 = 220$ V; *b* – $U_0 = 110$ V; *c* – $U_0 = 55$ V.

$$\Delta\Omega(s) = \frac{\Delta U}{\sqrt{L_{mq}T_{l0}}} \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n + \omega_n^2)}, \tag{14}$$

$$H_\omega(s) = \frac{\Delta\Omega(s)}{\Delta U(s)} = \frac{1}{\sqrt{L_{mq}T_{l0}}} \frac{\omega_n^2}{(s^2 + 2\zeta\omega_n + \omega_n^2)}. \tag{15}$$

As can be noted, not only the parameters ω_n and ζ depend on initial conditions but also the transfer coefficient k_ω:

$$k_\omega = \frac{\Delta\omega(\infty)}{\Delta u} = \lim_{s\to 0} H_\omega(s) = \frac{1}{\sqrt{L_{mq}T_{l0}}}. \tag{16}$$

Fig. 3 illustrates very well this result. If for $T_{l0} = 60$ Nm is obtained $k_\omega = 1.03$ rad/s·V and $\Delta\omega = 5.17$ rad/s, then for $T_{l0} = 15$ Nm a double value is obtained $k_\omega = 2.06$ rad/s·V and $\Delta\omega = 10.34$ rad/s respectively.

Similarly, for the current it results:

$$\Delta_I = -sJ\Delta U(s), \tag{17}$$

$$\Delta I(s) = \frac{\Delta_\Omega}{\Delta} = \frac{s}{L\left(s^2 + 2\zeta\omega_n + \omega_n^2\right)} \cdot \frac{\Delta U}{s}, \tag{18}$$

$$\Delta I(s) = \frac{1}{L} \cdot \frac{1}{s^2 + 2\zeta\omega_n + \omega_n^2} \Delta U, \tag{19}$$

$$H_I(s) = \frac{\Delta I(s)}{\Delta U(s)} = \frac{1}{L} \cdot \frac{s}{s^2 + 2\zeta\omega_n + \omega_n^2}, \tag{20}$$

$$k_I = \frac{\Delta I(\infty)}{\Delta u} = \lim_{s\to 0} H_I(s) = 0. \tag{21}$$

As expected, as long as the load torque does not change, according to eq. (10) the final value of the current is identical to the initial value. However the current is passing through an important transient regime until returns to initial value.

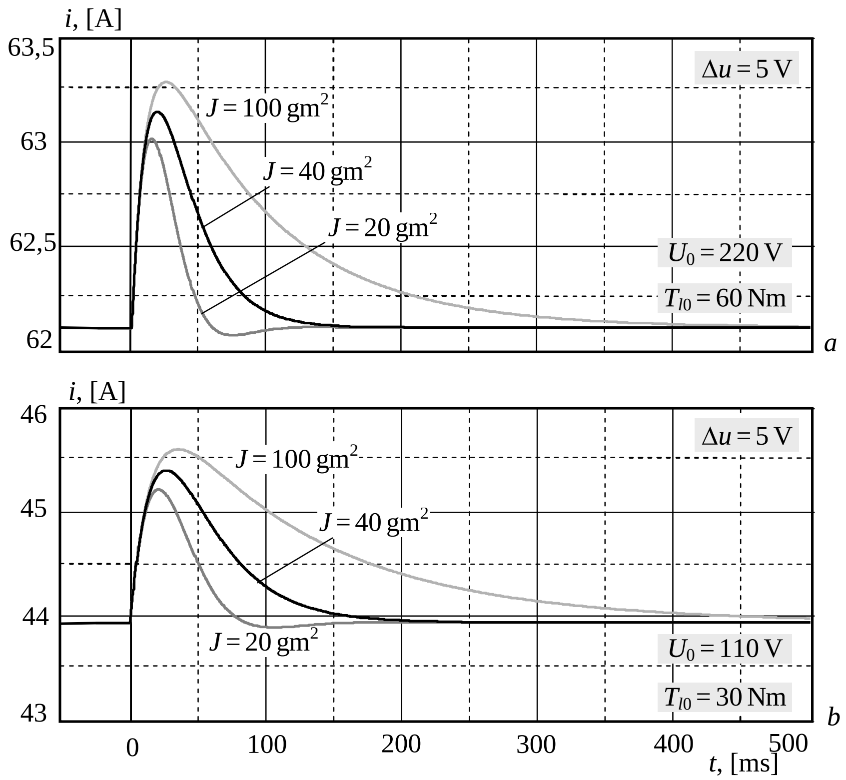

Fig. 4 – The transient response of the current for small step supply voltage: *a* – $U_0 = 220$ V; *b* – $U_0 = 110$ V.

4. The Transient at Small Variations of the Load Torque

At initial time, in steady state operating the machine is supplied by a constant voltage and loaded by the load torque T_{l0}. Supposing that suddenly the load changes with a small value Δt_l. From eq. (8), considering $\Delta \underline{u} = 0$ and using eq. (9) it follows:

$$\Delta_\Omega = (sL + R + \omega_{r0} L_{mq})\Delta T_l \, , \tag{22}$$

$$\Delta\Omega(s) = \frac{\Delta_\Omega}{\Delta} = -\frac{sL + R + \omega_{r0} L_{mq}}{JL\left(s^2 + 2\zeta\omega_n + \omega_n^2\right)} \cdot \frac{\Delta T_l}{s} \tag{23}$$

$$H_\omega(s) = -\frac{\Delta\Omega(s)}{\Delta T_l(s)} = -\frac{sL + R + \omega_{r0} L_{mq}}{JL(s^2 + 2\zeta\omega_n + \omega_n^2)} \, . \tag{24}$$

The transfer function shows one zero in addition to the two poles; its position is dependent on the initial angular speed. The minus sign indicates a decrease in angular velocity as load torque increases. Transfer factor becomes:

$$k_\omega = \frac{\Delta\omega(\infty)}{\Delta t_l} = \lim_{s\to 0} H_\omega(s) = -\frac{R + \omega_{r0} L_{mq}}{2 L_{mq} T_{l0}}. \qquad (25)$$

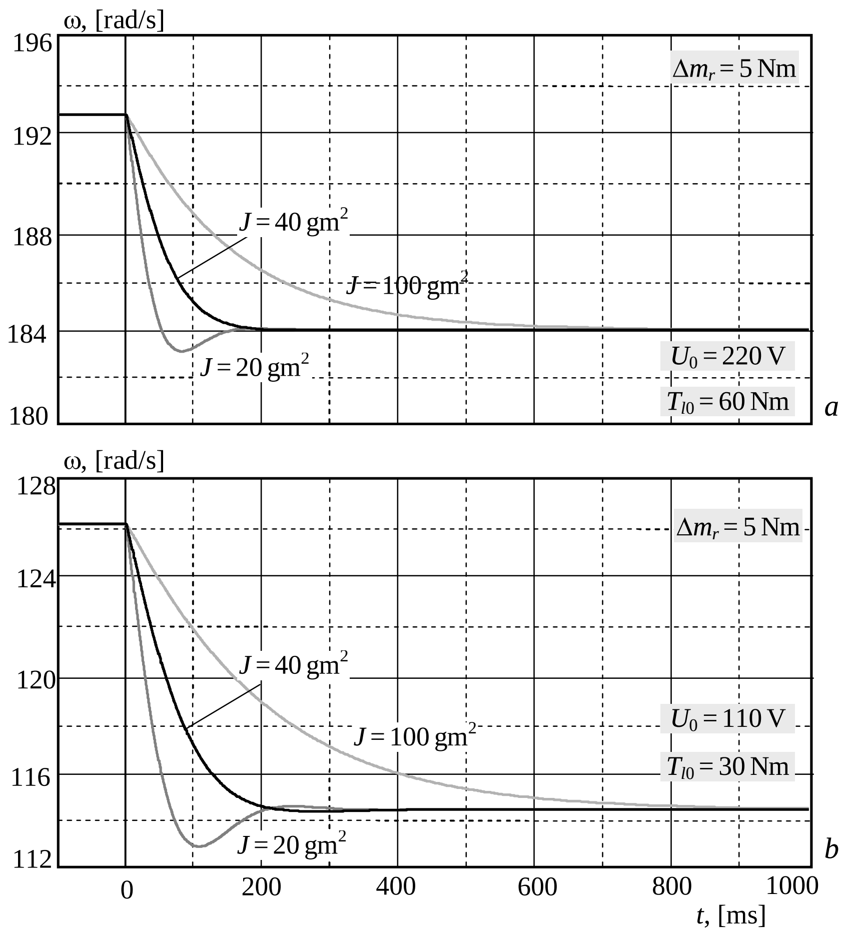

Fig. 5 – The transient response of the rotor angular speed for small step of the load: *a* – $U_0 = 220$ V; *b* – $U_0 = 110$ V.

Fig. 5 highlights this result. If for T_{l0} = 60 Nm are obtained the values k_ω = –1.89 rad/s.Nm and $\Delta\omega$ = –9.45 rad/s, then for T_{l0} = 30 Nm the new values are k_ω = –2.5 rad/s.Nm and $\Delta\omega$ = –12.52 rad/s respectively.

Regarding the current, a similar procedure gives:

$$\Delta_I = -L_{mq} I_0 \Delta T_l(s)\,, \tag{26}$$

$$\Delta I(s) = \frac{\Delta_I}{\Delta} = \frac{L_{mq} I_0}{JL\left(s^2 + 2\zeta\omega_n + \omega_n^2\right)} \cdot \frac{\Delta T_l}{s}\,, \tag{27}$$

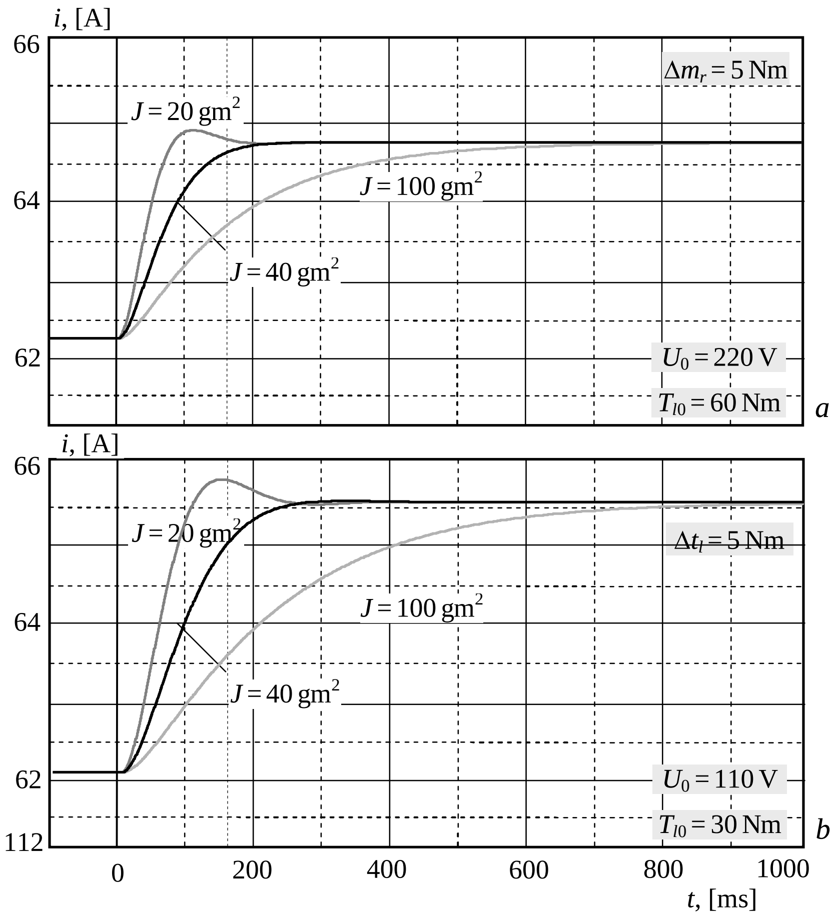

Fig. 6 – The transient response of the current for small step of the load: a – $U_0 = 220$ V; b – $U_0 = 110$ V.

$$\Delta I(s) = \frac{1}{2\sqrt{L_{mq} T_{l0}}} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \cdot \frac{\Delta T_l}{s}\,, \tag{28}$$

$$H_I(s) = \frac{\Delta I(s)}{\Delta M_r(s)} = \frac{1}{2\sqrt{L_{mq}T_{l0}}} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}. \tag{29}$$

Because the useful torque is proportional to the square of the current the load torque modification has as final effect (after the transient extinction) the corresponding modification of the current, as expected.

$$k_I = \frac{\Delta I(\infty)}{\Delta t_l} = \lim_{s\to 0} H_I(s) = \frac{1}{2\sqrt{L_{mq}T_{l0}}}. \tag{30}$$

Fig. 6 shows the transient current for small step variations of the load torque in two different situations: for T_{l0} = 60 Nm, k_I = 0.517 rad/s.Nm and ΔI = 2.58 rad/s values are obtained; for T_{l0} = 30 Nm the values obtained are k_I = 0.517 rad/s.Nm and ΔI = 3.66 rad/s. Both values calculated with eq. (30) correspond to those obtained by simulation.

5. Conclusions

The study of the transient of series dc motor is hampered by the nonlinearity of the equations that describe the machine behavior. The linearization around an operating point method can be applied to partial solve the problem.

For this, small variations of the input quantities □u şi □t_l were considered. Solving the system of equations leads to the output quantities: the rotor angular speed and the current through the machine.

In all cases, the denominator of the transfer function has two poles, pointing a second order system. The system can be underdamped or overdamped, depending on the constructive parameters and the initial steady state values as well.

The parameters describing the machine behavior change with operating mode. The steady state is characterized by the values of two quantities: supply voltage U_0 and load torque T_{l0}. The natural frequency ω_n is proportional to the root of load torque and independent of supply voltage. The damping coefficient is proportional to the supply voltage and inversely proportional to the load torque.

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CONSIDERAȚII ASUPRA REGIMULUI TRANZITORIU AL MAŞINII DE C.C. CU EXCITAȚIE SERIE

(Rezumat)

Sunt prezentate câteva considerații privind răspunsul tranzitoriu al maşinii de c.c. cu excitație serie. Pornind de la observația că sistemul de ecuații care descrie funcționarea maşinii este neliniar, se aplică metoda liniarizării caracteristicilor în jurul unui punct de funcționare din regimul permanent. La intrare au fost considerate pe rând mici variații ale tensiunii de alimentare şi ale cuplului rezistent. În aceste condiții au fost stabilite funcțiile de transfer pentru viteza unghiulară rotorică şi curent precum şi parametrii caracteristici sistemului de ordinul doi. Rezultatele analitice au fost validate prin simularea pe calculator utilizând programul Pspice.