BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LXI (LXV), Fasc. 1, 2015
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

ELECTRICAL PARAMETER ESTIMATION OF SUPERCAPACITOR WITH EQUIVALENT MODEL

BY

ELENA DĂNILĂ[*] and GHEORGHE LIVINŢ

"Gheorghe Asachi" Technical University of Iaşi
Faculty of Electrical Engineering

Received: February 20, 2015
Accepted for publication: March 10, 2015

Abstract. The paper continues a previous research regarding supercapacitors modeling on micro scale using equivalents circuits, by proposing a method of estimation for resistances and capacitances from the corresponding scheme. A supercapacitor model is very complex because of the distributed-parameter model, so the electrical parameter estimation involved three complementary mathematical methods and software simulations. The voltage characteristic of the obtained supercapacitor was similar to the voltage measured on a commercial supercapacitor taken as reference.

Key words: supercapacitor; equivalent circuit; transfer function.

1. Introduction

Renewable energy source are quickly accomplishing the need for superior energy storage devices. Due to their high reliability, efficiency and operating lifetime, supercapacitors are especially ideal for renewable energy applications. Design considerations are based on reliability and the need for maintenance free operations. Supercapacitors are fundamentally viewed as

[*]Corresponding author: *e-mail*: edanila@tuiasi.ro

maintenance-free devices that do not require costly test runs and expensive management systems versus batteries. In contrast with traditional capacitors, electric double-layer capacitors (supercapacitors) do not have a conventional dielectric. These capacitors use "plates" that are in fact two layers of the same substrate, and their electrical properties, the so-called "electrical double layer", result in the effective separation of charge despite the vanishingly thin (on the order of nanometers) physical separation of the layers. The lack of need for a bulky layer of dielectric permits the packing of "plates" with much larger surface area into a given size, resulting in their extraordinarily high capacitances in practical sized packages.

2. Supercapacitor Equivalent Model

Given the new equivalent model of a supercapacitor proposed by Dănilă *et al*., (2011), a nonlinear RC series-parallel circuit (Fig. 1), its electrical parameters will be estimated using numerical integration method and filtering. The obtained values must be as close as possible to the real parameters of a commercial supercapacitor, chosen as reference.

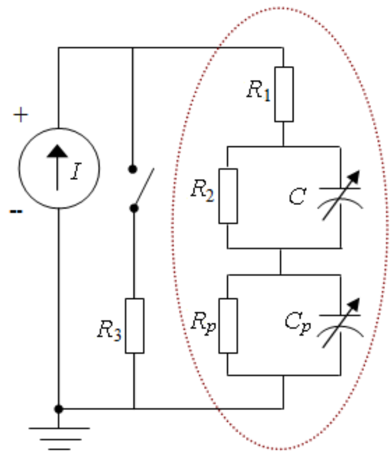

Fig. 1 – The proposed equivalent model for supercapacitor.

The two variable capacitances are nonlinear, dependent on the voltage applied to the entire circuit. Capacitance C is the one on which depends the behavior of the entire circuit, determining the maximum state of charge of the supercapacitor. The amount of stored energy and the value of energy levels variations are determined mainly by the capacitance (C is proportional to the stored energy). Resistance R_1 represents losses in charge/discharge cycle, arising due to resistance of the conductor element, as the process is not an ideal one. In case of connecting in series more than three supercapacitors with the proposed equivalent scheme (to increase the working voltage), an active voltage balancing circuit is needed to regulate the cell voltage. It is common to choose according to application a specific voltage, and thus, calculating the required capacitance (Thounthong *et al.*, 2009). Resistance R_2, connected in parallel with C, represents the quantification of auto discharge effect. Over voltage protection, provided by R_3 is necessary to prevent supercapacitor components damage, by passively balancing the voltage of the cells (otherwise, the voltage in an individual cell can increase more than the other cells values, leading to emission of gas or explosion). This difference in voltage can occur if a cell has a smaller capacitance than the other, which is reflected in the amount of stored energy (Cheng, 2010). R_p and C_p are included in the circuit to model the fastest processes in the supercapacitor behavior.

The proposed equivalent model has the following transfer function:

$$Z(s) = R_1 + \frac{R_p \frac{1}{sC_p}}{R_p + \frac{1}{sC_p}} + \frac{R_2 \frac{1}{sC}}{R_2 + \frac{1}{sC}} = R_1 + \frac{R_p}{sC_pR_p + 1} + \frac{R_p}{sC_pR_p + 1} \Leftrightarrow$$

$$Z(s) = \frac{s^2(CC_pR_pR_2R_1) + s(C_pR_1R_p + CR_1R_2 + CR_pR_2 + C_pR_pR_2) + R_1 + R_p + R_2}{s^2(CC_pR_pR_2) + s(C_pR_p + CR_2) + 1} \Leftrightarrow$$

$$Z(s) = \frac{R_1 s^2 + \frac{C_pR_1R_p + CR_1R_2 + CR_pR_2 + C_pR_pR_2}{CC_pR_pR_2} s + \frac{R_1 + R_p + R_2}{CC_pR_pR_2}}{s^2 + \frac{C_pR_p + CR_2}{CC_pR_pR_2} s + \frac{1}{CC_pR_pR_2}}, \tag{1}$$

3. Supercapacitor Electrical Parameters Estimation Using Numerical Integration Method

In case of estimating the parameters of a continuous model, the input vector $u(t)$ and output vector $y(t)$ contain also the terms with derivatives thereof, which are not available by measurement. The problem of these derivatives evaluation was solved by introducing a causal linear operator, stable and able to replace the differential operator preserving the exact transfer function or differential equation. Such a dynamic linear transformation (DLT) should ensure that the estimation model obtained after processing is equivalent to the original one. Due to the introduction of these operators that implement sampled signal processing techniques, in the secondary phase the parameters can be estimated by block calculation or by using a recursive algorithm.

The following analysis will be performed assuming that the supercapacitor operates at constant temperature and has the initial state “discharged”. At zero voltage, the supercapacitor has a base capacitance and as the voltage increases, the capacitance increases in an approximately linear fashion: $C(U) = C_0 + \text{const}\cdot U$, (Schönberger, 2009). This linear function, which is once determined for the prototype of a series of commercial supercapacitors, allows extrapolating the C_0 value for usual rated voltages.

Evaluation results for a variety of supercapacitor samples with different rated capacitance and voltage demonstrate that using the linear capacitance is more accurate than using the rated capacitance to estimate supercapacitor energy (Hengzhao *et al.*, 2015). Moreover, the results have shown that using the linear capacitance determined through a high power charge experiment can minimize the energy estimation error. The capacitance can be therefore considered linear.

The supercapacitor, mathematically modeled as a continuous linear dynamic and invariant system, can be generally described through a linear differential equation of *n* order, with constant and real coefficients:

$$\sum_{i=0}^{n} a_i \frac{\mathrm{d}^i y(t)}{\mathrm{d}t^i} = \sum_{j=0}^{m} b_j \frac{\mathrm{d}^j u(t)}{\mathrm{d}t^j},\ m \le n,\ a_n = 1, \tag{2}$$

where a_i, b_i are the transfer function numerator and denominator coefficients. For the supercapacitor, the equation becomes:

$$\begin{gathered}\frac{\mathrm{d}^2 v(t)}{\mathrm{d}t^2} + a_1 \frac{\mathrm{d}v(t)}{\mathrm{d}t} + a_0 v(t) = b_2 \frac{\mathrm{d}^2 i(t)}{\mathrm{d}t^2} + b_1 \frac{\mathrm{d}i(t)}{\mathrm{d}t} + b_0 i(t) \Leftrightarrow \\ \frac{1}{C \cdot C_p R_p R_2} v(t) + \frac{C_p R_p + C \cdot R_2}{C \cdot C_p R_p R_2} \cdot \frac{dv(t)}{dt} + \frac{d^2 v(t)}{dt^2} = \\ = \frac{R_1 + R_p + R_2}{C C_p R_p R_2} i(t) + \frac{C_p R_1 R_p + C R_1 R_2 + C R_p R_2 + C_p R_p R_2}{C C_p R_p R_2} \cdot \frac{\mathrm{d}i(t)}{\mathrm{d}t} + R_1 \frac{\mathrm{d}^2 i(t)}{\mathrm{d}t^2},\end{gathered} \tag{3}$$

where $v(t)$ is the voltage supplied by the supercapacitor, respectively the output of the system, and $i(t)$ is the charging current – the input of the system. Replacing the coefficients of the transfer function, the quadrature will be obtained:

$$\begin{gathered}\frac{1}{C C_p R_p R_2} \int \ldots \int_{t-hT_e}^{t} v(\tau) \mathrm{d}\tau^2 + \frac{C_p R_p + C R_2}{C C_p R_p R_2} \int \ldots \int_{t-hT_e}^{t} \frac{\mathrm{d}v(\tau)}{\mathrm{d}\tau} d\tau^2 + \int \ldots \int_{t-hT_e}^{t} \frac{\mathrm{d}^2 v(\tau)}{\mathrm{d}\tau^2} \mathrm{d}\tau^2 = \\ = \frac{R_1 + R_p + R_2}{C C_p R_p R_2} \int \ldots \int_{t-hT_e}^{t} i(\tau) \mathrm{d}\tau^2 + \\ + \frac{C_p R_1 R_p + C R_1 R_2 + C R_p R_2 + C_p R_p R_2}{C C_p R_p R_2} \int \ldots \int_{t-hT_e}^{t} \frac{\mathrm{d}i(\tau)}{\mathrm{d}\tau} \mathrm{d}\tau^2 + R_1 \int \ldots \int_{t-hT_e}^{t} \frac{\mathrm{d}^2 i(\tau)}{\mathrm{d}\tau^2} d\tau^2.\end{gathered} \tag{4}$$

For extracting spectral components of the signal and for linear predicting the integral, there are applied filtering specific operations (method of linear integral filters that uses for the integration domain a sliding temporal window [t–hT_e], where h is the synthesis parameter of the method and T_e is the sampling period). The recursive equation of supercapacitor is (Dougal *et al.*, 2004):

$$\begin{cases} i(nh) = \left(\dfrac{2\tau - h}{2\tau + h}\right)^n i(0) + g\sum_{k=1}^{n}\left\{\left(\dfrac{2\tau - h}{2\tau + h}\right)^{n-k}\left[v(kh) - v((k-1)h)\right]\right\}, \\ nh = t, \\ g = \dfrac{2C}{2\tau + h}. \end{cases} \tag{5}$$

Given the order of the resistances and that on R_2 the current can be neglected (very low self-discharge), the coefficients of the transfer function can be approximated as follows:

$$\begin{cases} a_1 = \dfrac{C_pR_p + C\cdot R_2}{C\cdot C_pR_pR_2} \cong \dfrac{1}{C_pR_p}, \\ a_0 = \dfrac{1}{C\cdot C_pR_pR_2}, \\ b_2 = R_1, \\ b_1 = \dfrac{C_p\cdot R_1R_p + C\cdot R_1R_2 + C\cdot R_p\cdot R_2 + C_pR_pR_2}{C\cdot C_pR_pR_2} \cong \\ \cong \dfrac{R_1}{C_pR_p} + \dfrac{1}{C_p} + \dfrac{1}{C}, \\ b_0 = \dfrac{R_1 + R_p + R_2}{C\cdot C_pR_pR_2} \cong \dfrac{R_1 + R_p}{C\cdot C_pR_p}, \end{cases} \Rightarrow \begin{cases} R_1 = b_2, \\ R_2 = \dfrac{b_0}{a_0}, \\ R_p = \dfrac{b_1}{a_1} - \dfrac{b_0}{a_1^2} - b_2, \\ C = \dfrac{a_1}{b_0}, \\ C_p = \dfrac{a_1}{-a_1^2b_2 - b_0 + a_1b_1}. \end{cases} \tag{6}$$

The vectors of dynamic systems coefficients will be identified using a parametric optimization method, *i.e.* the method of least squares.

4. Coefficients Identification Using the Method of Least Squares

In Matlab Simulink was build an estimation model with ARX block (autoregressive model with exogenous sizes) (Fig. 2).

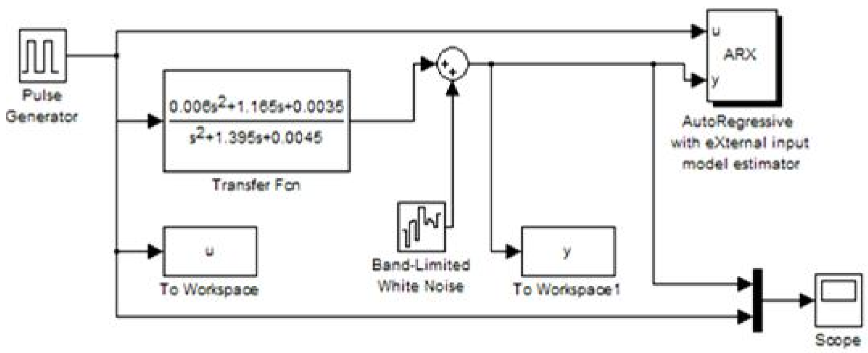

Fig. 2 – Matlab Simulink block diagram of the proposed model, with ARX estimator.

For the input and output vectors were introduced 1000 measured values in charging/discharging cycles of a commercial supercapacitor. Applying to u and y the function *tfestimate*, is obtained the transfer function:

$$Z_1(s) = \frac{0.006s^2 + 1.165s + 0.035}{s^2 + 1.643s + 0.0045} \tag{7}$$

The validation is made by simulation. It results that the error of the model with transfer function Z_1 (presented in Fig. 3 for each operating stage of the supercapacitor) is very low, varying between –0.1% and 0.6%, which means that the estimated model is very close to the real one. This can be also observed from the superposition of real (measured) voltage – pink- and the simulated one - blue.

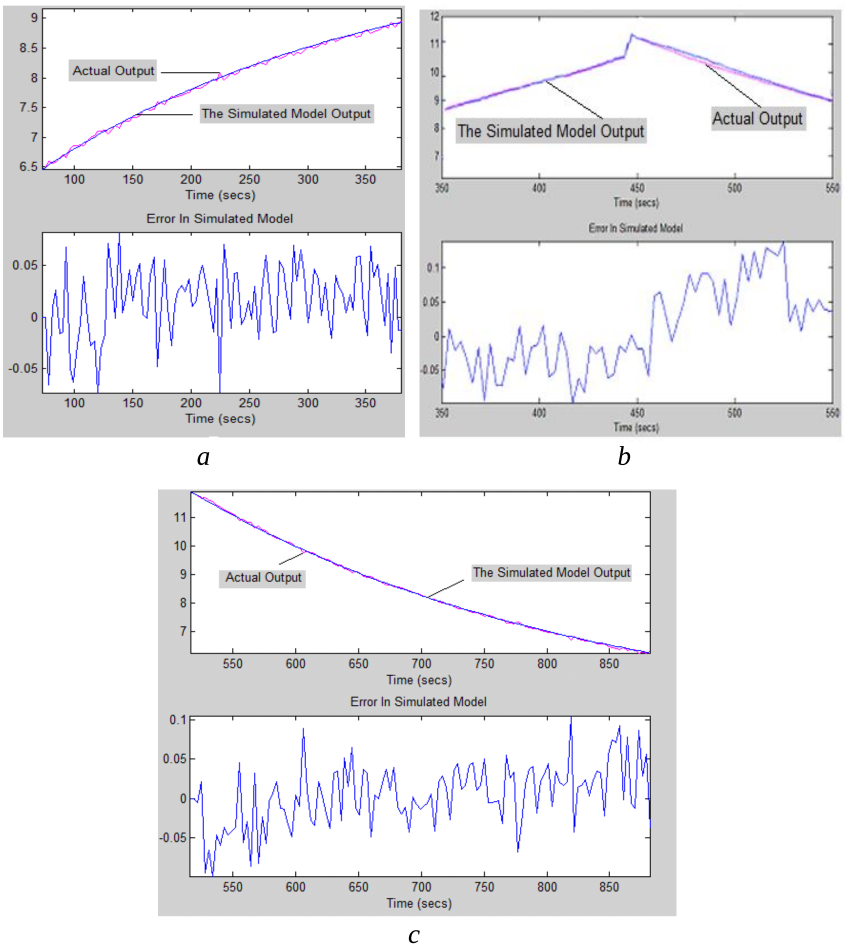

Fig. 3 – The error of simulated model: *a* – during charging, *b* – on the beginning of discharge and *c* – during discharging.

Replacing in (6) the obtained coefficients, for the supercapacitor results:

$$\begin{cases} R_1 = b_2 = 0.006 m\Omega \\ R_2 = \dfrac{b_0}{a_0} = 0.778\Omega \\ R_p = \dfrac{b_1}{a_1} - \dfrac{b_0}{a_1^2} - b_2 = 0.701\Omega \\ C = \dfrac{a_1}{b_0} = 398.57F \\ C_p = \dfrac{a_1}{-a_1^2 b_2 - b_0 + a_1 b_1} = 7.97F \end{cases} \tag{8}$$

Resistance R_2 doesn't vary, as the self-discharge of the analyzed supercapacitor is very slow (the electrode of the *PowerStor* supercapacitor - taken as reference – is of graphene). R_2 remains at the value of 0.8 Ω specified by the producer. R_p has a very low value and its influence on the voltage curve is not visible. R_1 is 6 mΩ, R_p is 1 Ω and C_p has to be equal to C/50 according to the supercapacitor's data sheet.

Assigning the estimated values to the components of the proposed model (represented in Simulink by the block scheme from Fig. 4), the input signal (current) and the output signal (voltage) for several operating cycles of supercapacitor are obtained.

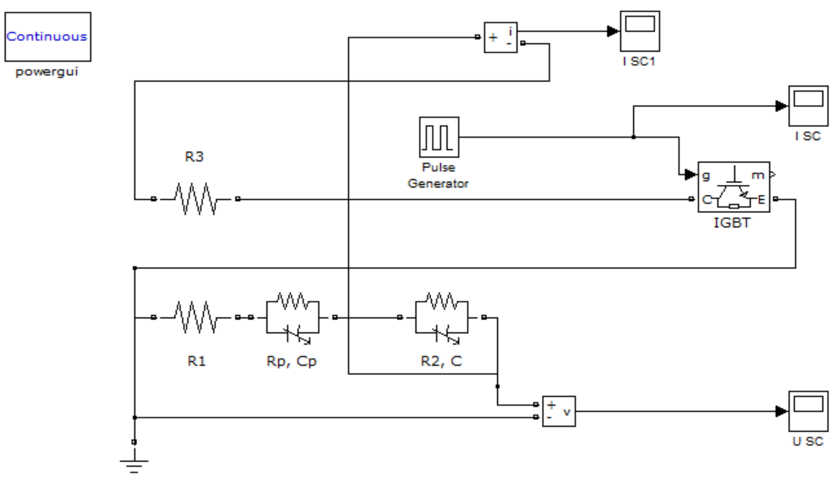

Fig. 4 –Simulink model of the proposed equivalent circuit.

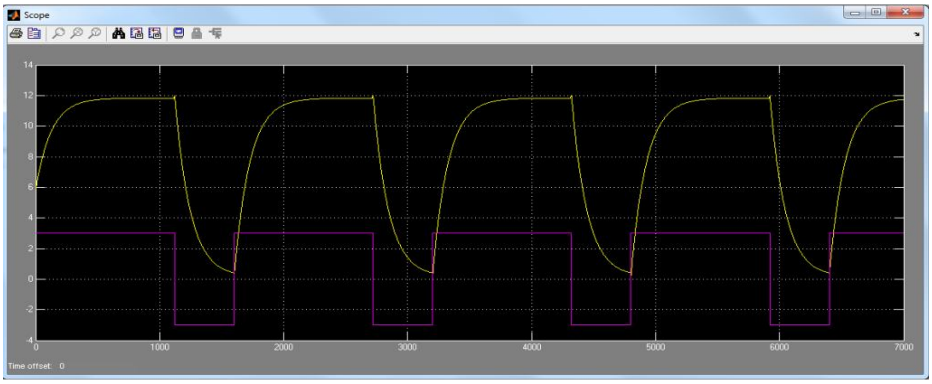

Fig. 5 – Supercapacitor’s voltage characteristic (yellow) on constant charging current (pink).

Based on Z_1 transfer function, the step response, polar characteristic (Nyquist diagram), pole-zero map and the Bode diagram of the estimated system are plotted using Matlab (Fig. 6).

The rise time is of 558 seconds, according to the step response. Theoretically, for a continuous system of order 2, the rise time is 1.8 higher than the time constant on charging, condition which is mathematically verified by the values from (8):

$$t_{\text{rise}} = 1.8\tau_{\text{charge}} = 1.8(398.57 \times 0.778) = 557.998 \text{ s}.$$

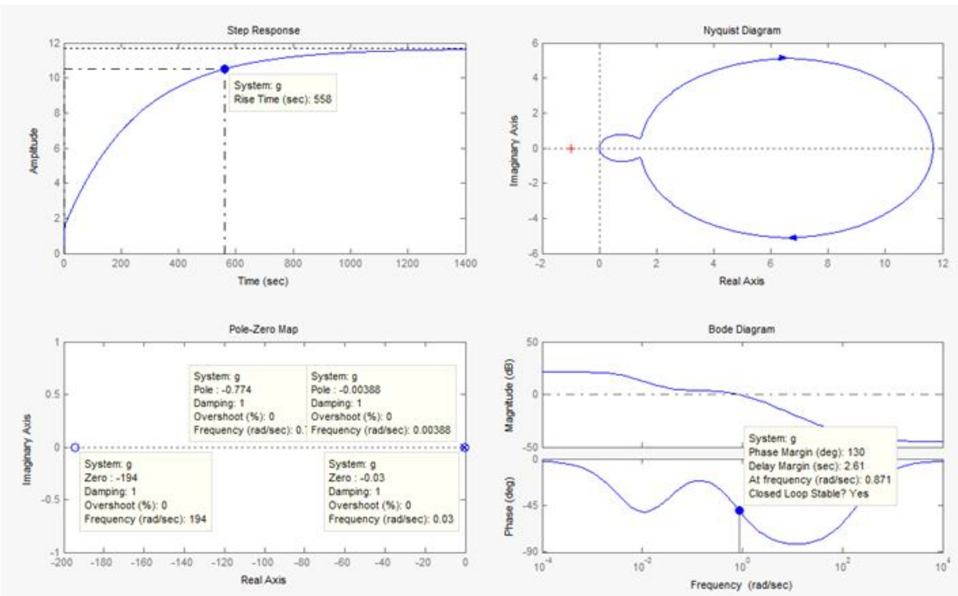

Fig. 6 – Time and frequency response of the equivalent model.

From pole-zero map results that the system is stable, as the poles and the zeros are in the left half-plane, on the negative real axis. There is one distant zero, but it doesn't influence the system performance. From the same map results that the system has a short time response, as the poles are distant from the imaginary axis. From Bode diagram appears also that the system is stable, because the phase margin is positive.

5. Conclusions

Supercapacitors superior characteristics compared to other storage devices in terms of energy density, efficiency, and lifespan open new directions for the development of micro-power sources. Modeling and parameterization of these storage units represents an important step in implementing a reliable supply circuit.

Sequential nature of the process of power supply allows the use of mathematical models for analysis in establishing optimal strategies to harmonize the demand with resources.

The presented method for electrical parameters estimation of a supercapacitor was validated by comparing, in Matlab, the simulated voltage curve with the voltage waveform provided by the commercial supercapacitors chosen for reference, the maximum error being of 0.15%.

REFERENCES

Cheng Y., *Assessments of Energy Capacity and Energy Losses of Supercapacitors in Fast Charging–Discharging Cycles*. IEEE Trans. on Energy Conversion, **25**, *1*, 253-261 (2010).

Dănilă E., Lucache D.D., Livinț G., *Models and Modelling Supercapacitors for a Defined Application*. 8th Internat. Conf. on Electromechan. a. Power Syst. SIELMEN 2011 – Ann. of the Univ. of Craiova, Electrical Engng. series, *35*, 200-205 (2011).

Dougal R.A., Gao L., Liu S., *Ultracapacitor Model with Automatic Order Selection and Capacity Scaling for Dynamic System Simulation*. Elsevier J. of Power Sources, *126*, 250-257 (2004).

Hengzhao Y., Ying Z., *Estimation of supercapacitor energy using a linear capacitance for applications in wireless sensor networks*. Elsevier Journal of Power Sources, *275*, 498–505 (2015).

Schönberger J., *Modeling a Supercapacitor using PLECS*, Plexim GmbH, version 4 (2010).

Thounthong P., Rael S., Davat B., *Analysis of Supercapacitor as Second Source Based on Fuel Cell Power Generation*. IEEE Trans. on Energy Conversion, **24**, *1*, 247-256 (2009).

ESTIMAREA PARAMETRILOR ELECTRICI AI SUPERCONDENSATORULUI CU MODEL ECHIVALENT

(Rezumat)

Lucrarea propune, în continuarea unor cercetări anterioare, în cadrul tehnicii de modelare dinamică a supercondensatoarelor prin circuite echivelente, un nou model căruia i s-au estimat parametrii şi i s-au calculat erorile raportat la valorile măsurate pe supercondensatori comerciali. Supercondensatorul modelat matematic ca un sistem dinamic liniar continuu şi invariant în timp, a fost descris general printr-o ecuație diferențială liniară de ordinul *n*, cu coeficienți constanți şi reali. Parametrii au fost estimați prin metoda integrării numerice şi printr-o metodă parametrică de optimizare implementată în Matlab Simulink, cu bloc ARX. Eroarea maximă a fost la identificare de 0,15%, iar forma de undă a tensiunii la ieşirea modelului cu parametrii determinați similară celei măsurate. Erorile au fost determinate prin compararea curbei de tensiune simulate cu cea de tensiune furnizată de producătorii supercondensatoarelor alese pentru referință, EPCOS şi CAP-XX.