BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LX (LXIV), Fasc. 4, 2014
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

SYMMERICAL COMPONENTS METHOD IN INDUCTION MACHINE BEHAVIOR ANALYSIS

BY

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Received: October 8, 2014
Accepted for publication: November 20, 2014

Abstract. Starting from the observation that generally induction machine is described by a system of nonlinear equations, we aim to determine the circumstances in which the symmetrical components method can be successfully used in studying the behavior of the induction machine. Using the PSpice implementation of the three-phase model of the induction machine, this paper carries out a study on the possibility of using the symmetrical components method in the induction machine behavior analysis. The steady-state, the quasi steady-state and transient are separately considered.

Key words: induction machine; symmetrical components; PSpice.

1. Introduction

Symmetrical components is a method discovered by Charles Legeyt Fortescue. Fortescue demonstrated that any set of unbalanced three-phase quantities could be expressed as the sum of three symmetrical sets of balanced phasors (Fortescue, 1918). The method of symmetrical components is usually used in power systems to simplify the analysis of unbalanced three-phase systems. In the most common case the resulting symmetrical components are referred as direct or positive, inverse or negative and zero or homopolar. The

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resulting transform equations are mutually linearly independent. Using this tool, unbalanced system conditions can be easily analyzed.

The basic idea is that an asymmetrical set of three phasors can be expressed as a linear combination of three symmetrical sets of phasors by means of a complex linear transformation. The method of symmetrical components is frequently used in power systems and requires linear loads. It is used in the study of networks which electrical machines are connected to, as well.

The first application of the symmetrical components to the electrical machines was described by Lyon (Lyon, 1954). In their work (White & Woodson, 1959), White and Woodson extended this transformation using the unitary form. In (Harris *et al.*, 1970) the transformations and per-unit (pu) systems are discussed in detail. In (Paap, 2000) a short historical overview of the symmetrical components transformation and applications of unitary and orthogonal transformations are presented. The analysis starts from the following statement: “However, in network calculations the internal relations of magnetic field quantities and currents in the separate machines are of less importance. The proper relations between terminal voltages, currents and power may prevail”.

This paper presents a study on the possibility of using symmetrical components method in the analysis of induction machine behavior. The effect of the machine on the supply network is particularly of no importance. Starting from the observation that induction machine is described by a system generally of nonlinear equations, we try to determine the circumstances in which this method can be used in studying the behavior of the induction machine. The steady-state, the quasi-steady-state and transient are separately considered. The results have been obtained by PSpice simulation (Justus, 1993) using the most general and appropriate induction machine model *i.e*. the model in phase coordinates, the three-phase model.

2. Pspice Implementation of Symmetrical Components Method

Consider a three-phase (asymmetric) system of sinusoidal quantities (voltages) and their corresponding complex phasors:

$$\begin{aligned} v_a &= \sqrt{2}V_a \sin(\omega t + \gamma_a) \ \leftrightarrow \ \underline{V}_a = V_a \mathrm{e}^{\mathrm{j}\gamma_a} \\ v_b &= \sqrt{2}V_b \sin(\omega t + \gamma_b) \ \leftrightarrow \ \underline{V}_b = V_b \mathrm{e}^{\mathrm{j}\gamma_b} \\ v_c &= \sqrt{2}V_c \sin(\omega t + \gamma_c) \ \leftrightarrow \ \underline{V}_c = V_c \mathrm{e}^{\mathrm{j}\gamma_c} \end{aligned} \tag{1}$$

According to symmetrical components method, it can be expressed as:

$$\underline{V}_a,\ \underline{V}_b,\ \underline{V}_c \quad \Leftrightarrow \quad \begin{array}{l} \underline{V}_{da},\ \underline{V}_{db},\ \underline{V}_{dc} \\ \underline{V}_{ia},\ \underline{V}_{ib},\ \underline{V}_{ic} \\ \underline{V}_{ha} = \underline{V}_{hb} = \underline{V}_{hc} = \underline{V}_h \end{array} \tag{2}$$

The system is uniquely determined by the origin vectors:

$$\begin{aligned} \underline{V}_d &= (\underline{V}_a + \underline{a}\cdot\underline{V}_b + \underline{a}^2\cdot\underline{V}_c)/3 = \underline{V}_{da} \\ \underline{V}_i &= (\underline{V}_a + \underline{a}^2\cdot\underline{V}_b + \underline{a}\cdot\underline{V}_c)/3 = \underline{V}_{ia} \\ \underline{V}_h &= (\underline{V}_a + \underline{V}_b + \underline{V}_c)/3 \end{aligned} \tag{3}$$

where $\underline{a} = \mathrm{e}^{\mathrm{j}2\pi/3}$ is a specific vector operator. Note that $\underline{a}^3 = 1$ so that $\underline{a}^{-1} = \underline{a}^2$. Based on these phasors we can determine all the components.

Because it follows a PSpice implementation we prefer to work in the time domain. The seven components (three direct, three indirect and one homopolar) can be calculated with the formulas below:

$$\begin{aligned} v_{da} &= (v_a + v_{a\cdot b} + v_{a^2\cdot c})/3 \\ v_{db} &= (v_{a^2\cdot a} + v_b + v_{a\cdot c})/3 \\ v_{dc} &= (v_{a\cdot a} + v_{a^2\cdot b} + v_c)/3 \end{aligned} \tag{4}$$

$$\begin{aligned} v_{ia} &= (v_a + v_{a^2\cdot b} + v_{a\cdot c})/3 \\ v_{ib} &= (v_{a\cdot a} + v_b + v_{a^2\cdot c})/3 \\ v_{ic} &= (v_{a^2\cdot a} + v_{a\cdot b} + v_c)/3 \end{aligned} \tag{5}$$

$$v_h = (v_a + v_b + v_c)/3 \tag{6}$$

The quantities in the components calculations are:

$$\begin{aligned} v_{a\cdot x} &= \sqrt{2}V_x \sin(\omega t + \gamma_x + 2\pi/3) \\ v_{a^2\cdot y} &= \sqrt{2}V_y \sin(\omega t + \gamma_y - 2\pi/3) \end{aligned} \tag{7}$$

where $x, y \in (a, b, c)$. These results are only peculiarities of PSpice implementation.

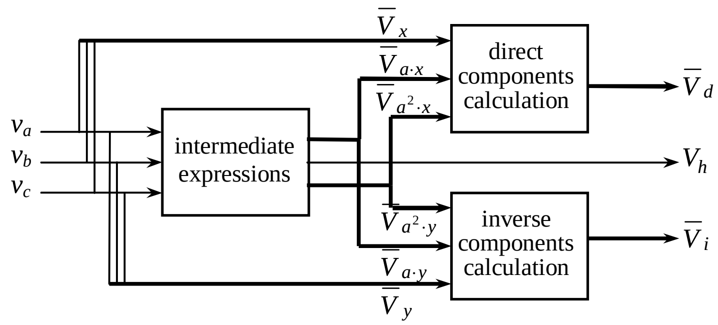

Fig. 1 – Block diagram of PSpice implementation of symmetrical components method.

According to the eqs. (3),…,(5) Fig. 1 shows schematically the way the symmetrical components have been determined by using PSpice. Starting from the expressions of the three unbalanced components, intermediate expressions (7) are calculated, which further allow determining all symmetrical components.

For instance, in Fig. 2 is shown an example of the unbalanced voltage supply system of a three-phase induction machine; the equivalent symmetrical components are presented in Fig. 3. This unbalanced voltage supply system is used in all examples in this paper.

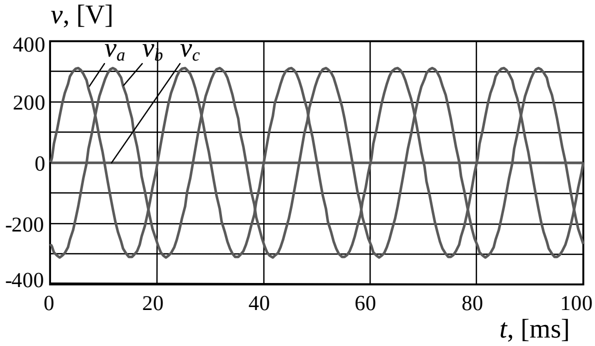

Fig. 2 – Unbalanced three-phase voltage system.

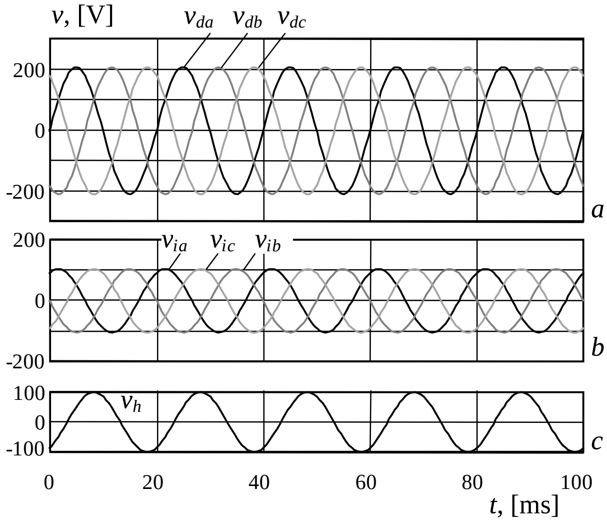

Fig. 3 – Voltage symmetrical components: *a* – direct; *b* – inverse; *c* – homopolar.

To describe the behavior of the induction machines, mathematical models with distributed or concentrated parameters have been suggested. For three-phase induction machines the natural model is the three-phase model. Most of the time however, the orthogonal (two-phase) model is used in the study of the three-phase induction machine operating as a motor or generator.

To allow for a more compact form of the equations, the following will use the spatial phasors to represent the quantities of interest in time domain. In a stationary reference frame the behavior of an induction machine can be described in the classical form by using the following equations:

a) voltage equations for stator and rotor;

b) stator and rotor fluxes;

c) electromagnetic torque expression (in terms of fluxes);

d) motion equation.

$$\begin{cases} \underline{v}_s = R_s \underline{i}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t}; \\ \underline{v}_r^{'} = R_r^{'} \underline{i}_r^{'} + \dfrac{\mathrm{d}\underline{\psi}_r^{'}}{\mathrm{d}t} - \mathrm{j}\omega_r \underline{\psi}_r^{'}; \end{cases} \tag{8}$$

$$\begin{cases} \underline{\psi}_s = L_s \underline{i}_s + L_m \underline{i}_r^{'}; & L_s = L_{\sigma s} + L_m; \\ \underline{\psi}_r^{'} = L_r^{'} \underline{i}_r^{'} + L_m \underline{i}_s; & L_r^{'} = L_{\sigma r}^{'} + L_m; \end{cases} \tag{9}$$

$$t_e = p \left| \underline{i}_r^{'} \times \underline{\psi}_r^{'} \right| \tag{10}$$

$$t_e - t_l = \frac{J}{p} \cdot \frac{\mathrm{d}\omega_r}{\mathrm{d}t} + F_\alpha \frac{\omega_r}{p} \tag{11}$$

All the rotor quantities are transformed to have the same frequency as the stator quantities, using well-known relations.

As can be seen, the system of equations describing the behavior of induction machine is nonlinear. Nonlinearity arises in two places, at terms $\omega_r(t) \times \underline{\psi}_r^{'}(t)$ and $\underline{i}_r^{'}(t) \times \underline{\psi}_r^{'}(t)$. But symmetrical components method can be used only in the case of linear loads. At first glance it seems that this method can not be applied in the case of electrical machines. Precisely because of this present paper aims to develop an analysis method from the viewpoint of induction machine.

The paper presents a comparative analysis of results obtained using the symmetrical components method and direct analysis of the unbalanced regimes. In both cases the simulation of the induction machine behavior has been accomplished on the basis of the three phase model.

The unbalanced three-phase voltage system used is shown in Fig. 2. As highlighted before, Pspice program has been used. There have been considered representative quantities: currents, magnetic fluxes and electromagnetic torque.

The numerical simulations have been performed for an induction machine rated as follows:

$$\begin{array}{llll} P_n = 5\,\mathrm{kW}; & R_s = 1\Omega; & L_{\sigma s} = 8\,\mathrm{mH}; & J = 30\ \mathrm{gm}^2; \\ U_{1\mathrm{ln}} = 380\,\mathrm{V}; & R_r^{'} = 0.9\Omega; & L_{\sigma r}^{'} = 7\,\mathrm{mH}; & F_\alpha \approx 0\ \mathrm{Nms/rad}; \\ f_1 = 50\ \mathrm{Hz} & Y\,\mathrm{connection}; & L_0 = 100\,\mathrm{mH}; & p = 1. \end{array}$$

3. Steady-State

For the beginning it is considered a particular steady-state, characterized by the zero speed of the rotor $\omega_r = 0$. Under these circumstances the nonlinear term $\omega_r(t)\times\underline{\psi}_r^{'}(t)$ disappears and the motion equations become useless. The machine behaves, as a linear device, at least from point of view of current and flux.

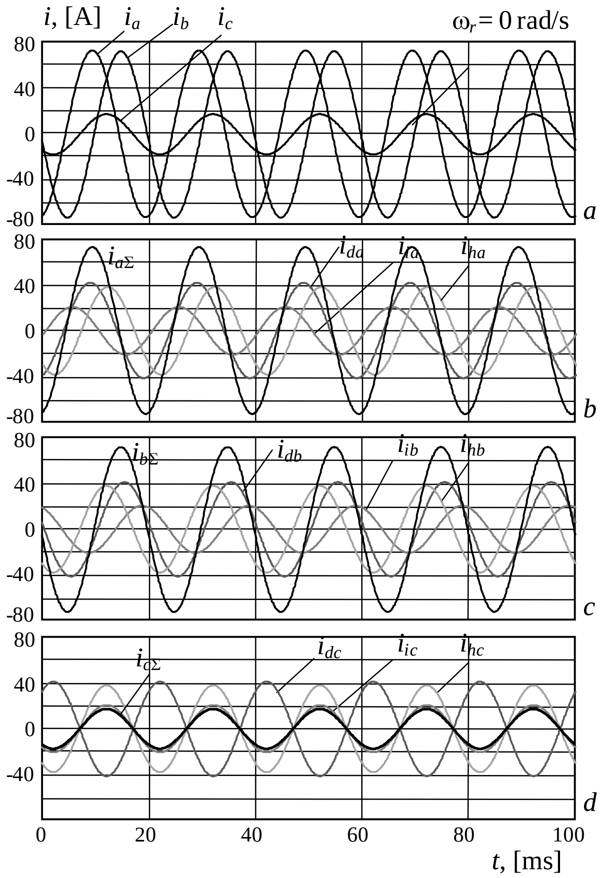

Fig. 4 – Stator currents in steady-state at $\omega_r = 0$ rad/s: *a* – stator currents; *b* –*a* winding current components; *c* – *b* winding current components; *d* – *c* winding current components.

The three stator currents i_a, i_b, i_c are shown in Fig. 4 *a*. In Fig. 4 *b*, *c* and *d* are presented the results obtained by using the symmetrical components method. Thus total stator currents are determined by adding the symmetrical components:

$$\begin{aligned} i_{a\Sigma} &= i_{ad} + i_{ai} + i_{ah}\,; \\ i_{b\Sigma} &= i_{bd} + i_{bi} + i_{bh}\,; \\ i_{c\Sigma} &= i_{cd} + i_{ci} + i_{ch}\,. \end{aligned} \tag{12}$$

As can be seen this results are identical to those evaluated by the direct method:

$$i_a = i_{a\Sigma}\,; \quad i_b = i_{b\Sigma}\,; \quad i_c = i_{c\Sigma}\,. \tag{13}$$

Fig. 5 shows the electromagnetic torque. As known, the homopolar components do not produce electromagnetic torque, so:

$$T_e = T_{e\Sigma} = T_{ed} + T_{ei}\,. \tag{14}$$

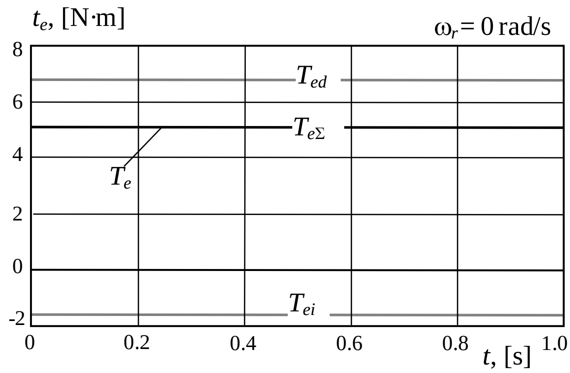

Fig. 5 – Electromagnetic torque components.

If the rotor angular speed is nonzero, but remains constant over time, the term $\omega_r(t) \times \underline{\psi}_r^{'}(t)$ is also nonzero and it is a linear term. It is expected that, at least in the currents and fluxes study, the symmetrical components method to give accurate results. In Fig. 6 is shown the simulation of the stator currents and in Fig. 7 the simulation of the main magnetic flux.

In both cases the summation of the direct, inverse and zero sequence components produces the same results as in the case of unbalanced supply:

$$\begin{aligned} i_a &= i_{a\Sigma} = i_{da} + i_{ia} + i_{ha}\,; \\ i_b &= i_{b\Sigma} = i_{db} + i_{ib} + i_{hb}\,; \\ i_c &= i_{c\Sigma} = i_{dc} + i_{ic} + i_{hc}\,. \end{aligned} \tag{15}$$

$$\begin{aligned} \Psi_{md} &= \Psi_{md\Sigma} = \Psi_{mdd} + \Psi_{mdi} + \Psi_{mdh}\,; \\ \Psi_{mq} &= \Psi_{mq\Sigma} = \Psi_{mqd} + \Psi_{mqi} + \Psi_{mqh}\,. \end{aligned} \tag{16}$$

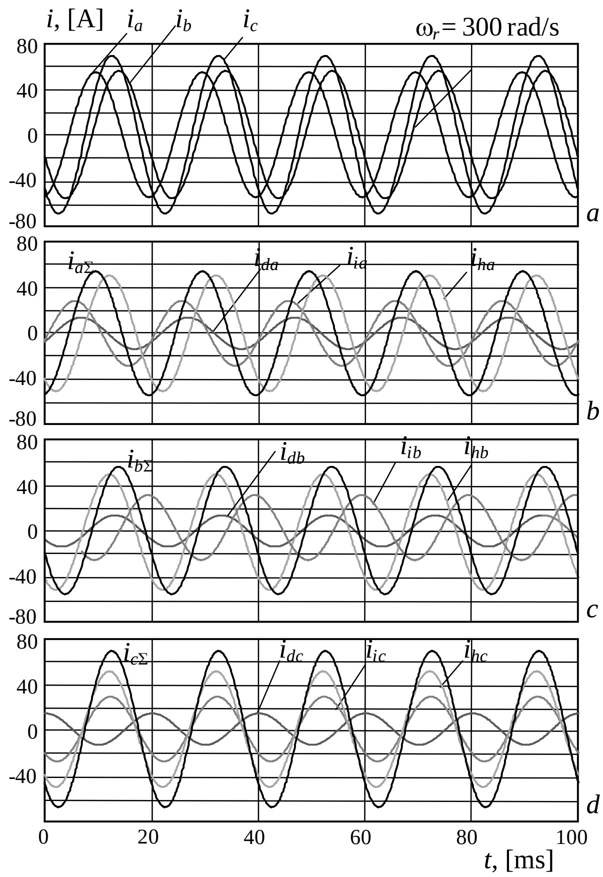

Fig. 6 – Stator currents in steady-state at $\omega_r = 300$ rad/s: *a* – stator currents; *b* – *a* winding current components; *c* – *b* winding current components; *d* – *c* winding current components.

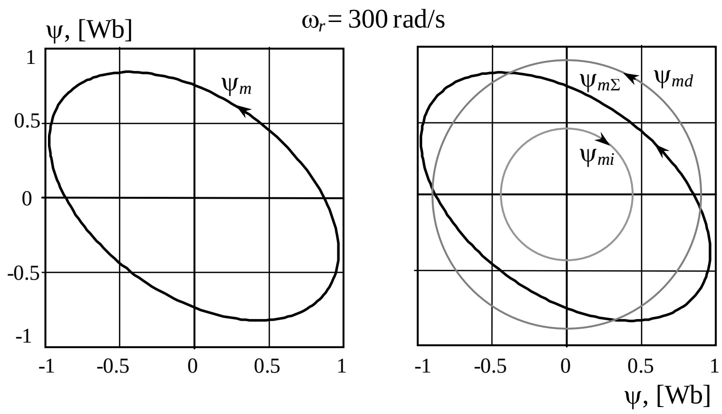

Fig. 7 – Main flux components in *dq* representation.

In the analysis of electromagnetic torque, by studying directly the unbalanced supply case (true result), the results differ from those achieved by applying symmetrical components method. In the first case the electromagnetic torque $t_e(t)$ has a sinusoidal variation superimposed over a continuous component $T_{e\ avg}$. In the second case, as natural, each component, direct and reverse, yields a constant torque T_{ed} and T_{ei} respectively. Their summation gives a constant value $T_{e\Sigma}$, with no sinusoidal variations superimposed. This value is identical to the average (continuous) value in the real case.

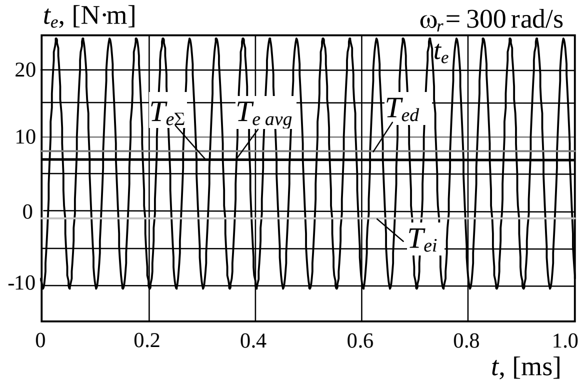

Fig. 8 – Electromagnetic torque components.

4. Quasi-Stationary Operation

If consider the quasi-stationary operating conditions, the rotor angular speed remains no longer constant, as in previous cases, but changes slowly over time. Thus it can be considered a steady-state in every moment which changes from a moment to another. Fig. 9 shows the waveforms of the stator current in the a axis when the angular speed ω_r varies from 0 to 314rad/s within 2s. Adding i_{ad}, i_{ai} and i_{ah} components gives $i_{a\Sigma}$ which is identical to i_a. In case of the other axes similar final results are found.

Fig. 10 shows the variation of the electromagnetic torque under quasi-stationary conditions. Within one second the angular rotor speed varies from 0 to 314rad/s. The electromagnetic torque t_e has the same characteristics found in previous case. Over an average value $t_{e\ avg}$ (t), sinusoidal variations overlap. Their amplitude depend on the rotor angular speed. For $\omega_r = 0$ oscillations amplitude is zero but increases constantly with angular speed. It should be noted that in this case the addition of the symmetrical components t_{ed} (t) and t_{ei} (t) gives again $t_{e\Sigma} = t_{e\ avg}$.

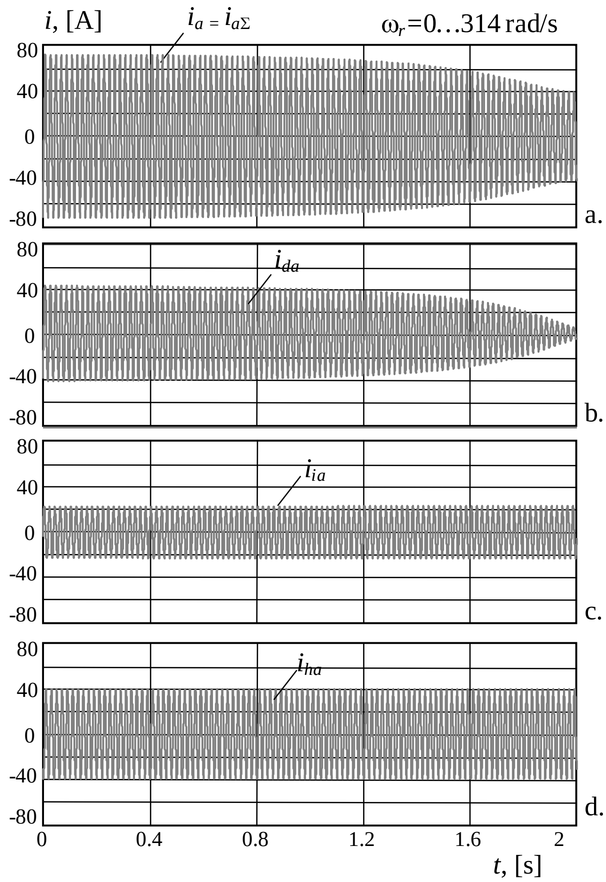

Fig. 9 – *a* winding stator current in quasi-stationary operation: a – stator current; b – direct component; c – inverse component; d – homopolar component.

5. Transient Operation

To evaluate the transient behavior three different situations, typical for induction machine operation, have been considered.

In the first situation, the machine, initially running at no-load, is suddenly loaded with a torque $T_r = 10$ Nm (Cociu *et al.*, 2011), (Cociu & Cociu, 2014). All quantities that characterize the machine are affected and a transient behavior occurs. Fig. 10 shows the stator current transient for *b* winding. The effect is a significant increase in the direct component i_{bd} (t), insignificant change in the inverse component i_{bi} (t) and whereas the zero component i_{bh} (t) remains constant. As can be observed the final result is $i_b = i_{b\Sigma}$.

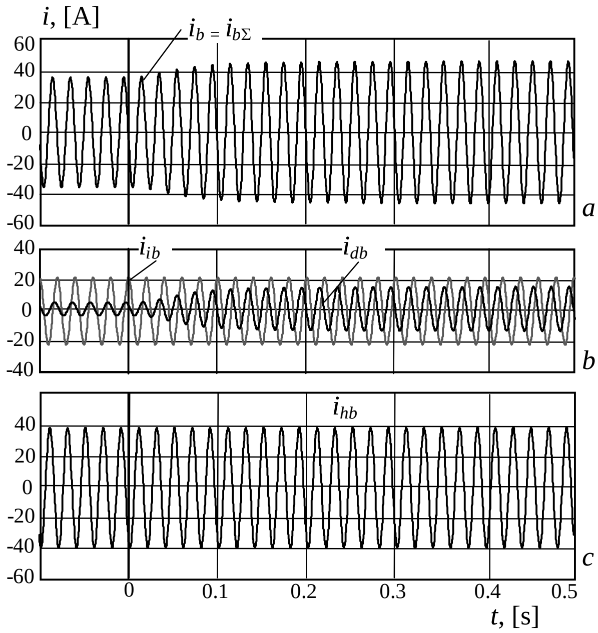

Fig. 10 – *b* winding stator current in transient: *a* – stator current; *b* – direct and inverse current components; *c* – homopolar current component.

In the case of the electromagnetic torque (Fig. 11) we found the same results as in the case of steady-state or quasi-stationary operation. The transient electromagnetic torque $t_e(t)$ shows sinusoidal oscillations of significant amplitude, superimposed over the average value $t_{e\,avg}(t)$. At any time the average torque (over a range of 10 ms) is equal to the sum of direct and inverse components: $t_{eavg} = t_{e\Sigma} = t_{ed} + t_{ei}$. Note that, in this case loading the machine causes the angular rotor speed to change, as well.

To demonstrate the second transient operation the rotor angular speed has been changed relatively rapid: it changes from 290 to 310 rad/s within 0.1 s, as shown in Fig. 12 *a*. Note that changing the angular speed causes the term $\omega_r(t)\cdot\underline{\psi}_r^{'}(t)$ to become nonlinear. Fig. 12 *b* shows the variation of the electromagnetic torque. Yet in this case we find that $t_{eavg} = t_{e\Sigma} = t_{ed} + t_{ei}$.

The third example is the most complex and it is the starting transient at no-load. Overcoming the practical difficulties of accurate evaluation for the average torque within 10 ms, in Fig. 13 it is demonstrated that, with a good approximation, one can consider $t_{eavg} \approx t_{e\Sigma}$.

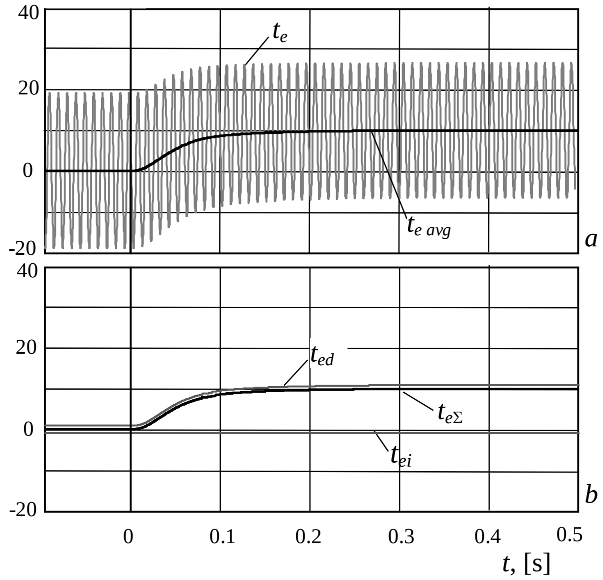

Fig. 11 – Electromagnetic torque in transient:
a – instantaneous and average torque; *b* – torque components.

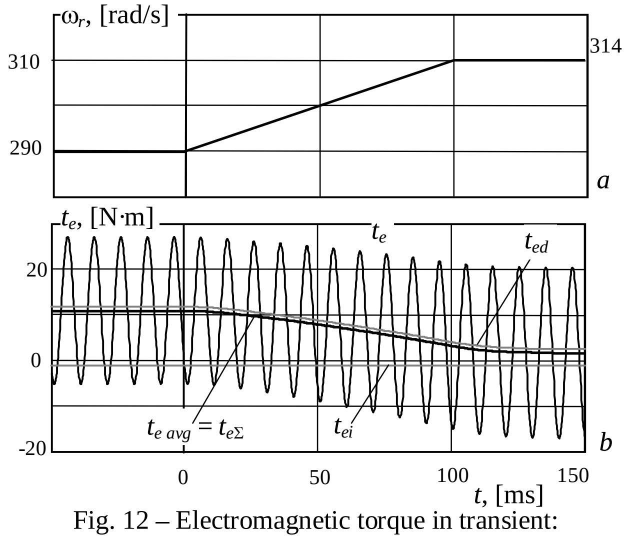

Fig. 12 – Electromagnetic torque in transient:
a – angular rotor speed; *b* – torque components.

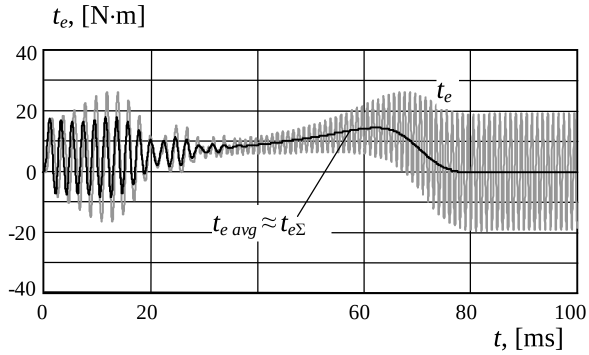

Fig. 13 – Electromagnetic torque in starting operation.

The influence of the electromagnetic torque can be clearly observed from the variation of the angular speed in starting operation, as shown in Fig. 14. Although the results are not identical in the two cases, they are very close.

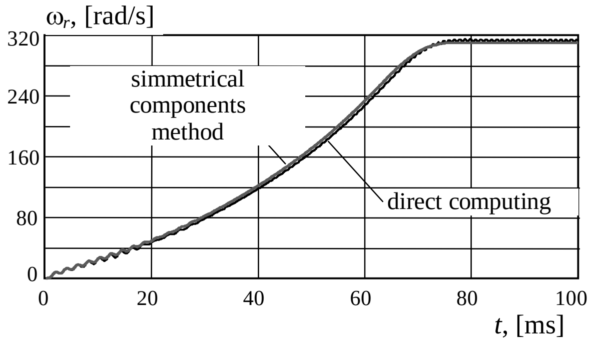

Fig. 14 – Angular rotor speed in starting operation.

6. The Electromagnetic Torque Analysis

According to eq. (10) gives:

$$t_e / p = \left| \underline{i}_r^{'} \times \underline{\psi}_r^{'} \right| \tag{17}$$

$$t_e / p = \left(\underline{i}_{rd}^{'} + \underline{i}_{ri}^{'} \right) \times \left(\underline{\psi}_{rd}^{'} + \underline{\psi}_{ri}^{'} \right) = \underline{i}_{rd}^{'} \times \underline{\psi}_{rd}^{'} + \underline{i}_{ri}^{'} \times \underline{\psi}_{ri}^{'} + \underline{i}_{rd}^{'} \times \underline{\psi}_{ri}^{'} + \underline{i}_{ri}^{'} \times \underline{\psi}_{rd}^{'} \tag{18}$$

The electromagnetic torque expression has four components of which the first two represent even direct torque and vice versa:

$$\underline{i}_{rd}^{'} \times \underline{\psi}_{rd}^{'} = t_{ed} / p ; \quad \underline{i}_{ri}^{'} \times \underline{\psi}_{ri}^{'} = t_{ei} / p . \tag{19}$$

To determine the other two components we consider:

$$\underline{i}'_{rd} = I_{rd} \cdot e^{\,\mathrm{j}(\omega t+\gamma_{id})}\,; \quad \underline{i}'_{ri} = I_{ri} \cdot e^{\,\mathrm{j}(-\omega t+\gamma_{ii})}\,;$$
$$\underline{\psi}'_{rd} = \Psi_{rd} \cdot e^{\,\mathrm{j}(\omega t+\gamma_{\psi d})}\,; \underline{\psi}'_{ri} = \Psi_{ri} \cdot e^{\,\mathrm{j}(-\omega t+\gamma_{\psi i})}\,. \tag{20}$$

$$\underline{i}'_{rd} \times \underline{\psi}'_{ri} = I_{rd}\Psi_{ri}\cos(2\omega t + \gamma_{id} - \gamma_{\psi i})$$
$$\underline{i}'_{ri} \times \underline{\psi}'_{rd} = I_{ri}\Psi_{rd}\cos(2\omega t - \gamma_{ii} + \gamma_{\psi d}) \tag{21}$$

Finally it results the electromagnetic torque by adding the two components of 2ω angular speed:

$$t_e = t_{ed} + t_{ei} + T\cos(2\omega t + \gamma_t)\,. \tag{22}$$

A thorough analysis leads to the conclusion that $T = T(\omega_r)$, which means that the amplitude of the component superimposed over average component changes with the rotor speed. Performing the average value over a period of π/ω gives:

$$t_{e\ avg} = t_{ed} + t_{ei} = t_{e\Sigma}\,. \tag{23}$$

7. Conclusions

Although the system of equations describing the induction machine behavior is nonlinear, the symmetrical components method can be successfully used. PSpice simulation and the three-phase model of the induction machine provided same results in both cases: direct analysis and symmetrical components method

Validation has been accomplished under steady-state, quasi-steady state and transient conditions. As expected, in terms of currents and magnetic fluxes the results has been identical in all situations.

Regarding the electromagnetic torque, in almost all situations some qualitative differences occur. The symmetrical components method considers that the machine is separately supplied by two symmetrical voltage systems: direct, and inverse. Each of them yields a specific electromagnetic torque, constant if the angular speed is constant. In the real situation, when the machine is supplied by an unbalanced voltage system, oscillations of 2ω angular frequency arise, overlapping the average torque. But the average torque is equal to the result obtained by symmetrical components method.

To conclude, the symmetrical components method can be used in the analysis of induction machine behavior if ignore the sinusoidal electromagnetic torque component, which occurs in the real case.

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METODA COMPONENTELOR SIMETRICE ÎN STUDIUL COMPORTĂRII MAŞINII DE INDUCŢIE

(Rezumat)

Pornind de la observaţia că o maşină de inducţie este descrisă de un sistem de ecuaţii în general neliniar, se încearcă determinarea situaţiilor în care se poate utiliza cu succes metoda componentelor simetrice în studiul comportării maşinii de inducţie. Utilizând implementarea în PSpice a modelului trifazat (în coordonatele fazelor) al maşinii de inducţie, lucrarea prezintă un studiu privind posibilităţile utilizării acestei metode în analiza comportării maşinii. S-au avut în vedere, separat, regimul staţionar, regimul cvasistaţionar şi regimul tranzitoriu. S-au considerat categorii reprezentative de mărimi: curenţi, fluxuri magnetice şi cuplul electromagnetic.