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DETERMINING THE MATHEMATICAL EXPRESSION OF THE STATOR CURRENTS BASED ON MEASURED VALUES

BY

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Abstract. The paper aims to determine the expressions of an induction machine stator currents from their measured (recorded) values. No matter the variation of the current, it is considered the same angular frequency as the sinusoidal supply voltage. In these circumstances, the currents are uniquely determined by the amplitude and phase.

Based on the measured values the amplitude $I_{sm}(t)$ of the current components can be determined. By calculating intermediate quantities $P^*(t)$ and $Q^*(t)$ it is possible to determine the shift phase $\varphi(t)$. The method can be applied to three or two-phase induction machines, no matter the operating conditions. $I_{sm}(t)$ and $\varphi(t)$ are quantities useful in further analysis.

The method was validated by using data acquired from simulation. The PSpice *d-q* model has been used.

Key words: induction machine; stator current; active power; reactive power; shift phase.

1. Introduction

Modeling of induction machine (three-phase or two-phase) is based on several hypotheses which allow the electrical and magnetic behaviors to be

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considered as linear – under certain conditions, and simplify the mathematical study (Ong, 1997; Lyshevsky, 1999).

To analyze the phenomena of induction machine as a system, its block diagram is shown in Fig. 1 (Cociu *et al.*, 2012).

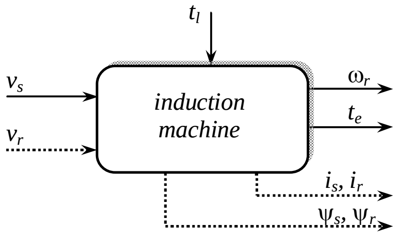

Fig. 1 - Induction machine as a block diagram.

The basic input and output quantities can be three-phase, two-phase or single-phase quantities. Input quantities are stator supply voltages v_s, rotor supply voltages, v_r and load torque t_l. Output quantities are electromagnetic torque t_e and rotor angular speed ω_r. Internal unknown quantities are stator and rotor currents i_s, i_r, stator and rotor fluxes ψ_s, ψ_r.

To allow for a more compact form of the equations, the following will use spatial phasors to represent the machine quantities of interest. In a *d*-*q* stationary reference frame, stator and rotor voltage equations and flux equations are the following:

$$\begin{cases} \underline{v}_s = R_s \underline{i}_s + \dfrac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t}; \\ \underline{v}_r' = R_r' \underline{i}_r' + \dfrac{\mathrm{d}\underline{\psi}_r'}{\mathrm{d}t} - \mathrm{j}\,\omega_r \underline{\psi}_r'; \end{cases} \tag{1}$$

$$\begin{cases} \underline{\psi}_s = L_s \underline{i}_s + L_m \underline{i}_r'; & L_s = L_{\sigma s} + L_m; \\ \underline{\psi}_r' = L_s \underline{i}_r' + L_m \underline{i}_s; & L_r' = L_{\sigma r}' + L_m. \end{cases} \tag{2}$$

All the rotor quantities are transformed to have the same frequency as the stator quantities by using well-known relations.

In steady state the angular frequency is constant (ω_r = const.), so the equation system is linear. Given the stator voltage and possibly the rotor voltage, the currents and fluxes result as solutions of a linear system of equations.

2. Theoretical Expression of Stator Currents

Considering the supply voltage as sinusoidal the corresponding spatial phasor results as:

$$\underline{v}_s = \sqrt{2}V_s \mathrm{e}^{\,\mathrm{j}(\omega_1 t+\gamma_v)} . \tag{3}$$

Solving the system of equations in the case of squirrel cage rotor ($\underline{v}_r = 0$) gives for stator currents:

$$\underline{i}_s = \underline{i}_{sn} + \underline{i}_{sf} , \tag{4}$$

where: $\underline{i}_{sn}$ is the zero-input component and $\underline{i}_{sf}$ the forced component. The first one, if induction machine, has generally two aperiodic sub-components decreasing exponentially in time:

$$\underline{i}_{sn} = \sum_k \underline{I}_{snk} \mathrm{e}^{-t/\tau_k} . \tag{5}$$

The forced component has the same waveform as the input function, that is sinusoidal.

$$\underline{i}_{sf} = I_{sf} \mathrm{e}^{\,\mathrm{j}(\omega_1 t+\gamma_i)} . \tag{6}$$

The spatial phasor is a rotating vector with ω_1 angular frequency; its projections on the *d*-*q* orthogonal axes are sinusoidal with the same angular frequency as the supply voltage. In dynamic conditions $\omega_r = \omega_r(t)$ and the resulting currents and fluxes (the system solution) have more complicated forms.

As shown by Cociu & Cociu (2011), the forced response component has the same waveform with the excitation and the natural components are characterized by time constants as:

$$\tau = \tau[\omega(t)] = \tau(t) . \tag{7}$$

The main practical result of above presented considerations is the fact that supplying an induction machine with sinusoidal voltage of ω_1 angular frequency, the resulting stator currents have the forced component also sinusoidal of the same angular frequency ω_1.

In steady state, when the rotor angular frequency ω_r is constant the spatial phasor of stator current rotates synchronously with the supply voltage phasor $\underline{v}_s$; it is of constant magnitude and lags $\underline{v}_s$ by an angle $\varphi = \gamma_v - \gamma_i$. Thus:

$$\begin{gathered} \underline{i}_{sf} = I_{sf} \mathrm{e}^{\,\mathrm{j}(\omega_1 t+\gamma_i)}; \\ I_{sf} = \mathrm{const.};\ \omega_1 = \mathrm{const.};\ \gamma_i = \gamma_v - \varphi. \end{gathered} \tag{8}$$

The components of the stator current on the orthogonal axes *d*-*q* are:

$$\begin{gathered} i_{sfd} = I_{sf} \cos(\omega_1 t + \gamma_i), \\ i_{sfq} = I_{sf} \sin(\omega_1 t + \gamma_i). \end{gathered} \tag{9}$$

Under dynamic conditions the magnitude of the stator current spatial phasor changes over time, being mainly affected by the rotor angular speed:

$$I_{sf} = I_{sf}[\omega_r(t)] = I_{sf}(t)\,. \tag{10}$$

Regarding the angular frequency and phase of the *d-q* components, both quantities are present in the expression of the sin (or cosine) angle. Both change over time but it is well known that changing the angular speed with time involves the phase change and vice-versa.

Therefore, to keep unity in expressing stator currents for both steady state and transient conditions and since there are no physical arguments to consider the frequency to change over time, contrary to (Doglas *et al.*, 2004), we propose to maintain the frequency at a constant value and to consider the angle modification in the variation of phase.

$$\omega_1 = \text{const.};\ \gamma_i = \gamma_i(t) = \gamma_v - \varphi(t)\,. \tag{11}$$

Thus the *d* and *q* stator currents can be expressed as:

$$\begin{aligned} i_{sfd} &= I_{sf}(t)\cos[\omega_1 t + \gamma_i(t)], \\ i_{sfq} &= I_{sf}(t)\sin[\omega_1 t + \gamma_i(t)]. \end{aligned} \tag{12}$$

3. Determining the Mathematical Expression of the Stator Currents Based on Measured Values

Consider a two-phase model of an induction machine supplied by orthogonal sinusoidal voltages. In steady-state, all the machine quantities of interest (voltages, currents, magnetic fluxes) are also sinusoidal and orthogonal. If the voltage corresponding to the *d*-axis is considered to be as phase origin, then the expressions of the *d-q* voltages and currents become:

$$\begin{cases} v_{sd} = \sqrt{2}V_s\cos(\omega t); & i_{sd} = \sqrt{2}I_s\cos(\omega t - \varphi); \\ v_{sq} = \sqrt{2}V_s\sin(\omega t); & i_{sq} = \sqrt{2}I_s\sin(\omega t - \varphi). \end{cases} \tag{13}$$

By evaluating the expression:

$$\begin{gathered} v_{sd}i_{sd} + v_{sq}i_{sq} = \\ = 2V_x I_x\left[\sin\omega t\sin(\omega t - \varphi) + \sin(\omega t - \pi/2)\sin(\omega t - \pi/2 - \varphi)\right] = \\ = 2V_s I_s\cos\varphi \end{gathered} \tag{14}$$

we obtain the active power as:

$$P = 2V_s I_s\cos\varphi = v_{xd}i_{xd} + v_{xq}i_{xq}\,. \tag{15}$$

Similarly, the reactive power, at a given moment, can be determined as follows:

$$\begin{gathered} v_{sq}i_{sd} - v_{sd}i_{sq} = \\ = 2V_x I_x\left[\sin(\omega t - \pi/2)\sin(\omega t - \varphi) - \sin\omega t \sin(\omega t - \pi/2 - \varphi)\right] = \\ = 2V_s I_s \sin\varphi \end{gathered} \quad (16)$$

$$Q = 2V_s I_s \sin\varphi = v_{sq}i_{sd} - v_{sd}i_{sq}. \quad (17)$$

By using eqs. (3) and (5) the active and reactive power at a given moment can be calculated and can be used in the computer aided study of the electrical machines, particularly the induction machine (Akagi & Kanawasa, 1984; Kim & Akagi, 1997). We notice that these expressions are valid only in steady-state operation.

The amount of shift phase between voltage and current can be obtained at any moment of time based on the active and reactive power values at a given moment:

$$\mathrm{tg}(\varphi) = \sin\varphi / \cos\varphi = P/Q. \quad (18)$$

In transient, the matter is more complex. In Cociu & Cociu (2011), we considered an induction machine supplied by orthogonal voltages and the rotor speed increasing at constant acceleration. Speed changing causes a transient state having forced components due to the supply voltages and natural components that usually decay exponentially. Since the speed change is of ramp type (constant acceleration) the magnitude of natural components is proportional to the rate of speed change. If the acceleration is reduced, the natural components are negligible and the operating mode is close to the steady-state, *i.e.* quasi-stationary. Increasing acceleration the natural components become important and can no longer be neglected.

Correspondingly to eqs. (15) and (17) we define the following calculating quantities (Cociu & Cociu, 2012):

$$P^*(t) = v_{sd}i_{xd} + v_{sq}i_{xq}, \quad (19)$$

$$Q^*(t) = v_{sq}i_{xd} - v_{sd}i_{xq}. \quad (20)$$

In quasi-stationary state, up to a certain speed slope limit, we expect the values obtained using eqs. (7) and (8) to be similar to the values of the active and reactive powers defined in steady state at corresponding speed.

$$P^* \approx P, \quad Q^* \approx Q. \quad (21)$$

It is expected that the amount of shift phase between voltage and current in quasi-stationary state to be determined in the same way as in steady state:

$$\mathrm{tg}[\varphi(t)] = \sin\varphi(t) / \cos\varphi(t) \approx Q^*(t) / P^*(t). \quad (22)$$

Hence, the maximum value or r.m.s. value of the stator current can be obtained if the instantaneous value at a given moment is known:

$$I_{sm}(t)=\sqrt{i_{sd}^2(t)+i_{sq}^2(t)}\,. \qquad (23)$$

From eq. (12) it follows that to determine the stator currents is necessary to know the expression of currents amplitude at any time and also $P^*(t)$ and $Q^*(t)$. Therefore the stator currents can be expressed as:

$$\begin{aligned} i_{sd}^* &= \sqrt{2}I_{sm}(t)\cos[\omega t-\varphi(t)], \\ i_{sq}^* &= \sqrt{2}I_{sm}(t)\sin[\omega t-\varphi(t)]. \end{aligned} \qquad (24)$$

Notice that these expressions can be obtained from any data file, no matter how it has been acquired: by simulating or from practical experiment.

5. Method Validation

We validated the method previously described by using data acquired from simulation but any other method to get required data could be used. To simulate the machine behavior in the above three cases, a three-phase induction machine is considered, rated at:

$$\begin{aligned} &P_n=5\ \text{kW};\ U_{1n}=400\ \text{V};\ f_1=50\,\text{Hz};\ Y-\text{conn.} \\ &R_s=1.1\ \Omega;\ R_r=1.1\ \Omega; \\ &L_{\sigma s}=10\ \text{mH};\ L_{\sigma r}=10\ \text{mH};\ L_m=130\ \text{mH}; \\ &F_\alpha=10^{-3}\text{Nms/rad};\ J=20\text{gm}^2. \end{aligned}$$

The PSpice *d-q* model (Justus, 1997; Cociu & Cociu 1997, 2012) has been used. The currents i_{sd}, i_{sq} have been evaluated ("measured") from simulation. Then we calculated from eq. (23) the amplitude $I_{sm}(t)$ by using the values thus obtained; we used eqs. (19) and (20) to calculate $P^*(t)$ and $Q^*(t)$ respectively and the phase shift $\varphi(t)$ from eq. (22).

Once $I_{sm}(t)$ and $\varphi(t)$ were determined, the currents i^*_{sd} and i^*_{sq} can be written as in eq. (24) and then graphically represented. Finally the currents thus obtained have been compared to the measured currents i_{sd} and i_{sq}.

For the beginning assume the machine operates at no-load, in steady state. The results are shown in Figs. 2 and 3. As expected, under steady state conditions the stator currents are characterized by constant parameters, so it is simple to get their mathematical expression, according to eqs. (18) and (23).

The currents resulted in quasi-steady state operation are shown in Fig.4 and 5. It has been considered the rotor speed to vary slightly. Within 1s, the rotor is accelerated from 0 to synchronous speed. Consequently I_{sm}, P^*, Q^* and φ vary with rotor speed. As observed even in this case there are no differences between the currents "measured" and those calculated from eq. (24).

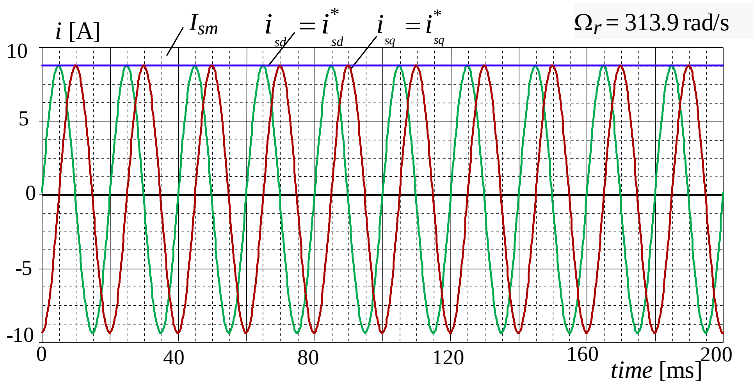

Fig. 2 – Stator currents in steady state. Amplitude.

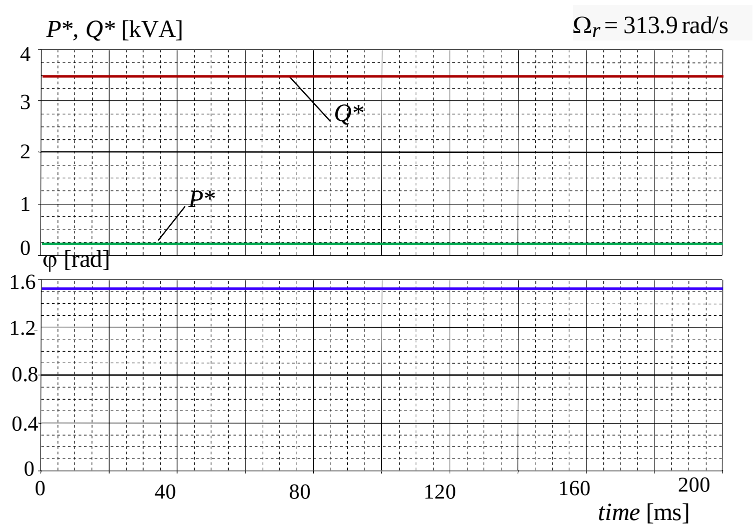

Fig. 3 – P^*, Q^* and shift phase in steady state.

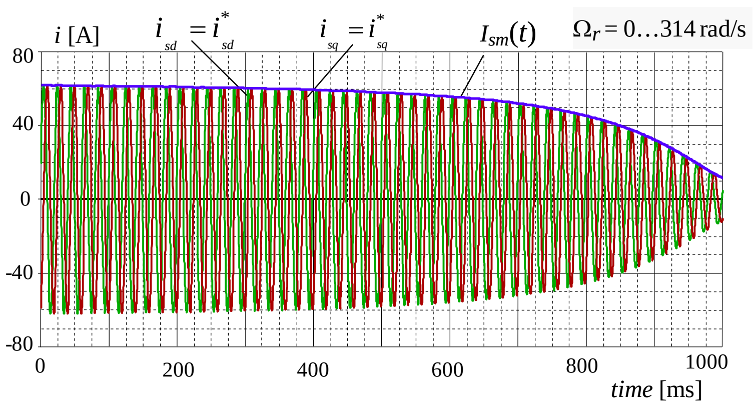

Fig. 4 – Stator currents in quasi-steady state. Amplitude variation.

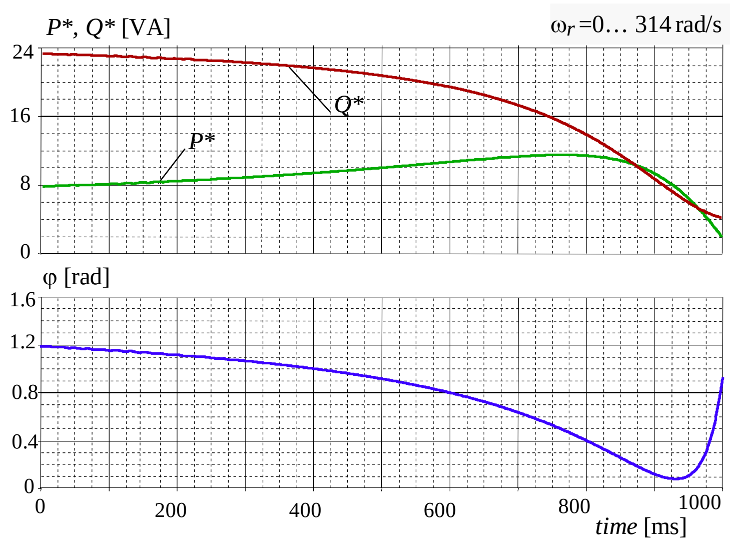

Fig. 5 – P^*, Q^* and shift phase in quasi-steady state.

The results under dynamic conditions are given in Fig. 6 and 7. The motor is supplied by sinusoidal voltages and starts on no load, thus resulting the most complex transient operation. Variation with time of the quantities calculated based on currents "measured" is presented. As observed there are no differences between the "measured" currents and those calculated with eq. (24).

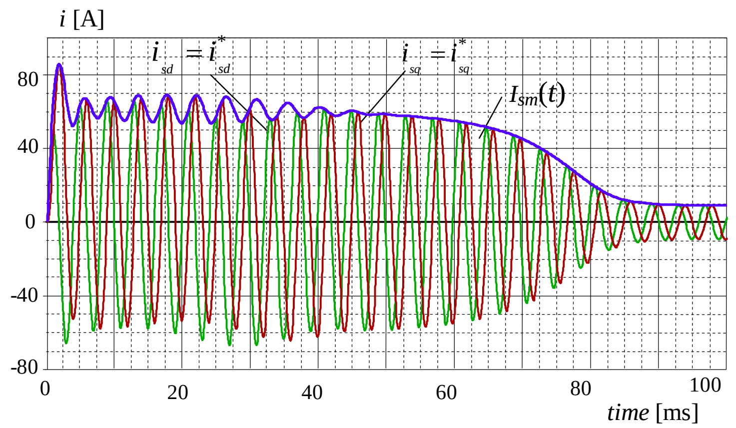

Fig. 6 – Stator currents. Amplitude variation under dynamic conditions.

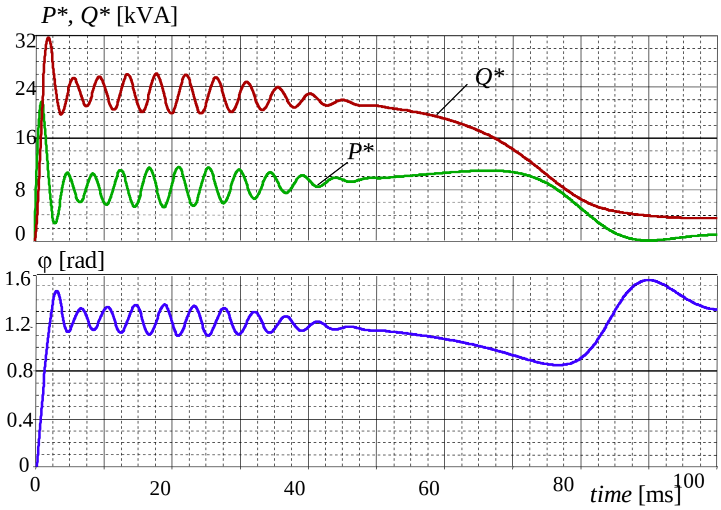

Fig. 7 – *P**, *Q** and shift phase under dynamic conditions (starting).

7. Conclusions

The paper aims to determine the expression of an induction machine stator currents from their measured (recorded) values. To keep unity in expressing for both steady state and transient conditions and since there are no physical arguments to consider the frequency to change over time, we propose to maintain the frequency at a constant value and to consider the angle modification in the variation of phase. In these circumstances, the currents are uniquely determined by the amplitude and phase.

Based on the measured values of the current components, the amplitude $I_{sm}(t)$ can be determined. By calculating intermediate quantities $P^*(t)$ and $Q^*(t)$ it is possible to determine the shift phase $\varphi(t)$. So, the current expression is uniquely determined.

The method can be applied to three or two-phase induction machines, no matter the operating conditions. $I_{sm}(t)$ and $\varphi(t)$ are quantities very useful in further analysis.

The method was validated by using data acquired from simulation but any other method to get required data could be used. The PSpice *d-q* model has been used.

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DETERMINAREA EXPRESIEI MATEMATICE A CURENȚILOR STATORICI PE BAZA VALORILOR MĂSURATE

(Rezumat)

Lucrarea îşi propune determinarea expresiei curenților statorici ai unei maşini asincrone pornind de la valorile lor măsurate (înregistrate). Indiferent de modul de variație al curenților se consideră cunoscută pulsația, aceeaşi cu cea a tensiunii sinusoidale de alimentare. În aceste condiții, curenții sunt unic determinați de către amplitudine şi faza inițială.

Pe baza valorilor măsurate se poate determina amplitudinea $I_{sm}(t)$ a componentelor trifazate sau bifazate ale curentului statoric. Prin calcularea unor mărimi intermediare $P^*(t)$ şi $Q^*(t)$ se determină în final faza ințială $\varphi(t)$. Metoda se poate aplica maşinilor trifazate sau bifazate indiferent de regimul de funcționare şi permite determinarea $I_{sm}(t)$ şi $\varphi(t)$, mărimi utile în analize ulterioare.

Verificarea metodei a fost realizată prin utilizarea valorilor componentelor bifazate ale curentului statoric obținute prin simularea în PSpice pe baza modelului ortogonal.