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INFLUENCE OF TRELLIS TERMINATION METHODS ON TURBO CODES PERFORMANCES OVER MIDDLETON CLASS-A CHANNELS

BY

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Abstract. Turbo codes occupy a special place in the field of wireless communications for their outstanding performances in correcting errors that appear on transmission channels. This paper analyzes the influence of trellis termination methods on turbo codes performances, over a channel affected by impulsive noise and Binary Phase-Shift Keying modulation. The statistical model used for the impulsive noise is Middleton Class-A. The evaluation of turbo code performances was done in terms of bit error rate (BER) and frame error rate (FER), for two lengths of S-random interleaver. The simulations were performed for different values of the parameters that describe the impulsive noise model. The used decoding algorithm is Max-Log-MAP.

Key words: impulsive noise; Middleton Class-A; trellis termination; turbo codes.

1. Introduction

Turbo codes (Berrou *et al.*, 1993) are a class of error-correcting codes used nowadays especially in wireless communication systems, because of their performances in applications as: 3GPP – Group Radio Access Network (3gpp.org), deep space (Divsalar *et al.*, 1995) or wireless metropolitan network

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standard IEEE 802.16 (WiMAX) (Kovaci *et al.*, 2012). Turbo codes consist of two recursive systematic convolutional codes parallel concatenated, with an interleaver placed between them, such that the input of the second convolutional encoder is an interleaved version of the input sequence.

The dimension and type of interleaver influences the turbo code performances and for optimal decoding the two trellises must begin from and end in the same state (usually, the null state) for every input data block. To achieve this, a trellis termination operation is required. There are many methods for trellis termination (Hokfelt *et al.*, 2001a): termination of the first encoder, post-interleaver flushing, dual termination, tail-biting. In the original version of turbo codes proposed by Berrou in (Berrou *et al.*, 1993), the trellis of the first encoder was terminated in null state, while the second one was truncated into an unknown state.

Many times, the main strategies for trellis terminating are compared to the situation when no encoders are terminated (Hokfelt *et al.*, 2001b). For a uniform interleaver with two different lengths: 100 and 500, the constraint lengths 3 and 4, and Bahl-Cocke-Jelinek-Raviv (BCJR) decoding algorithm, the performance differences between different methods are small, excepting the case of no trellis termination. The most efficient methods are post-interleaver flushing and dual termination.

These investigations were also made for one-binary turbo code with coding rates $1/3$ and memory 3, defined in 3GPP standard for two lengths of interleaver, 752 and 48, respectively (Kovaci *et al.*, 2012) and for multi-non-binary turbo codes (Balta *et al.*, 2014), with Max-Log-MAP (Kock *et al.*, 1990) decoding algorithm. Generally, the conclusion is the same we mentioned before: no termination of either component encoder causes severe performance degradation, while the differences are small between the other investigated termination strategies and the smaller the interleaver length is, the more consistent is the effect of terminating methods on the Bit Error Rate (BER) performance.

Although there already exists a rich literature on turbo codes and trellis termination methods, current results are mainly restricted to Additive White Gaussian Noise (AWGN) channel, ignoring other sources of noise, like industrial noise, man-made activities such as automobile spark plugs, microwave ovens (Middleton, 1977) and network interference (Kanemoto *et al.*, 1998), noises known to be non-Gaussian (or impulse noise).

The Middleton Class-A model is frequently used to describe the impulsive noise. This was used to investigate the performances of turbo codes over channels affected by impulsive noise versus AWGN, when the encoder has two identical recursive systematic convolutional encoders with constraint length 5, rate $1/2$, generator matrix $G = [1, 23/25]$ (in octal form) and Binary Phase Shift Keying (BPSK) modulation. These are significantly weaker than the ones for Gaussian noise (Umehara *et al.*, 2004a). Most of the systems affected by non-Gaussian noise suffer performance degradation for high SNR values.

This paper investigates the influence of trellis termination methods on turbo code performances for a channel affected by Middleton Class-A impulsive noise (AWCN), for various values of parameters that describe the impulsive noise model. We investigated the turbo code performances under the following circumstances: no trellis termination, termination only of the first trellis, post-interleaver flushing and dual termination. We considered the case of Max-Log-MAP decoding algorithms, with an extrinsic information scaling coefficient $s = 0.7$. The interleaver used is of S -random type, of length 1,024 and 16,384, respectively.

The paper is structured as follows. Section 2 describes the Middleton Class-A impulse noise model and Section 3 presents the system model. The simulation results are shown in Section 4 and conclusions are highlighted in Section 5.

2. Middleton Class-A Model

In many applications, non-Gaussian noise appears in addition to Gaussian noise. Some of its sources are: automotive ignition noise, power transmission lines, devices with electromechanical switches (photocopy machines, printers), microwave ovens etc. There are many statistical models for impulsive noise; in this study we assume the Middleton Class-A model. This type of noise has two components: a Gaussian one, with variance σ_g^2 , and an impulsive one, with variance σ_i^2 . The probability density function (PDF) of impulsive noise is a Poisson weighted sum of Gaussian distributions and it is given by (Umehara *et al.*, 2004a).

$$p(n) = \sum_{m=0}^{\infty} \frac{A^m e^{-A}}{\sqrt{2\pi} m! \sigma_m} \exp\left(-\frac{n^2}{2\sigma_m^2}\right). \quad (1)$$

The significance of quantities in eq. (1) is as follows: m is the number of active interferences (or impulses), A – the impulse index and it indicates the average number of impulses during the interference time. This parameter describes the noise as follows: as A decreases, the noise gets more impulsive; conversely, as A increases, the noise tends towards AWGN (Andrei *et al.*, 2014a). σ_m^2 is given by:

$$\sigma_m^2 = \sigma^2 \cdot \frac{\frac{m}{A} + T}{1 + T}, \quad (2)$$

where: $\sigma^2 = \sigma_g^2 + \sigma_i^2$ is the total noise power and

$$T = \frac{\sigma_g^2}{\sigma_i^2} \quad (3)$$

is the Gaussian factor. From eq. (3), we can observe that for low T values, the impulsive component prevails, and that for high values, the AWGN component.

An impulsive noise sample is given by (Andreadou *et al.*, 2009):

$$n = x_g + \sqrt{K_m} \cdot w \quad (4)$$

where: x_g is the white Gaussian background noise sequence with zero mean and variance σ_g^2 , w – the white Gaussian sequence with zero mean and variance σ_i^2 / A and K_m – the Poisson distributed sequence, whose PDF is characterized by the impulsive index A . If SNR_G is the value of signal-to-Gaussian noise power ratio and R_c is the coding rate, the variance σ_g^2 is obtained from:

$$\sigma_g = 1 / \sqrt{2R_c \text{SNR}_G} \quad (5)$$

3. System Model

A. The turbo code structure

The structure of the turbo encoder and the corresponding turbo decoder is given in Fig. 1. RSC1, RSC2 are the component recursive systematic convolutional codes, of memory 3 and generating matrix $G = [1, 15/13]$ (in octal form). The global coding rate of the turbo encoder is $1/3$. The interleaver π is of S -random type. The design of this interleaver is based on its random choice with a constraint imposed on its spread. The permutation is generated as follows (Divsalar *et al.*, 1995): each randomly selected integer is compared to S previously selected integers. If the current selection is equal to any S previous selections within a distance of $\pm S$, then the current selection is rejected. This process is repeated until all L integers are selected. This implies that the following condition has to be fulfilled: $(\forall) i, j \in \{0, 1, \dots, L-1\}$, with $|i - j| \leq S$, we have

$$|\pi(i) - \pi(j)| > S, \quad (6)$$

where: π represents the permutation describing the interleaver.

The search time for this interleaver generation algorithm is reasonable, if we choose $S < \sqrt{L/2}$. The simulations were done for the two of its lengths: 1,024 and 16,384.

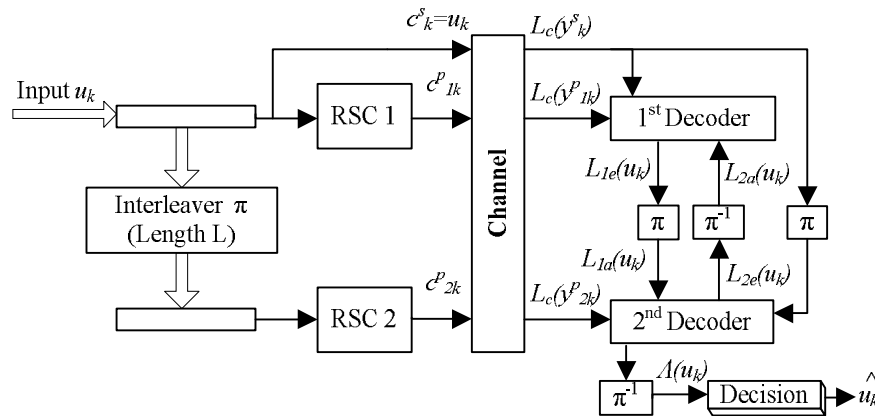


Fig. 1 – The structure of a turbo code.

$c_k^s = u_k$, c_{1k}^p and c_{2k}^p are the outputs of the turbo encoder, having the following meaning: $c_k^s = u_k$ is the systematic bit, and c_{1k}^p , c_{2k}^p are the parity check bits from the current trellis section, corresponding to encoders RSC1 and RSC2, respectively. These bits are BPSK (Binary Phase-Shift Keying) modulated. The constellation for BPSK modulation has two real points $\{-1, +1\}$, where -1 value corresponds to the bit 0 and $+1$ value corresponds to the bit 1. The BPSK symbols are transmitted over the AWCN channel. The received version of the transmitted symbol c_k is given by:

$$y_k = c_k + n, \tag{7}$$

where n is the noise sample generated as in eq. (4).

The turbo decoder includes two MAP decoders, one for each of the RSCs, a S -random interleaver (π) and the corresponding deinterleaver (π^{-1}). The entries in the two decoders are: $L_c(y_k^s)$ – the channel values for the received systematic bits, $L_c(y_{ik}^p)$ – the channel values for the received parity check bits, where $i = 1$ indicates the first decoder and $i = 2$, the second one. $L_{ie}(u_k)$ are the extrinsic information for each decoder, and $L_{ia}(u_k)$ are the a priori logarithmic likelihood ratios (LLRs), $i = 1, 2$. After a number of iterations, based on the calculated LLR- $A(u_k)$, the decoder will decide on the bit \hat{u}_k .

For turbo decoding we used Max-Log-MAP algorithm with an extrinsic information scaling coefficient $s = 0.7$. The used iteration stopping criterion is

based on the magnitude of LLR corresponding to a transmitted bit frame (Trifina *et al.*, 2005), and the chosen LLR threshold is $LLR_{Thresh} = 15$. The Max-Log-MAP decoding algorithm was detailed described for AWGN channel in (Andrei *et al.*, 2013). The difference for AWCN channel is the way how we compute reliabilities for the received symbols. That was described in (Andrei *et al.*, 2014b).

For AWGN channel, the reliability L_c is given by (Umehara *et al.*, 2004a):

$$L_c(y_k) = 4R_c \text{SNR}_G y_k. \quad (8)$$

For impulsive noise, the reliability is defined by (Umehara *et al.*, 2004a,b):

$$L_c(y_k) = \ln \frac{\sum_{m=0}^{\infty} \frac{A^m}{m! \sigma_m} \exp\left(-\frac{(y_k - 1)^2}{2\sigma_m^2}\right)}{\sum_{m=0}^{\infty} \frac{A^m}{m! \sigma_m} \exp\left(-\frac{(y_k + 1)^2}{2\sigma_m^2}\right)} \quad (9)$$

where: y_k is the sample received at time moment k .

B. Methods for trellis termination

For good decoding performances, the trellises of the two codes must begin from and end in the same state (usually, the null state) for each of the input data blocks. In order to accomplish this aim, the “trellis termination” operation is needed. It consists in adding a number of bits equal to at least the maximum of encoders’ memories, in order to bring the trellises to the null state.

In our simulations, we used 3 primary trellis termination classes that were detailed described in (Andrei *et al.*, 2013): termination of the first encoder, dual termination and post-interleaver flushing. The system performances obtained with these methods were compared with the case when no trellises are terminated. We assume the two encoders have m_1 and m_2 memory elements, respectively, (Hokfelt *et al.*, 2001a).

B1. No - trellis - termination

In this case, both encoders are left un-terminated and, as it was expected, the decoding performances are the weakest.

B2. Termination of the first encoder

This method consists in adding m_1 tail bits to the input sequence, so that the first encoder is brought in the null state. These bits are included in the interleaver input sequence and this is why, after permutation, they do no longer correspond to the termination bits of the second encoder.

B3. *Dual termination*

This method can be achieved by identifying the specific input positions, depending on the interleaver, in order to bring the encoders to the null state, independent of each other. This is accomplished without imposing constraints to interleaver, but with a slight increase of the number of input bits required for trellis termination (m_t bits), where

$$\max(m_1, m_2) \leq m_t \leq m_1 + m_2. \quad (10)$$

B4. *Post-interleaver flushing*

Using this method, both encoders are brought into the same state, independent from each other, after coding the input sequence of L bits. In this paper, we consider the case when the termination bits of the second trellis are transmitted.

4. Simulation Results

The simulations were performed using a turbo code with the structure in Fig. 1, with a global coding rate of $1/3$. The generator matrix for the two component convolutional codes is $G = [1, 15/13]$. The interleaver used is of S -random type. We considered two interleaver lengths, 1,024 and 16,384, respectively, with S -parameter 22 and 90, respectively. In this section, we analyzed the performances of turbo codes with the above features over a AWCN channel, with BPSK modulation and Max-Log-MAP decoding algorithm with an extrinsic information scaling factor $s = 0.7$. The used iteration stopping criterion is based on the magnitude of LLR corresponding to a transmitted bit frame and the chosen LLR threshold is $LLR_{Thresh} = 15$. The parameters for the Middleton Class-A impulsive noise were varied between $(A; T) = (0,01; 0,01)$, corresponding to a strongly impulsive noise and $(0,1; 0,1)$, corresponding to a weakly impulsive noise. Only the first $M = 2$ terms are considered in sum from eq. (1), for generating the impulsive noise samples, for all cases considered (Umehara *et al.*, 2004a; Umehara *et al.*, 2004b).

From all figures it can be observed that when trellis termination does not occur, we obtain the poorest performances and when the both trellises are terminated, the best performances are obtained. For higher interleaver length ($L = 16,384$), there are very small differences between the last three considered methods for trellis termination.

Case A: $(A; T) = (0,01; 0,01)$, $L = 1,024$

For the interleaver length of $L = 1,024$ and parameters $A = 0,01$ (highly impulsive noise), $T = 0,01$, the BER and FER curves depending on the value of SNR were represented in Figs. 2 *a* and *b*, respectively. Compared to no trellis-termination method, the termination of the first trellis offers coding gains of 0.4 dB and 0.35 dB for BER = 2×10^{-7} and FER = 2×10^{-4} , respectively.

The dual and post-interleaver termination offer the best performances and similarly, an additional coding gain of 0.25 dB, in the BER domain, and 0.35 dB in FER domain, against the case when only the first trellis is terminated. These coding gains are higher than those on the AWGN channel (Andrei *et al.*, 2013).

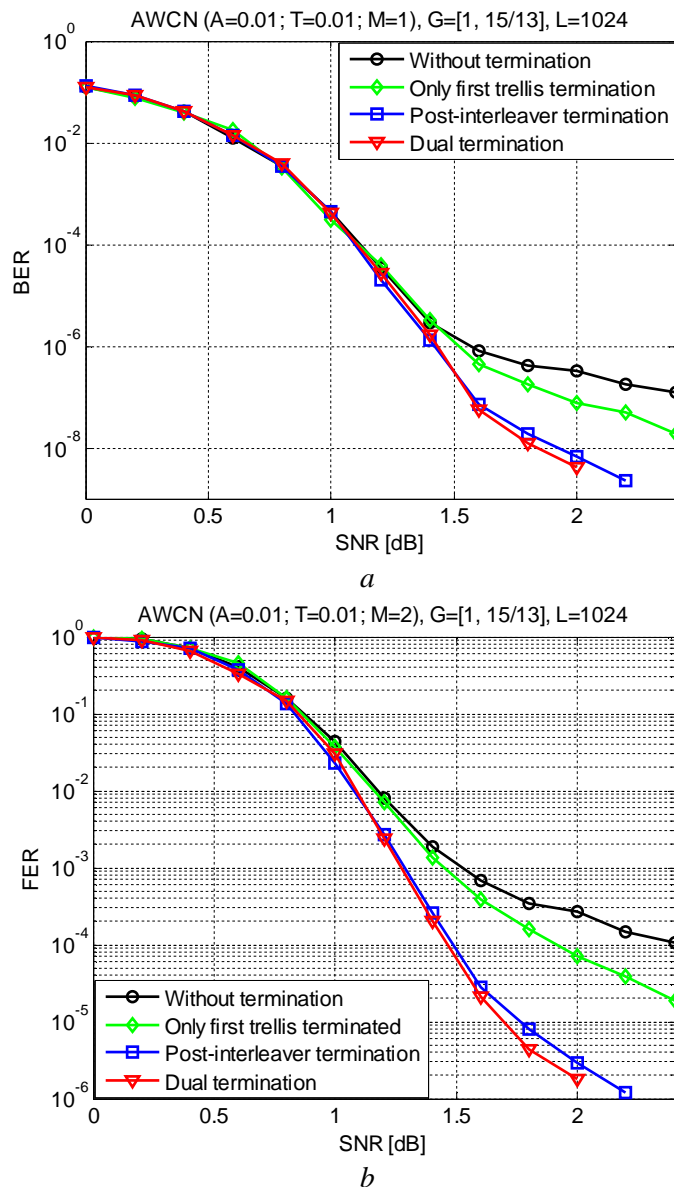


Fig. 2 – *a* – BER and *b* – FER curves for the turbo code over AWGN channel with $A = 0.01$ and $T = 0.01$, $L = 1,024$.

Case B: $(A; T) = (0.1; 0.1)$, $L = 1,024$

For parameters $A = 0.1$, $T = 0.1$ (weakly impulsive noise), the BER and FER curves depending on the value of SNR were represented in Figs. 3 *a* and *b*, respectively. In the case of AWGN channel, in the BER domain, the four methods used for trellis termination lead to similar performances until SNR = 1 dB (Andrei *et. al.*, 2013), while for AWCN channel, the SNR increases to 2 dB.

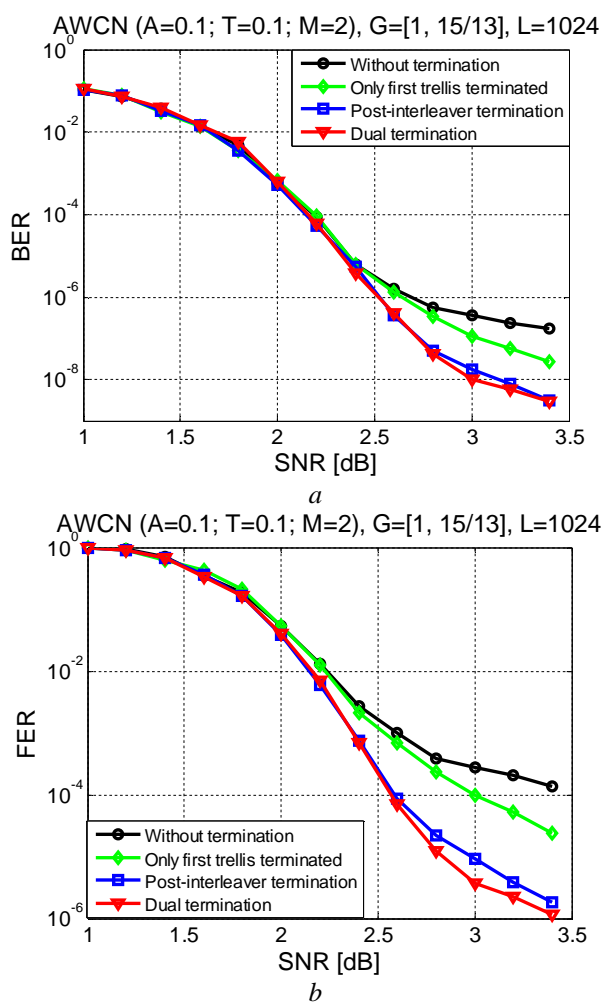


Fig. 3 – *a* – BER and *b* – FER curves for the turbo code over AWCN channel with $A = 0.1$ and $T = 0.1$, $L = 1,024$.

After this value, the results for a channel affected by impulsive noise are comparable to those on the channel affected by white Gaussian noise. The best performances are obtained for post-interleaver and dual termination,

bringing an additional coding gain of about 0.25 dB against when the first trellis is terminated, for $\text{BER} = 2 \times 10^{-7}$ and about 0.65 dB for the case when no trellis is terminated. This method (without termination) has the weakest performances for all the cases considered. In the FER domain, the additional coding gain brought by the two last methods considered is about 0.35 dB against the second one, and 0.7 dB for the case without termination, for $\text{FER} = 2 \times 10^{-4}$.

Case C: $(A; T) = (0.01; 0.01)$, $L = 16,384$

Figs. 4 *a* and *b* show the simulation results for the interleaver length of $L = 16,384$ and impulsive noise model parameters $A = 0.01$ and $T = 0.01$.

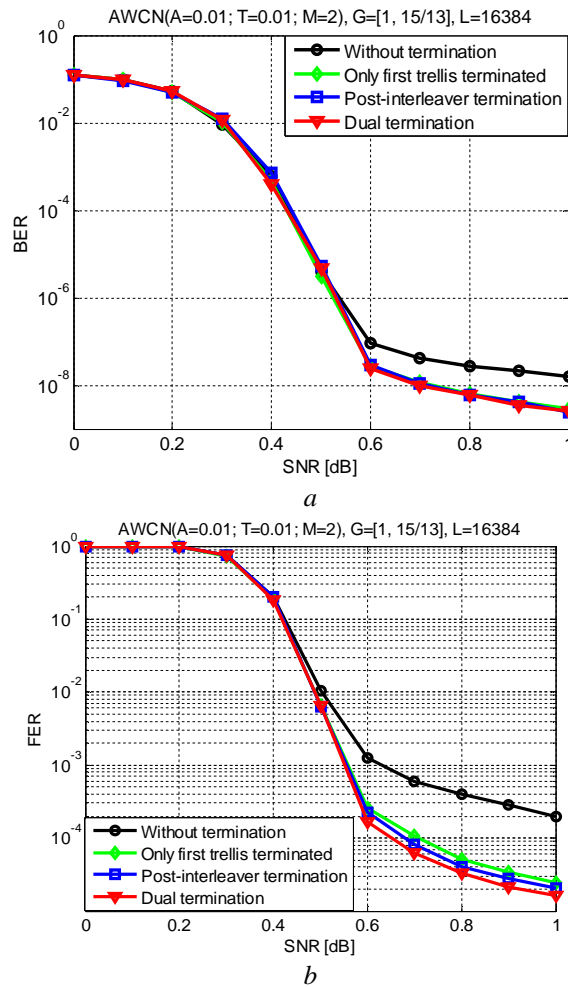


Fig. 4 – *a* – BER and *b* – FER curves for the turbo code over AWCN channel with $A = 0.01$ and $T = 0.01$, $L = 16,384$.

In this case, both in BER and FER domain, no trellis termination has the worst performances and the other methods have similar results, bringing an

additional coding gain of about 0.35 dB for $\text{BER} = 2 \times 10^{-8}$ and 0.4 dB for $\text{FER} = 2 \times 10^{-4}$. In the FER domain there are some differences between the last three methods, but they are very small, dual termination leading to the best results. Thus, for high interleaver length, it is essential that the trellises are terminated, irrespective of the means used. In this case, SNR has lower values compared to the above situations.

Case D: $(A; T) = (0.1; 0.1)$, $L = 16,384$

In the case of interleaver length $L = 16,384$ and noise model parameters $A = 0.1$, $T = 0.1$, the results are shown in Figs. 5 *a* and *b*. The same observation as in previous case is valid for the BER domain, except the values of SNR that are higher.

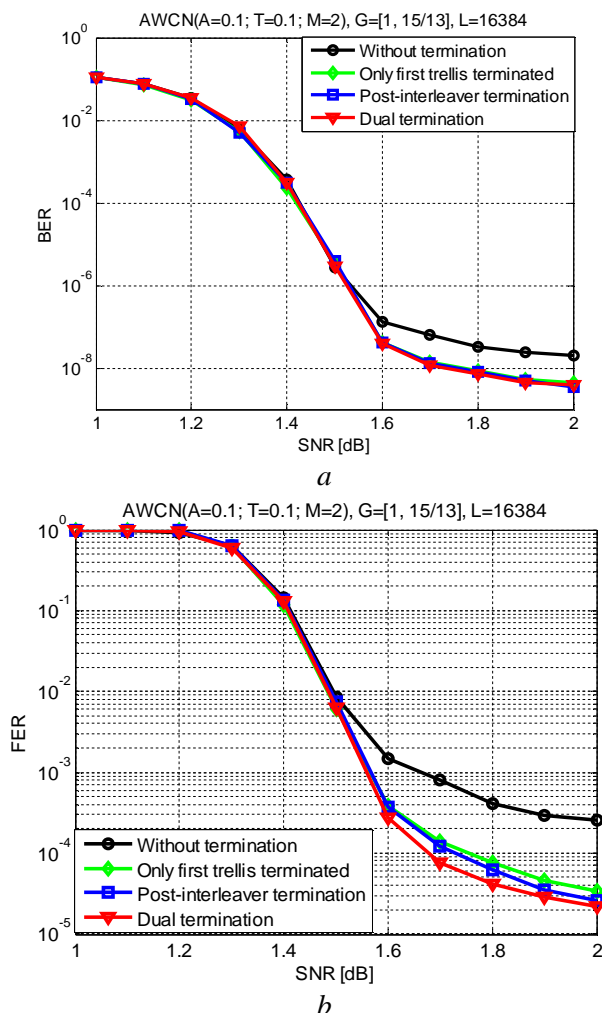


Fig. 5 – *a* – BER and *b* – FER curves for the turbo code over AWCN channel with $A = 0.1$ and $T = 0.1$, $L = 16,384$.

In the FER domain, the best results are obtained for dual termination, the additional coding gain brought by this method being 0.28 dB against the case when no trellises are finished, for $\text{FER} = 3 \times 10^{-4}$. Only the first trellis termination and post-interleaver termination have similar performances, bringing a coding gain about 0.35 dB against the first method.

5. Conclusions

In this paper, we investigated the influence of four methods for trellis termination on the BER/FER performances of turbo codes on channels affected by impulsive noise of Middleton type class A. The modulation used was of BPSK type. We considered two interleaver lengths: one for small to medium lengths (1,024) and one for large lengths (16,384). For the AWCN channel, we considered two parameter sets: ($A = 0.01$; $T = 0.01$), meaning a highly impulsive noise and ($A = 0.1$; $T = 0.1$), meaning a low impulsive noise. These result in four study cases. The decoding algorithm used is Max-Log-MAP with the extrinsic information-scaling factor 0.7.

According to simulation results from section 4, in all cases the poorest performances was obtained for “without termination”. The best method was found to be the dual termination as for the AWGN channel. Post-interleaver termination leads also to good results, similar to the above method.

The length of interleaver is important in the case of a channel affected by impulsive noise: for higher interleaver length ($L = 16,384$), there are very small differences between the last three considered methods for trellis termination.

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INFLUENȚA METODEI DE TERMINARE A TRELLIS-ULUI ASUPRA
CODURILOR TURBO CU TRANSMISIE PE CANAL CU ZGOMOT IMPULSIV DE
TIP MIDDLETON DE CLASA-A

(Rezumat)

Codurile Turbo ocupă un loc special în domeniul comunicațiilor wireless datorită performanțelor deosebite pe care le au în corectarea erorilor ce apar pe canalele de transmisiune. Lucrarea prezintă analiza influenței metodelor de terminare a trellis-ului asupra performanțelor codurilor turbo, pe un canal afectat de zgomot impulsiv, cu modulație binară de fază (Binary Phase-Shift Keying). Modelul statistic folosit pentru zgomotul impulsiv este Middleton Class-A. Evaluarea performanțelor s-a efectuat prin considerarea erorii de bit și de cadru, pentru două lungimi ale interleaver-ului S-aleator. Simulările au fost realizate pentru diferite valori ale parametrilor ce descriu modelul Middleton Class-A. Algoritmul de decodare folosit a fost Max-Log-MAP.

