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MATHEMATIC MODEL AND APLICATION FOR HARMONIC STEADY-STATE OF THE POWER TRANSMISSION LINE

BY

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Abstract. For electricity transmission on long or very long distances, closer to large consumption centres, a transmission system (electricity transmission networks – ETN) is used. In essence, electricity transmission means propagation of the electromagnetic field which is dependent on the electric conduction and radio currents (convection and Roentgen current). Until now electrical energy transmission uses only alternating and direct conduction currents. These conduction currents are determined by the flow through the conductor of the load wearers. The physical medium of this energy transfer, using the technique of conduction currents, is the conductor’s overhead or cable lines. Based on information from the literature, the paper presents a mathematical model and a computer program, intended for a complete analysis of the ETN symmetrical operation steady-state.

Key words: harmonic steady-state; transmission network.

1. Introduction

In power sector, the economic benefits impose the interconnection of all power installations from a country, also developing the national power system

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(NPS). The main elements of the NPS are: power generators from power plants, power transformers, autotransformers, power transmission and distribution lines, and electricity customers. In addition to these main components, the NPS also includes other elements such as, for example, switching, automation and security elements. Basically, the NPS comprises three components: generation, transmission and distribution power subsystems.

In order to transfer the produced electricity from power plants, at long distances, closer to major consumption centres, the transport subsystem is used. Therefore, the power plants and large consumption centres are interconnected via AC power lines of high voltage (HV) and extra high voltage (EHV), *i.e.* 220 kV, 400 kV and 750 kV used in Romania, forming in this way, the electricity transmission network (ETN). ETN contains the following main elements: HV and EHV power lines; interconnection stations with primary voltage of HV and EHV. In our country, the ETN are coordinated and managed by the National Company for Power Transmission (CN Transelectrica SA), which plays the Transmission System Operator (TSO) role, having the following attributions: responsible for safely operating the NPS; administrator of electricity markets (day-ahead market, balancing market); responsible for safely operating of interconnections and therefore the international transactions; responsible for compliance with European standards imposed by ENTSO – E (European Network of Transmission System Operators for Electricity); responsible for developing and implementing Romania's energy policy infrastructure on medium and long term.

Currently, the paradigm regarding the steady or transitory state computation of power installations is recognized by the most specialists as a fundamental problem in the complex activities regarding the power networks analysis. Accordingly, the steady-state computation is widely used for planning, development, design, modernization, reconstruction and rational operation of power networks from technical and economical point of view.

2. Considerations for ETN Computation. Computing Hypotheses

Electromagnetic energy is a form of free energy, taking into accounts that their expression contains exclusive local and instantaneous quantities of the electromagnetic field. The electromagnetic energy theorem is important for the consequences regarding the existence and uniqueness deduction conditions of non-steady state field (Mocanu, 1981; Rosman & Savin, 1974; Rosman, 2008).

Electricity transmission means the propagation of electromagnetic field in the current flow existence. According with the information from the literature (Georgescu & Gavrilaş, 1992; Georgescu & Neagu, 2014; Rosman, 2011), the developed form of the magnetic circuit law emphasize the existence of conduction and radio currents, the latter being due to electric induction variations: conduction, convection and Roentgen currents. Conduction current is determined by the flow through the conductor of the load. These current can have two components, one induced by an applied electric field and the other induced by real electric field, due to some non-electrical physic-chemical conditions, such as material non-homogeneity, uneven temperature distribution etc. Radio components are determined mainly by electrical induction variation in time and space. The applications that use Roentgen and convection currents transmit only low power, with no prospect for electricity transmission over long and very long distances (Eremia *et al.*, 2006).

Until now, for electricity transmission only alternating and direct conduction currents were used. From this perspective, the electricity transmission at distance can be defined as the process of high power flows from hundreds even to thousands kilometres. The physical medium of power transfer is the conductors of overhead or cable lines. They play the role of the waveguides which "*defines the path of minimum effort*" where the waves of the electromagnetic field will propagate. Power lines for high power transit can be divided into the following categories (Georgescu & Gavrilaş, 1992): evacuation power lines from power plants; power lines which link the subsystems; power transmission lines and interconnection lines between large power systems. The transmission power lines must meet certain requirements, the most important being to ensure sufficient transmission capacity in relation with the power flow.

Regarding the mathematical model of the electromagnetic waves propagation along a transmission line, it can be expressed in terms of global (voltage and current) or local parameters of the electromagnetic field. Usually, for electricity transmission at low or medium frequency the electromagnetic wave propagation phenomenon is expressed in terms of global parameters, simplifying the computation. The transversal currents reach important values in the ETN with lengths over 250 km, and an accurate modeling of the phenomena that occur on the transmission line does not allow the concentration of its parameters. Consequently, the long lines were studied using electromagnetic wave propagation theory, using uniformly distributed parameters. The mathematical model is based on the following simplifying assumptions: the line is considered homogeneous regarding the conductor material and symmetrical regarding its geometry; the supply voltages are equal in phase and shifted with 120º, and the load distribution in three-phase system is symmetrical; the end, skin and proximity effect are neglected; the line is characterized by four electrical parameters namely: r_0 – resistance, l_0 – inductance, g_0 – conductance, c_0 – capacity; all of these electrical parameters are reported at 1 km of line.

The loads accumulation longwise of the circuit, characterized by electrical capacity makes that in time-varying states the current vary along the phase conductors. Due to imperfection of insulation line the current that flow between the phase conductors contributes at the current intensity variation along the transmission line. In fact, the transmission lines are constituted from parallel threadlike conductors with a very high length compared to the distance between conductors, being used, practically, either for remote of the electromagnetic energy transmission in power engineering, either for electromagnetic signals transmission in telecommunications (Rosman, 2011; Rosman, 2011).

3. Mathematical Model for Transmission Lines Steady-State Analysis

Electromagnetic phenomena in a section of transmission lines depends both on the space variable (*x*) which separates the section from line output (considered origin for this variable), and time variable (*t*). Thus, the instantaneous voltage and current values in a line section, at a time *t*, for a distance *x* from the end line are $u(x, t)$ and $i(x, t)$.

In order to establish a mathematical model which describes the ETN electromagnetic waves propagation, taking into account the above simplifying assumptions, a mono-phase fictive power line is considered. This line is composed of a phase conductor and a fictive neutral conductor, being electrically equivalent with a real AC three phase power line.

In this purpose, from the equivalent fictive line a small portion of infinitesimal length (Δx), situated at distance *x* to the line end and x' from the line begin is analyzed. By applying the electromagnetic induction law and the energy conservation theorem on the circuit portion with infinitesimal length Δx, the equations that describe the distribution and the time evolution of voltage and current along the power line are obtained. The differential equations obtained have first and second order partial derivatives and are known in the literature as the telegraph equations (Gomez-Aguilar & Baleanu, 2014; Ashyralyev & Modanli, 2015). These operation equations forms were established for the first time by Kelvin, since 1855. As regards the second-order differential equations system, they are not independent, being connected with the first-order differential equations, depending both of the initial conditions from the propagation phenomenon appearance and boundary conditions, generally imposed at the power line ends. The mathematical model with partial derivative equations, allow the study of both transients and harmonic stationary states for a transmission lines in order to obtain a transfer of electromagnetic energy at industrial frequency (Georgescu & Neagu, 2014; Rosman & Savin, 1974).

Solving the equations system for the general case of transient state, the obtained solutions highlight a multiple series of waves which are reflected and refracted, being then attenuated. They depend on the propagation medium of the power line formed by the conductors and dielectric. The voltage and current in any point of a transmission lines occur as a result of the *incident* and *reflected waves* that overlap in the considered point.

Also, the mathematical model can be interpreted in the sense of sin-phase waves in all line points, but with different amplitudes, depending on space variable *x*. In this way, voltage and current, from any point along the transmission lines are resultant of two stationary waves, namely (Rosman & Savin, 1974): a wave for goal state of the power line; a wave for short-circuit state of the power line.

In harmonic steady-state case, the integration of first-order differential equations is made using a complex transformation, taking into account the main property of this transformation, respectively:

$$\begin{aligned}\frac{\mathrm{d}\underline{U}(x)}{\mathrm{d}x} &= (r_0 + \mathrm{j}\,\omega l_0)\underline{I}(x) = \underline{z}_0\underline{I}(x),\\ \frac{\mathrm{d}\underline{I}(x)}{\mathrm{d}x} &= (g_0 + \mathrm{j}\,\omega c_0)\underline{U}(x) = \underline{y}_0\underline{U}(x),\end{aligned} \tag{1}$$

where: $\underline{U}(x)$ is the phase voltage of the transmission lines in the x section:

$$\underline{U}(x) = \frac{\mathrm{j}\sqrt{2}}{T}\int_0^T e^{-\mathrm{j}\omega t}u(x,t)\,\mathrm{d}t, \tag{2}$$

$\underline{I}(x)$ – current of the transmission lines in the x section:

$$\underline{I}(x) = \frac{\mathrm{j}\sqrt{2}}{T}\int_0^T e^{-\mathrm{j}\omega t}i(x,t)\,\mathrm{d}t, \tag{3}$$

T – the time of forced phenomenon, imposed at the input line terminals; $\underline{z}_o$, $\underline{y}_o$ – the specific impedance and admittance of power line; $\omega = 2\pi/T$ – the pulsation of the voltage and current electromagnetic waves on transmission lines.

Previous equations represent a complex form of the first-order differential equations that characterize the electromagnetic wave propagation along the electricity transmission lines. This equation system solution, taking into account that for the determination of complex integration constants are used the boundary conditions (imposed at one of the line terminals) and unknown functions (voltage $\underline{U}(x)$ and current $\underline{I}(x)$ complex wave), are the following:

a) knowing the power line output voltage $\underline{U}_2$ and current $\underline{I}_2$:

$$\begin{aligned}\underline{U}(x) &= \underline{U}_2\cosh\underline{\gamma}x + \underline{Z}_c\underline{I}_2\sinh\underline{\gamma}x,\\ \underline{I}(x) &= \underline{Y}_c\underline{U}_2\sinh\underline{\gamma}x + \underline{I}_2\cosh\underline{\gamma}x,\end{aligned} \tag{4}$$

b) knowing the power line input voltage $\underline{U}_1$ and current $\underline{I}_1$:

$$\begin{aligned}\underline{U}(x) &= \underline{U}_1\cosh\underline{\gamma}x - \underline{Z}_c\underline{I}_1\sinh\underline{\gamma}x,\\ \underline{I}(x) &= -\underline{Y}_c\underline{U}_1\sinh\underline{\gamma}x + \underline{I}_1\cosh\underline{\gamma}x,\end{aligned} \tag{5}$$

where: $\underline{Y}_c = 1/\underline{Z}_c$ is the characteristic admittance of long transmission line.

In this way, for a transmission line with uniformly distributed parameters the systems (4) and (5) represent the fundamental equations used to study the ETN steady-state.

The fundamental equations can be simplified by considering the load flows from power lines as active and reactive power reported to the natural power ($P_{\text{nat}} = U_n^2 / Z_c$) of studied lines. For a lossless line ($r_0 \cong 0, g_0 \cong 0$), the attenuation coefficient of voltage and current waves can be neglected ($\alpha \cong 0$), and the steady-state equations in relative values are the following:

$$\begin{aligned} \underline{u}(x) &= \cos\varphi + q_2 \sin\varphi + \mathrm{j} p_2 \sin\varphi, \\ \underline{i}(x) &= p_2 \cos\varphi + \mathrm{j}(\sin\varphi - q_2 \cos\varphi), \end{aligned} \tag{6}$$

where: φ is the angle in degrees, corresponding to the length x, considered at the line end, *i.e.* from source to consumer.

The operating equations (5) are the basic equations of transmission lines without losses, expressed in relative values and used in practice to analyze the specific electricity transmission lines steady-states (Neagu *et al.*, 2012; Eremia *et al.*, 2006; Georgescu & Neagu, 2014; Rosman & Savin, 1974).

4. Software for Harmonic Steady-State Analysis of the Power Transmission Line

ANRETRANS (Romanian abbreviation from ANaliza REtelelor de TRANSport) software was written using the aforementioned mathematical model being used for a complex and complete analysis of electricity transmission networks (ETN) with high or very high voltage (U_n =220 kV, U_n = = 400 kV), with hundreds of kilometers lengths. ANRETRANS was written in C Sharp (C#) language and can run on any Win 32 or 64 systems. The executable file is *anretrans.exe*, being in Romanian language.

According to the software general menu (Fig. 1), for power transmission line analysis the user can select one of the ANRETRANS possibilities, as follows:

1º Specific parameters computation.

2º Characteristic parameters computation.

3º Steady-state computation (only with active power transfer or with active and reactive power transfer).

4º Active/reactive power losses and efficiency of the long transmission line.

5º Series compensation with capacitors.

6º Shunt compensation with reactors;

7º Mixed compensation with shunt reactors and series capacitors.

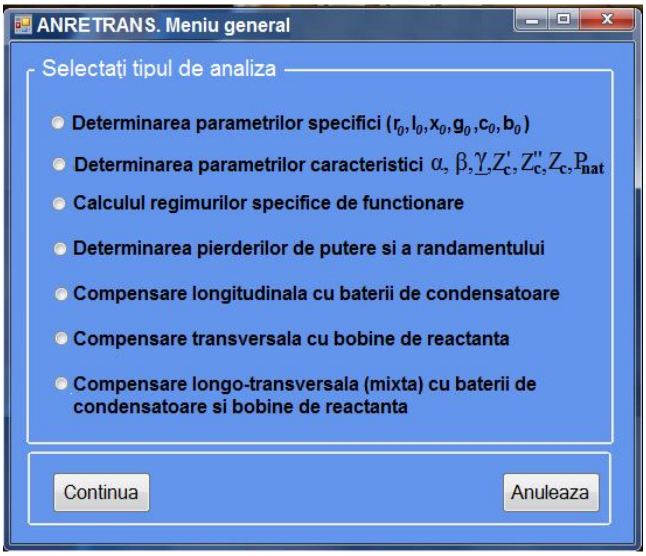


Fig. 1 – The general menu of ANRETRANS.

ANRETRANS has a graphical user interface (GUI), to allow the user to easily input and output data. The input data necessary for the software operation are introduced in modular form. Thus, after introduction of the input data in a window (Fig. 2), the software provides options to selectively modify the data.

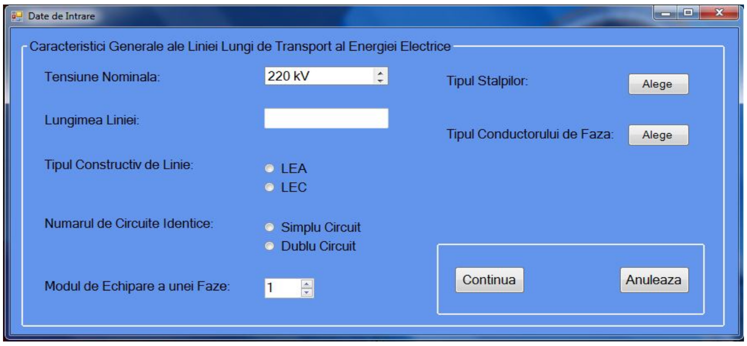


Fig. 2 – Window for input data introduction.

The ETN general characteristics depend on the chosen option. These are the following: nominal voltage; length; tower or post type; phase conductors type; ETN type (overhead or cable power lines); circuits' number (single or double circuit). Also, Fig. 3 represents an ETN constructive type selection window, taking into account the following characteristics: nominal voltage, ETN type (overhead or cable power lines), and posts type.

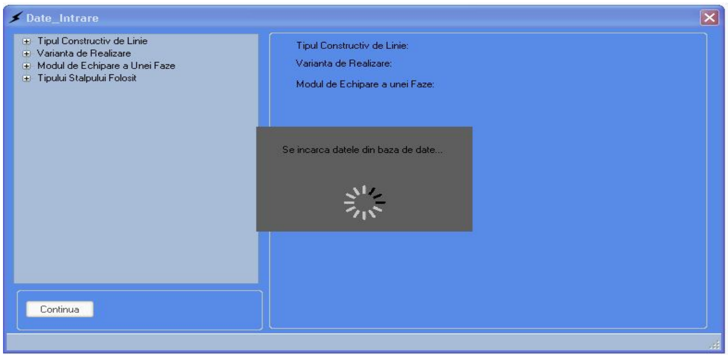


Fig. 3 – Window with input data and the database loading.

After a short time foe charging the data from databases, the user can select the options from navigation tree, the changes being reflected to the right side of the window, as shown in Fig. 4. It should be noted that in these windows any necessary changes can be made independent of previous randomly selections.

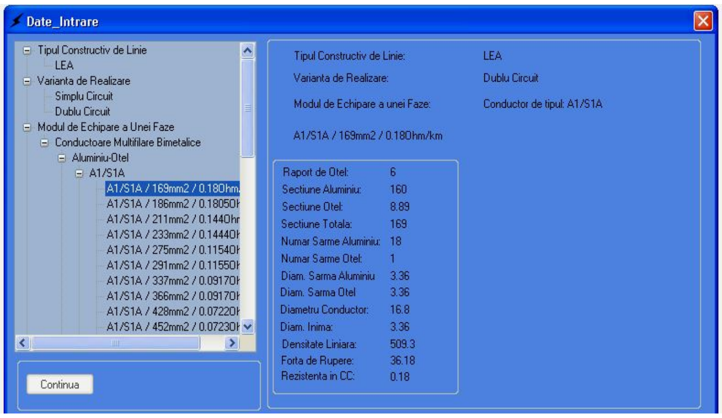


Fig. 4 – Details regarding the navigation tree and data selection mode.

For example, in Fig. 4 is presented the electrical, mechanical and material characteristics for OL-Al bimetallic multi-wire conductor (A1/S1A type), with total sections of 169 mm^2. Important to note is that both for the cable section and the metal posts, new values can be entered by editing directly in the database files or using option "+*Adauga (Add)*". Although the direct editing of data files is possible, is not recommended because a mistake can lead to syntax errors, therefore the "+ *Adauga*" is preferred. This option is suggestively illustrated in Fig. 5. If this option is selected, the user must enter the new conductor characteristics.

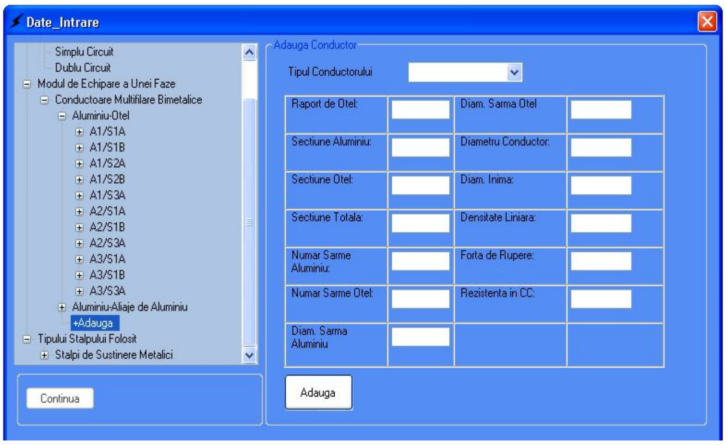


Fig. 5 – Window for adding a new conductor in the database.

Taking into account the aforementioned easy adding of a new active or protection conductor, analogously, a function for adding the new metallic posts (by introducing the new post parameters) was introduced, Fig. 6.

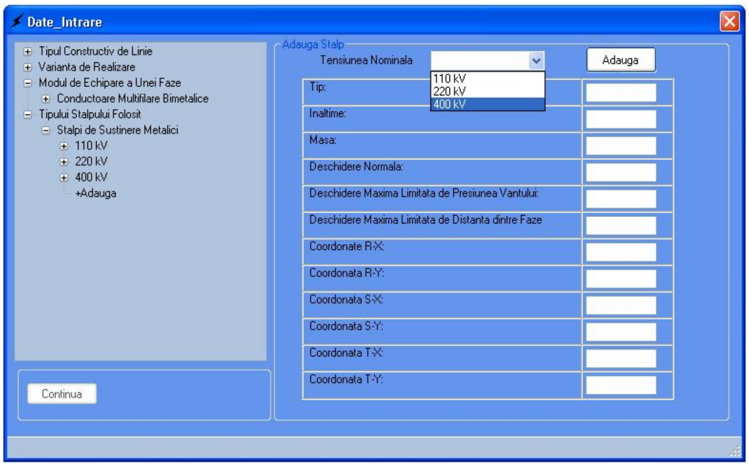


Fig. 6 – Window for adding a new post in the database.

It must be mentioned that our software allows the electrical parameters computation for each phase of the long line (when the conductors were untransposed), or for entire long line (when the conductors were transposed). Also, ANRETRANS allows the parameters of an ETN double circuit, with a phase shift θ between the two circuit voltages of transmission lines.

In the proposed software, for computation of the transmission lines characteristic parameters (complex propagation coefficient with attenuation coefficient – α and phase coefficient – β, the wavelength which characterize the electromagnetic wave propagation along the line, characteristic impedance, complex coefficients and natural power of the long line) the relations given in the literature were used (Rosman, 1981; Linqvist, 2011; Neagu & Georgescu, 2012). Selecting the specific parameters or characteristic parameters computation option, with general data of the introduced transmission lines, the software will generate a window like the one shown in Fig. 7.

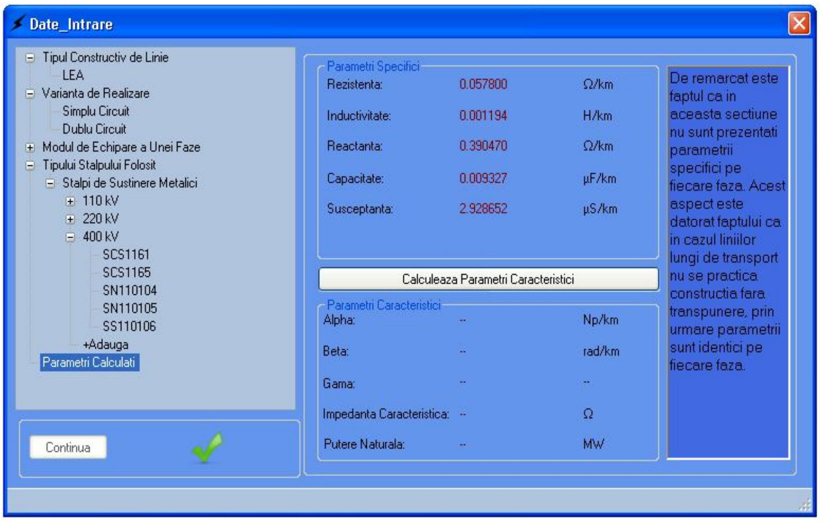


Fig. 7 – Specific parameters for a transmission line when the conductor transpose was made.

If the user has opted for steady-state computation options then a new window appears, Fig. 8. The new window allows the input data strictly necessary for ETN steady-state computation (*i.e.* active and reactive loads at the transmission line ends) in the following variants: active (MW) and reactive (MVAr) power, Fig. 8; active power (MW), and power factor (cos φ), Fig. 9.

The required data for ETN steady-state computation is introduced in the tabs form, each tab allows selection of needed parameters. Initially, the "*Caracteristici generale - General Characteristics*" tab is selected.

After data completion in the Figs. 8 and 9 windows, by accessing the "OK" button, automatically new windows will be open, and the user can choose the steady-state computation method: by using the operating equations in per unit of the transmission lines or by considering the long lines modeled through *T* or Π elementary quadripole chains. If the user chooses the last method, it must specify the quadripole type (Π or T) and the modeled line length. The steady-state computation results (voltage and current values) are displayed both in tabular and graphical form.

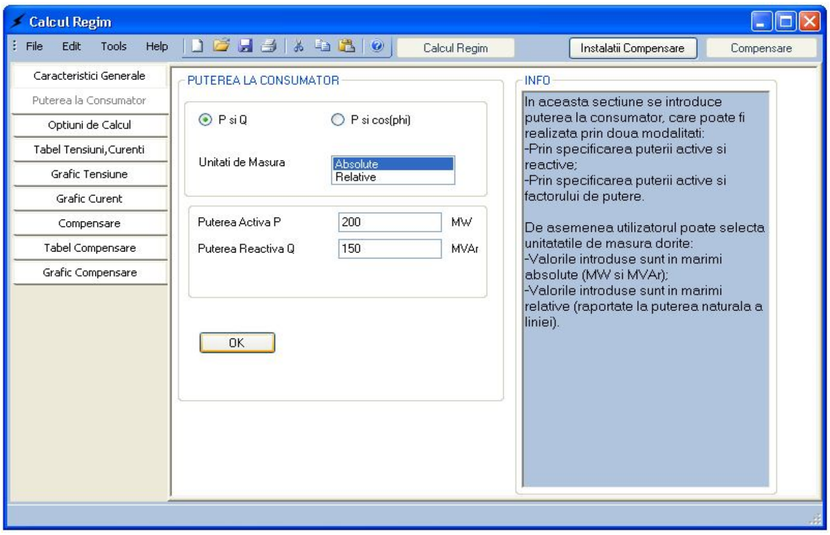


Fig. 8 – Introduction of active and reactive power for the long line steady-state computation.

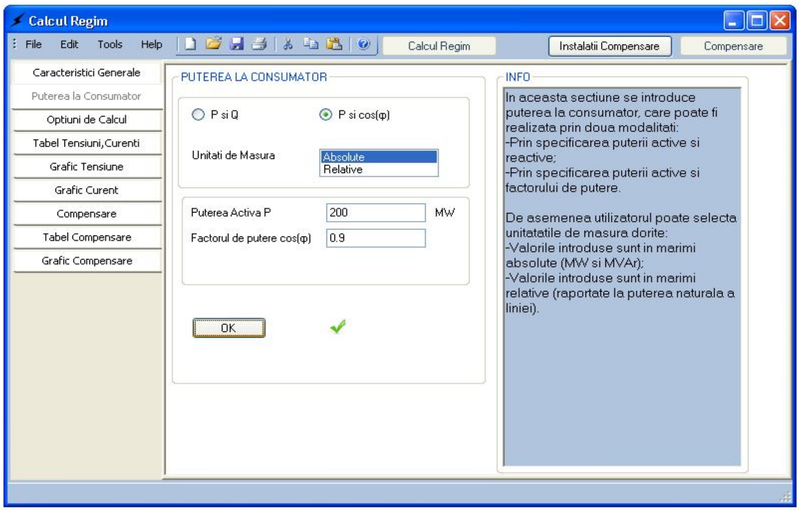


Fig. 9 – Introduction of active power and power factor for the long line steady-state computation.

By clicking the "*Instalații compensare (Compensation installation)*" button the software automatically generates a window with all ETN compensation possibilities, *i.e.* longitudinal, transversal and mixed compensation. The choice of compensation mode is performed in accordance with the "INFO" tab located on the right side of the window, as shown in Fig. 10.

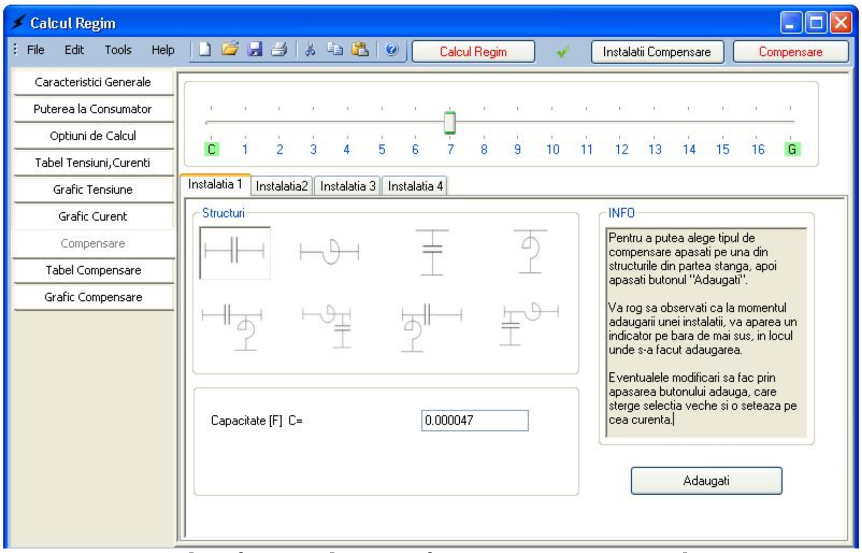
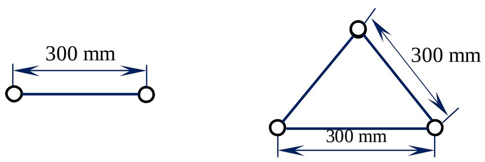


Fig. 10 – Window for introduction of compensation station characteristics for long lines.

If the user wants the installation of a new compensation substation, she/he must activate the "*Adăugați – Add* " button and continue analogously as in the first compensation station case (add the electrical parameters of compensation station). After entering all input data and using the „*Compensare – Compensation*" button, it will result a compensated ETN. Next, for determination of voltage and current variation through the transmission line in compensated case, the user accesses "*Calcul regim*" button in the same manner as for aforementioned uncompensated transmission lines.

5. Study Case

To highlight the possibilities of the presented mathematical model and software, an electricity transmission line was considered. The 400 kV analyzed line area is an overhead single circuit transmission line, for which the along the phase conductor transposistion is made. The posts are portal type and the three phases coordinates (in meters) are the following: *R*(–11, 18), *S*(0; 18) and *T*(11; 18). For 400 kV long lines analysis two variant were considered:



a *b*

Fig. 11 – Variants for equipping the phase conductors of the analyzed 400 kV long line.

a) Two normal steel – aluminum conductors for each power line phase (2×450 mm^2), as shown in Fig. 11 *a*.

b) Three normal steel – aluminum conductors for each power line phase (3×450 mm^2), as shown in Fig. 11 *b*.

By using the ANRETRANS software modules, in the following the results for the 400 kV transmission line are presented.

1. Specific parameters computation of the analysed long line in the two considered variations (Tables 1 and 2).

Table 1

Specific Parameters of the 400 kV Power Line in the Two Conductors Variant, Both for Phase Conductors Untransposed and Transposed Cases

Parameters	Phase		
	R	*S*	*T*
Untransposed phase conductors			
l_0, [mH/km]	1.123	1.047	1.123
x_0, [Ω/km]	0.353	0.329	0.353
c_0, [nF/km]	10.405	11.154	10.405
b_0, [μS/km]	3.269	3.504	3.269
Transposed phase conductors			
l_0, [mH/km]	1.094		
x_0, [Ω/km]	0.344		
c_0, [nF/km]	10.628		
b_0, [μS/km]	3.339		

Table 2

Specific Parameters of the 400 kV Power Line in the Three Conductors Variant, Both for Phase Conductors Untransposed and Transposed Cases

Parameters	Phase		
	R	*S*	*T*
Untransposed phase conductors			
l_0, [mH/km]	1.015	0.939	1.015
x_0, [Ω/km]	0.319	0.295	0.319
c_0, [nF/km]	11.493	12.427	11.493
b_0, [μS/km]	3.611	3.902	3.611
Transposed phase conductors			
l_0, [mH/km]	0.985		
x_0, [Ω/km]	0.309		
c_0, [nF/km]	11.767		
b_0, [μS/km]	3.697		

Analyzing the results shown in Tables 1 and 2 we can see that the transition from two to three conductors per phase leads to the inductance and reactance reduction and the increase of capacity and susceptance regarding the analysed power line, with approximately 10%.

2. Characteristic parameters (complex propagation coefficient, characteristic impedance and natural power) computation of the analysed long line in the two considered variations (Tables 3,...,5).

Table 3
Propagation Coefficient of the Electromagnetic Waves on 400 kV Overhead Line in the Two Analyzed Variant

Phase construction	α, [Np/km]	β, [rad/km]
2 × 450 mm^2	5.46×10^{-5}	1.074×10^{-3}
3 × 450 mm^2	3.97×10^{-5}	1.069×10^{-3}

Table 4
Characteristic Impedance of the 400 kV Overhead Line in the Two Analyzed Variant

Phase construction	$\Re e(\underline{Z}_c)$, [Ω]	$\Im m(\underline{Z}_c)$, [Ω]	$\lvert\underline{Z}_c\rvert$, [Ω]	Arg($\underline{Z}_c$), [rad]
2 × 450 mm^2	321.5	–16.3	321.9	–0.051
3 × 450 mm^2	289.4	–10.7	289.6	–0.037

Table 5
Natural Power of the 400 kV Overhead Line in the Two Analyzed Variant

Phase construction	$\Re e(\underline{S}_n)$, [MW]	$\Im m(\underline{S}_n)$, [MVAr]	$\lvert\underline{S}_n\rvert$, [MVA]	Arg($\underline{S}_n$), [rad]
2 × 450 mm^2	496.3	–25.3	497.0	–0.051
3 × 450 mm^2	552.1	–20.4	552.5	–0.037

3. The steady-state computation for 400 kV single circuit line with two active conductor on the phases, section of each phase 450 mm^2 and length 800 km, in two particular situations, respectively:

A. The lossless transmission line (without longitudinal and transversal losses), namely $r_0 = 0$ and $g_0 = 0$.

B. The transmission line with longitudinal losses ($r_0 \neq 0$), but without transversal losses ($g_0 = 0$).

For each of the two variants the following specific steady-states for the transmission line were analyzed:

a) Steady-state only with active power transfer:

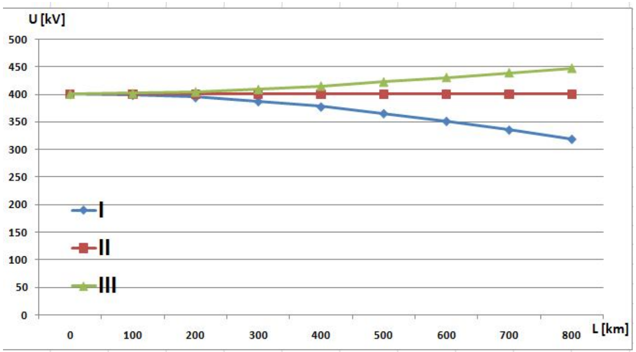
$$\text{I. } \begin{cases} p_2 = 0.6 \\ \cos\varphi_2 = 1 \end{cases} \qquad \text{II. } \begin{cases} p_2 = 1.0 \\ \cos\varphi_2 = 1 \end{cases} \qquad \text{III. } \begin{cases} p_2 = 1.2 \\ \cos\varphi_2 = 1 \end{cases}$$

b) Steady-state with active and reactive power transfer: $p_2 = 0.6$; $q_2 = 0.26$; $\cos\varphi_2 = 0.9$.

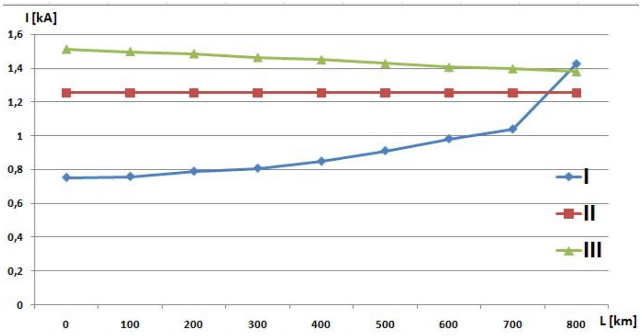
The results regarding the voltage and current variation along the analysed long line for each proposed state are presented below.

A. The lossless transmission line case.

a) For steady-state only with active power transfer, voltage and current variation along the transmission line are presented in Fig. 12.



a



b

Fig. 12 – Voltage (*a*) and current (*b*) variation along the transmission line, for steady state only with active power transfer.

b) For steady-state with active and reactive power transfer, the voltage variation along the transmission line is presented in Fig. 13.

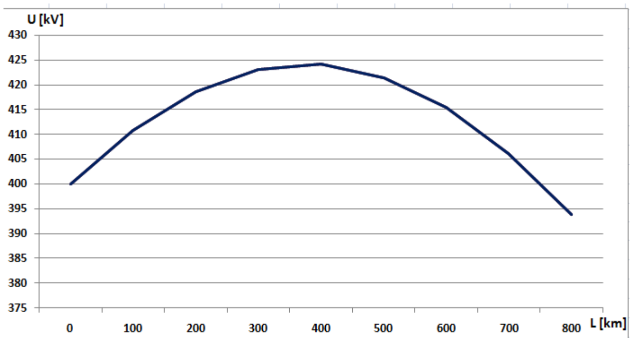


Fig. 13 – Voltage variation along the transmission line, for steady state with active and reactive power transfer.

B. The case of transmission line with longitudinal losses, but without transversal losses.

a) For steady-state only with active power transfer, the voltage variation along the transmission line is presented in Fig. 14.

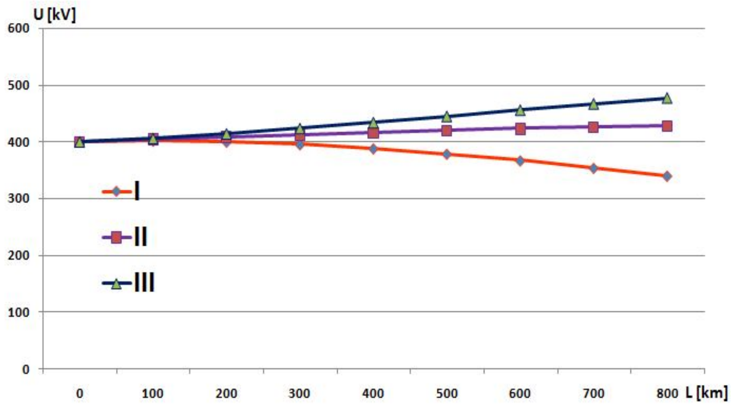


Fig. 14 – Voltage variation along the transmission line, for steady state only with active power transfer.

b) For steady-state with active and reactive power transfer, the voltage variation along the transmission line is presented in Fig. 15.

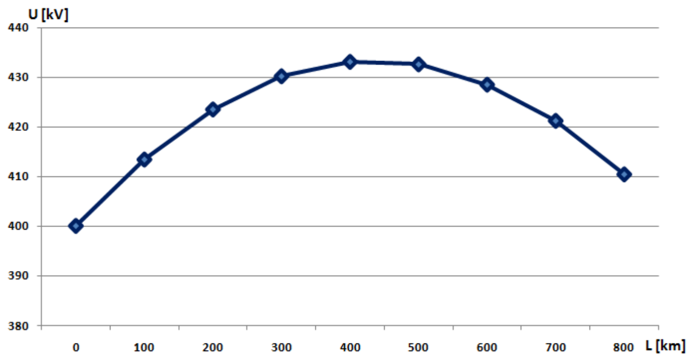


Fig. 15 – Voltage variation along the transmission line, for steady state with active and reactive power transfer.

4. The active power losses and yield computation for the 400 kV overhead line by using both exact (the difference between the powers at the line ends) and approximate method (mean square current) for the following steady-state only with active power transfer: $p_2 = 0$, $p_2 = 0.6$, $p_2 = 1.0$ and $p_2 = 1.2$. The results regarding the active power losses and yield of the analyzed transmission line are presented in Tables 6 and 7.

Table 6
The Active Power Losses and Yield for the Transmission Line, Using the Exact Method

$p_2 = P_2/P_{nat}$	$\cos\varphi_2$	P_1, [MW]	P_2, [MW]	ΔP, [MW]	η, [%]
0	1	9.194	0	9.194	0
0.6	1	319.635	297.936	21.698	93.21
1.0	1	540.209	496.561	43.648	92.96
1.2	1	654.580	595.873	58.707	91.03

Table 7
The Active Power Losses and the Yield for the Transmission Line, Using the Mean Square Current Method

$p_2 = P_2/P_{nat}$	$\cos\varphi_2$	P_1, [MW]	P_2, [MW]	ΔP, [MW]	η, [%]
0	1	9.194	0	9.194	0
0.6	1	320.513	297.936	22.576	92.96
1.0	1	541.673	496.561	45.112	91.67
1.2	1	656.336	595.873	60.463	90.79

From the results analysis, it is found that, for 400 kV ETN with 800 km length, the active power losses and yield values using the exact method (Table 6) and mean square current method (Table 7), are very close.

5. To highlight the effects of longitudinal and transversal compensation of the studied overhead transmission lines, different variants were analyzed. Thus, the following variants of longitudinal compensation (with capacitor) and transverse (with reactors) were analyzed:

A. A series compensation station with a capacity of C_n = 46.26 μF, which practically compensates a quarter of the inductive reactance of analyzed power line ($X_C = X_L/4$), located at 600 km from the line end.

B. Two series compensation stations, each of them with a nominal capacity of capacitors of C_n = 46.26 μF, symmetrically located at 200 km from the analyzed line ends.

C. Two shunt compensation stations, with capacitive reactance of reactors X_b =1,404.6 Ω, symmetrically located at 200 km from the analyzed line ends.

The results from ANRETRANS software, regarding the voltage variation for the three longitudinal and transverse compensation variants (A, B and C) are shown in Figs. 16,…,18. From the results provided by ANRETRANS software regarding the different steady state specific for electricity transmission lines longitudinal and transverse compensated, the mathematical model theoretical considerations are confirmed, namely: the transport capacity modification of high and very high voltage transmission lines; the improvement of voltage along the electricity transmission lines. However, should be noted that the voltage improvement in the long lines, is significantly affected by judicious choice of compensation stations parameters and location along the transmission lines.

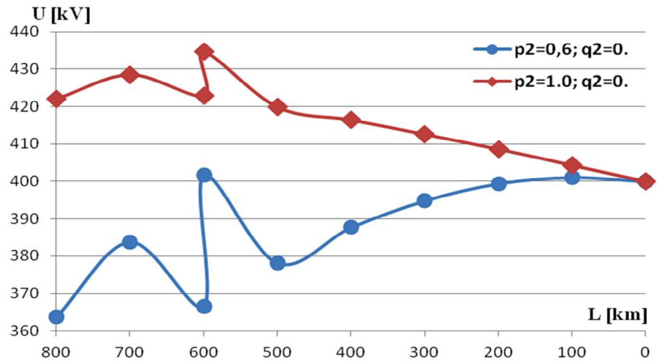


Fig. 16 – The voltage variation for a compensated line with a single series capacitor (C_n = 46.26 μF), the line is transferring only active power: p_2 = 0.6, q_2 = 0; p_2 = 1, q_2 = 0.

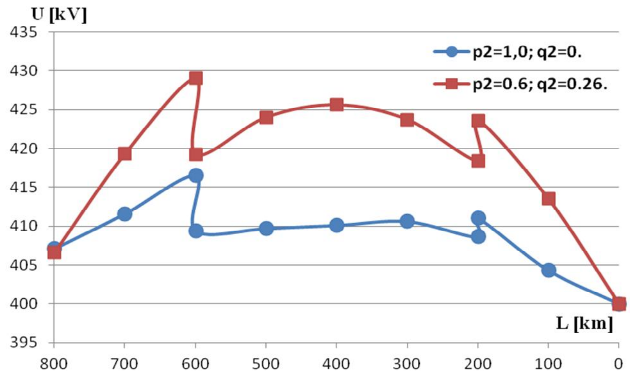


Fig. 17 – The voltage variation for a compensated line with two series capacitor, located symmetrically at line ends, for the following steady state: p_2 = 0.6, q_2 = 0; p_2 = 1, q_2 = 0.

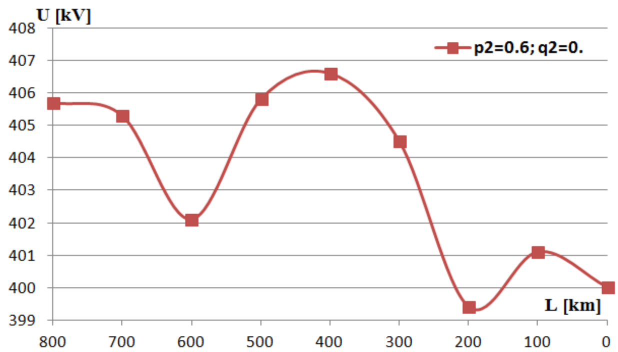


Fig. 18 –Voltage variation for a compensated line with two reactors, located symmetrically at the line ends, for one steady state: p_2 = 0.6, q_2 = 0, cosφ_2 = 1.

6. Conclusions

The steady-state computation is widely used for planning, development, design, modernization, reconstruction and rational operation of electricity transmission networks from technical and economical point of view. The mathematical model and software presented in the paper allow a more complete and comprehensive analysis of the symmetrical steady-state of the electricity transmission networks. The results provided by the ANRETRANS software refers to the specific parameters computation, characteristic parameters (complex propagation coefficient, characteristic impedance, natural power) computation, the voltage and current electromagnetic wave propagation along the transmission lines, the power losses and yield evaluation, longitudinal and transversal compensation and optimal location of the compensation stations along the power transmission lines.

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MODEL MATEMATIC ŞI PROGRAM DE CALCUL PENTRU ANALIZA REGIMURILOR ARMONICE STAȚIONARE DE FUNCȚIONARE ALE LINIILOR DE TRANSPORT A ENERGIEI ELECTRICE

(Rezumat)

Pentru transferul energiei electrice produse în centralele electrice, pe distanțe mari şi foarte mari, până în apropierea centrelor importante de consum, este utilizat sistemul de transport, respectiv rețelele electrice de transport (RET) din sistemul electroenergetic (SEE). În esență, transmiterea energiei electrice înseamnă, de fapt, propagarea câmpului electromagnetic care, la rândul său, este legată de existența, sub o formă sau alta, în principal a curentului electric de conducție, precum şi a unor curenți electrici hertzieni (curent de deplasare, curent de convecție şi curent Roentgen). Până în prezent transmiterea energiei electrice, aşa cum este înțeles acest termen în electroenergetică, a folosit numai curenții de conducție alternativi şi continui. Acești curenți de conducție sunt determinați de deplasarea prin conductor a purtătorilor de sarcină. Suportul fizic al acestui transfer de energie, folosind tehnica curenților de conducție, îl reprezintă conductoarele liniilor electrice aeriene sau în cablu. Pe baza informațiilor din literatura de specialitate în lucrare este descris un model matematic şi un program de calcul, destinat unei analize complexe a regimurilor permanente simetrice de funcționare a RET.