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THE INFLUENCE OF THE TIME AT WHICH A SHORT-CIRCUIT OCCURS ON THE INDUCTION MACHINE TRANSIENT

BY

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Abstract. The time at which a short-circuit occurs affects the characteristics of the transient of the induction machine. The paper presents a few considerations regarding this topic, focusing on both the transient response of the spatial phasor of the stator current and the change in its orthogonal components. The analysis is based on the influence of the supply voltage phase on the mathematical expression of stator current, validated by PSpice simulation of the transient process using the orthogonal model of the induction machine.

Key words: induction machine; transient; short-circuit; starting point.

1. Introduction

Short-circuit studies usually involve a three-phase fault condition. In this case, we can consider that all three phases create a zero impedance connection. The initial conditions take into account an induction motor normally supplied, in steady state operation and no load, rotating at a speed close to synchronism. At some instant a three-phase short circuit occurs on the supply.

Simplified expressions of current and transient fluxes linkage were deduced in Cociu & Cociu, (2014). The mathematical expressions have a simple

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form and can easily be used in an analytical study. They illustrate main aspects of the short-circuit transient state. In addition to previous work in the field, *e.g.* (Covacs, 1980; Das, 2011), the precise mathematical expressions for currents and fluxes were presented in Cociu & Cociu, (2014). The results they yield are very close to reality, but the complexity of the final expressions limits the possibility of performing an analytical study.

In both cases simplifying hypotheses have been used: constant angular speed during the process ω = cont., and equality of stator and rotor impedances. The calculations in the Laplace domain gave the expressions for currents and fluxes linkage (Cociu & Cociu, 2014). The analysis was based on the spatial phasor machine model (Ong, 1997; Lyshevsky, 1999).

Using a stationary reference frame, the supply voltage was considered to be:

$$\underline{u}_s = U_{sm}\,\mathrm{e}^{\omega t}\ . \tag{1}$$

In this reference frame the expression of the stator current is (Cociu & Cociu, 2014):

$$\underline{i}_s = \frac{U_{sm}}{\omega L'}\mathrm{e}^{-\frac{R}{L'}t}\left[\left(\underline{A}+\underline{B}\right)\mathrm{e}^{\,\mathrm{j}\omega' t} - \underline{B}\mathrm{e}^{\,\mathrm{j}\omega'' t}\right], \tag{2}$$

where:

$$\omega' = \frac{\omega}{2}\left(1-\sqrt{1-\frac{4k^2}{\omega^2}\cdot\frac{R^2}{L'^2}}\right);\ \ \omega'' = \frac{\omega}{2}\left(1+\sqrt{1-\frac{4k^2}{\omega^2}\cdot\frac{R^2}{L'^2}}\right), \tag{3}$$

$$\underline{A} = \frac{\omega L'}{R+\mathrm{j}\omega L};\ \ \underline{B} = \frac{\omega}{\mathrm{j}\left(\omega''-\omega'\right)}\cdot\frac{Rk^2+\mathrm{j}\,\omega' L'}{\left(R+\mathrm{j}\,\omega' L'\right)}\,. \tag{4}$$

Stator current has two different components, characterized by two different angular frequencies ω' and ω'':

$$\underline{i}_s = \underline{i}_s' + \underline{i}_s''\,, \tag{5}$$

$$\underline{i}_s' = \frac{U_{sm}}{\omega L'}e^{-\frac{R}{L'}t}\left(\underline{A}+\underline{B}\right)\mathrm{e}^{\,\mathrm{j}\omega' t}\,, \tag{6}$$

$$\underline{i}_s'' = -\frac{U_{sm}}{\omega L'}\mathrm{e}^{-\frac{R}{L'}t}\underline{B}\mathrm{e}^{\,\mathrm{j}\omega'' t}\,. \tag{7}$$

2. The Influence of the Short-Circuit Time

The paper aims to study the influence of the instant at which a short-circuit occurs on the stator current transient. We can study the influence on the transient current by examining the voltage phase. For this purpose, a supply voltage in a general case is considered:

$$\underline{u}_{s\gamma} = U_{sm}\,\mathrm{e}^{\mathrm{j}(\omega t+\gamma)} = \underline{u}_s \mathrm{e}^{\mathrm{j}\gamma}\,. \tag{8}$$

characterized by the phase γ .

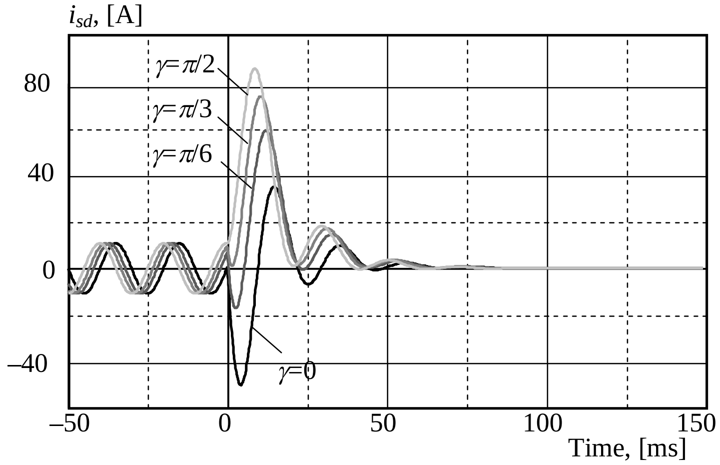

Fig. 1 – The *d* component of the stator current.

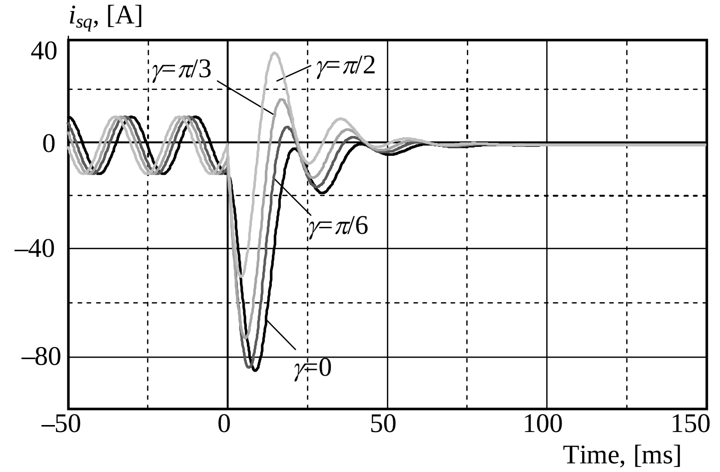

Fig. 2 – The *q* component of the stator current.

Figs. 1 and 2 show the variation of the stator *d* and *q* currents for different starting points of the transient, involving different values of the phase of the supply voltage. Representative values γ = 0, $\pi/6$, $\pi/3$, $\pi/2$ were considered. We notice a major change of the stator current aspect when the phase changes. The positive and negative peak values are affected. However, changes in the overall shape are unlikely.

Previous studies, performed in the Laplace domain, lead to the following expression for stator current:

$$I_s(s) = H(s)U_s(s)\,, \tag{9}$$

where: $H(s)$ is the transfer function current-voltage. With the new conditions $\gamma \neq \neq 0$, using the new expression for voltage supply (8) it results:

$$I_{s\gamma}(s) = H(s)U_{s\gamma}(s) = H(s)U_s(s)\mathrm{e}^{\mathrm{j}\gamma} = I_{s\gamma}(s)\mathrm{e}^{\mathrm{j}\gamma}\,. \tag{10}$$

In the time domain, the two components of the stator current become:

$$\underline{i}_{s\gamma} = \underline{i}_s\mathrm{e}^{\mathrm{j}\gamma} = \underline{i}_s^{'}\mathrm{e}^{\mathrm{j}\gamma} + \underline{i}_s^{''}\mathrm{e}^{\mathrm{j}\gamma}\,, \tag{11}$$

$$\underline{i}_{s\gamma}^{'} = \underline{i}_s^{'}\mathrm{e}^{\mathrm{j}\gamma}\,,\ \ \underline{i}_{s\gamma}^{''} = \underline{i}_s^{''}\mathrm{e}^{\mathrm{j}\gamma}\,. \tag{12}$$

The new expressions of the current components are:

$$\underline{i}_s^{'} = \frac{U_{sm}}{\omega L'}\mathrm{e}^{-\frac{R}{L'}t}\left(\underline{A}+\underline{B}\right)\mathrm{e}^{(\mathrm{j}\omega' t+\gamma)}\,, \tag{13}$$

$$\underline{i}_s^{''} = -\frac{U_{sm}}{\omega L'}\mathrm{e}^{-\frac{R}{L'}t}\underline{B}\mathrm{e}^{\mathrm{j}(\omega'' t+\gamma)}\,. \tag{14}$$

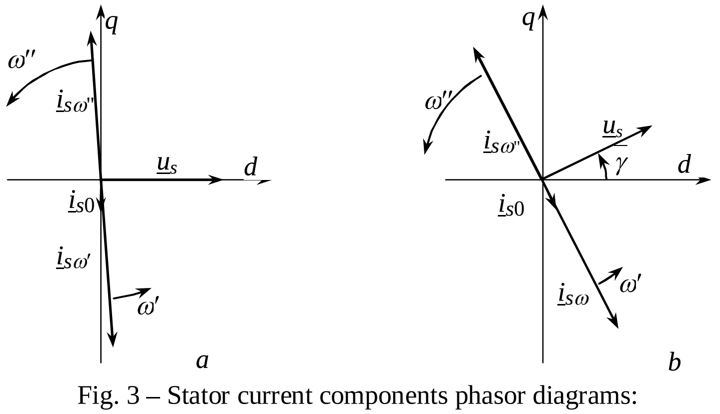

Fig. 3 – Stator current components phasor diagrams:
a – voltage supply with zero phase; *b* – voltage supply with nonzero phase

To better understand the current waveforme modification, consider Fig. 3. A change in the voltage phase leads to a rotation of the phase diagram by the angle γ. The relative position of the phasors remains unchanged. This is expressed by relations (11), (13) and (14).

Both components of the stator current have the same initial phase γ. However, equal phases produce different time delays for the two current components.

$$\Delta t' = \frac{\gamma}{\omega'}, \quad \Delta t'' = \frac{\gamma}{\omega''}. \tag{15}$$

Therefore, when summing the two components the current waveform is altered.

3. Simulation and Numerical Analysis

To validate the theoretical results obtained in this work, an orthogonal model of an induction machine has been simulated. Numerical values for the machine parameters, the same as in (Cociu & Cociu, 2014), are rated as follows:

$$\begin{array}{lll} P_n = 5\,\mathrm{kW}; & U_{1ln} = 400\,\mathrm{V}; & f_1 = 50\,\mathrm{Hz}; \\ R_s = 1.1\,\Omega; & R_r' = 1.1\,\Omega; & L_m = 100\,\mathrm{mH}; \\ L_{\sigma s} = 8\,\mathrm{mH}; & L_{\sigma r}' = 8\,\mathrm{mH}; & J = 30\,\mathrm{g}\cdot\mathrm{m}^2; \\ Y_{\mathrm{conn.}}; & p = 1; & F_\alpha = 2\times10^{-3}\,\mathrm{N}\cdot\mathrm{m}\cdot\mathrm{s}/\mathrm{rad}. \end{array}$$

The simulation is based on the two-phase model implemented in PSpice (Justus, 1993; Cociu & Cociu, 1997; Ong, 1997). For the numerical examples the values $\omega' = 14.67\ \mathrm{rad/s}$ and $\omega'' = 299.48\ \mathrm{rad/s}$ are obtained.

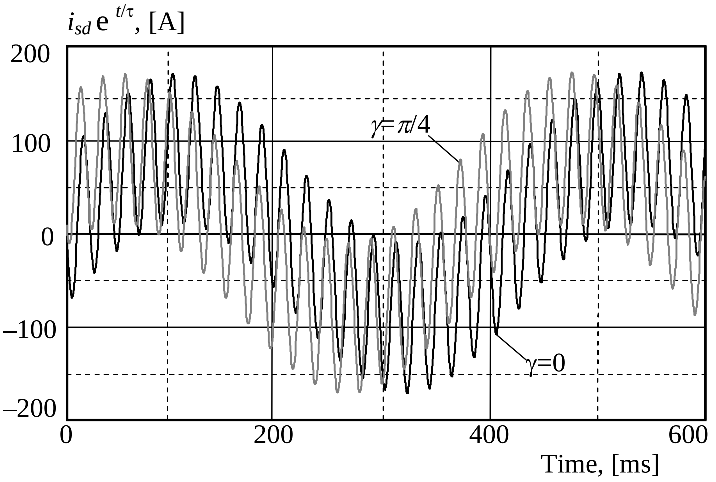

Fig. 4 – Non-damped stator currents for $\gamma = 0$ and $\gamma = \pi/4$.

In order to highlight the shape of the stator current in different conditions, the damping process can be compensated by multiplying the stator current expression by $\exp(t/\tau)$. The results obtained using PSpice simulation are shown in Fig. 4 for $\gamma = 0$ and $\gamma = \pi/4$. A wider time range was used, to show at least a full period of both components. The general waveform appears to be the same; the time evolution is similar, but there is a difference in time delay.

Fig. 5 presents the results obtained by decomposing the stator current into its components.

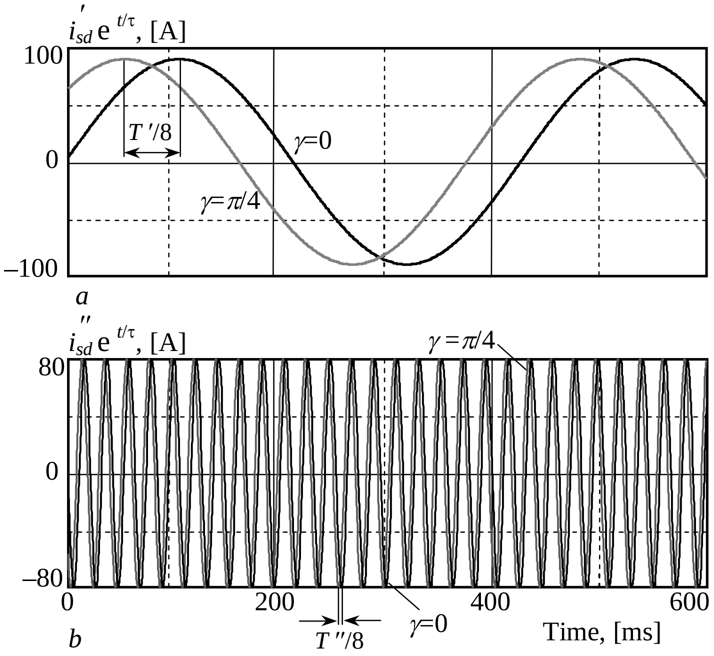

Fig. 5 – Stator current components:
a – low frequency components: *b* – high frequency components.

Both components have the same initial phase $\gamma = \pi/4$ corresponding to a delay of $T/8$. But for the low frequency component the delay is $\Delta t' = \gamma/\omega' =$ $= 53.54$ ms, while for the high frequency component the delay is $\Delta t'' = \gamma/\omega'' =$ $= 2.62$ ms. The summation of the two frequency components in the time range of interest is shown in Fig. 6 for the *d* current.

In the first 20 ms, the low frequency component has a rather slow variation. It can therefore be approximated by a constant (Covacs, 1980; Das, 2010). The value of this constant is strongly influenced by the phase γ. The high frequency component is faster and its phase is strongly influenced by γ. The variation of this component gives the transient stator current variation. The phase of the supply voltage γ determines when the stator current takes the maximum value.

Fig. 6 shows how the two non-damped components, corresponding to low and high frequency, sum in the time range of interest. The phase of the supply voltage strongly influences the waveform of the stator current.

A similar analysis can be carried out for the q stator current, with similar results: changing the supply voltage phase leads to a significantly different current waveform.

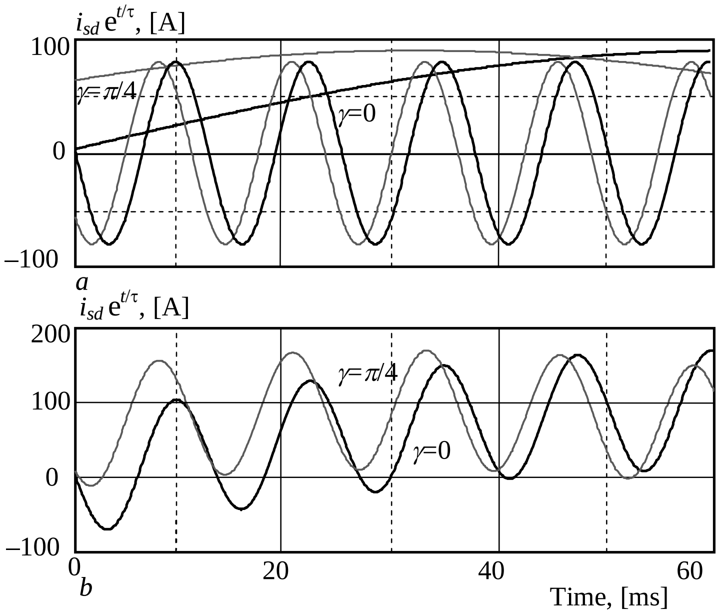

Fig. 6 – Non-damped stator currents:
a – frequency components of the stator current; *b* – stator current.

Changing the phase of the supply voltage leads to a change in the shapes of both d and q currents. By analyzing Fig. 3 one might conclude at first glance that we are dealing with a unique phenomenon that is reflected different on both axes and, therefore, the two orthogonal current components as well.

In fact, rotating the phasor diagram by γ does not change the transient of the spatial phasor of stator current; it is only the projections of this phasor on the d and q axes, that are changed.

The above is confirmed by Fig. 7, which shows the evolution of the absolute value of the stator current spatial phasor. This is unaltered, regardless of the phase of the supply voltage. What is changed is the position of the spatial phasor relative to the reference system, which leads to a change in the projections on the two axes and therefore in the two orthogonal components of stator current. However, their cumulated effect, *i.e.* the amplitude of the spatial phasor, stays the same.

The moment in which the spatial phasor of the stator current reaches its peak value is measured from the moment of the short-circuit onset. It has the value (Cociu & Cociu, 2014):

$$t_m = \frac{2\,\mathrm{atg}\left[\left(\omega'' - \omega'\right)\tau/2\right]}{\omega'' - \omega'}, \tag{16}$$

regardless of the initial position, which is dependent on the supply voltage phase.

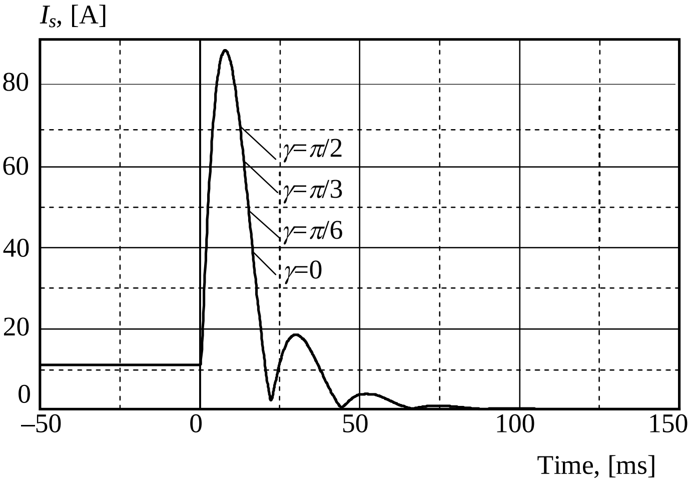

Fig. 7 – Transient stator current magnitude.

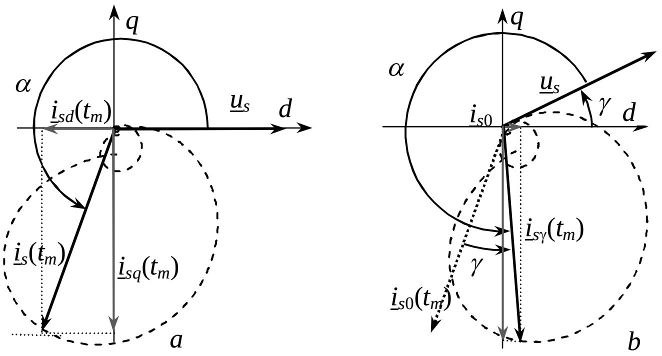

Fig. 8 – Maximum magnitude of stator current phasor:
a – voltage supply with zero phase; *b* – voltage supply with phase.

Fig. 8 summarizes what was presented before. When the stator current reaches its peak value, the phase difference between the supply voltage phasor $\underline{u}_s$ and the stator current phasor $\underline{i}_s$ is the same, regardless of the phase of the supply voltage. Whereas in case *a* the projection of $\underline{i}_s(t_m)$ on the *d* axis is negative, in case *b* it is positive and deeply reduced. On the other hand, the projection on the *q* axis is negative, and its value changed only by a very small amount.

4. Conclusions

The time at which a short-circuit occurs does not influence the transient phenomena of the induction machine. The nature of the transient and the parameters describing it are unchanged if the phenomenon is studied at the current spatial phasor level. There is only a simple rotation of the phasor diagram, and the relative positions of the phasors remain the same. The time instant when stator current reaches its peak is independent of the starting point of the transient.

Changing the starting point of the transient is mathematically equivalent to changing the phase of the supply voltage. This results in a change in the phase of the stator current and, therefore, of each of its components. Each of the two frequency components is delayed differently, leading to an overall change in the shape of the stator currents projected on the two orthogonal axes.

However, the changes on the orthogonal current components are only due to how the projections of the stator current phasor are affected by rotating the phasor diagram. The maximum value of each component changes significantly. Apparently, the shape of each component is modified, but their net effect on the spatial phasor of stator current is null.

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INFLUENȚA MOMENTULUI PRODUCERII UNUI SCURTCIRCUIT ASUPRA REGIMULUI TRANZITORIU AL MAŞINII ASINCRONE

(Rezumat)

Sunt prezentate câteva considerații privind influența pe care o are momentul producerii scurtcircuitului trifazat la bornele de alimentare asupra caracteristicilor regimului tranzitoriu. Se au în vedere atât răspunsul tranzitoriu al fazorului spațial al curentului statoric cât şi modificarea componentelor sale ortogonale. Analiza se bazează pe influența fazei inițiale a tensiunii de alimentare asupra expresiei matematice a curentului statoric, validată prin simularea în PSpice a procesului tranzitoriu utilizând modelul ortogonal al maşinii asincrone.