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## SOFT ESTIMATES FOR DOUBLY ITERATIVE DECODING FOR 16–QAM AND 64–QAM MODULATION

BY

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**Abstract.** In order to increase the spectral efficiency, a bit interleaved coded modulation can be combined with a high order modulation scheme like Phase Shift Keying (PSK) and Quadrature Amplitude Modulation (QAM), improving the communication system’s performance. In this paper, we derive and explain the soft estimates for a doubly iterative decoder using space-time turbo codes and a large number of transmit and receive antennas for 16–QAM and 64–QAM modulation used in a mobile communication system.

**Key words:** space-time turbo codes; doubly iterative decoder; soft estimates; QAM; FER performances.

### 1. Introduction

The use of multiple antenna transmission techniques increases the spectral efficiency of wireless systems. (Biglieri *et al.*, 2005) presented a block scheme for a doubly iterative receiver, based on the minimum mean square error (MMSE) criteria (Biglieri *et al.*, 2003). A new scheme has been presented with a significantly reduced complexity for a large number of transmit and receive antennas, compared to previous scheme proposed by Stefanov and Duman (Stefanov & Duman, 2001). Since the large number of antennas

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increases the receiver complexity, a spatial interference canceling scheme is used, ensuring a good compromise between complexity and performance.

A higher order modulation scheme gives the advantage that larger data rates and better spectral efficiency for radio communications systems are obtained. The disadvantage is that the performance of the iterative receivers depends critically on the size of the signal constellation and a high order modulation scheme is less robust to noise and interference. This leads to performance degradation.

In Section 2 we recall the system model consisting of the transmitter and the receiver block scheme. In Section 3 we analyze 16-QAM and 64-QAM modulation schemes, specifying their characteristics, advantages and disadvantages. In Section 4, we derive the soft estimates for 16-QAM and 64-QAM modulations. In Section 5 we analyze the simulation results and Section 6 concludes the paper.

## 2. System Model

We consider the same mobile communication system as in (Rotopanesu *et al.*, 2012) with  $N_t$  transmit and  $N_r$  receive antennas. The information bits are turbo-coded with coding rate  $R_c$  and block size of  $N_t N$  symbols, where  $N$  is the number of successive transmissions from the transmit antennas, corresponding to a codeword. The input signal from the modulator output, is transmitted by antenna  $i$ ,  $1 \leq i \leq N$ , at each time instant. The spectral efficiency refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. The spectral efficiency is equal to  $R_c M N_t$ , where  $R_c$  is the coding rate of the turbo code.

The transmitter block scheme is presented in Fig. 1 and performs a coded modulation with bit interleaving and antenna diversity as described in (Caire *et al.*, 1998). The receiver block scheme (Trifina *et al.*, 2011) uses MMSE iterative algorithm and a linear MMSE interface (Rotopanesu *et al.*, 2012) and it is given in Fig. 2. The used turbo decoding algorithm is the Max-Log-APP.

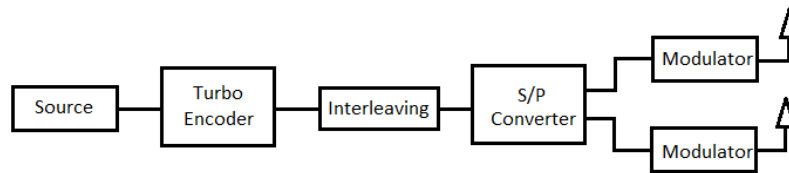


Fig. 1 – Transmitter block scheme.

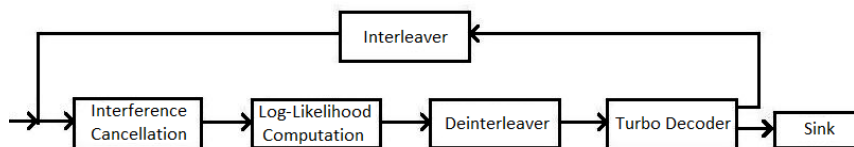


Fig. 2 – Receiver block scheme.

The linear MMSE interface used by the receiver consists in a linear filter modeled by a matrix that minimizes the mean square error. The filtered signal is transmitted to the interference canceling block.

The extrinsic soft estimates of transmitted bits are provided by the turbo decoder as a logarithm likelihood ratio (*i.e.* the logarithm of the ratio of the probabilities that the bit is either 1 or 0):

$$L(c_{i,t}) = \ln \left( \frac{P(c_{i,t} = 1)}{P(c_{i,t} = 0)} \right) \quad (1)$$

and knowing that

$$P(c_{i,t} = 0) = 1 - P(c_{i,t} = 1), \quad (2)$$

we have

$$2P(c_{i,t} = 1) - 1 = \tanh \left( \frac{L(c_{i,t})}{2} \right), \quad (3)$$

where

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (4)$$

is the hyperbolic tangent function.

### 3. QAM Constellation

QAM is extensively used as a modulation scheme for digital telecommunication systems. Arbitrarily high spectral efficiencies can be achieved with QAM by setting a suitable constellation size, limited only by the noise level and linearity of the communications channel.

A motivation for the use of QAM comes from the fact that QAM schemes are more bandwidth efficient. Considering a higher-order QAM constellation, it is possible to transmit more bits per symbol. However, if the mean energy of the constellation remains the same, the points must be closer to each other and thus they are more susceptible to noise and other corruption.

The QAM constellation points are normally arranged in a square grid with equal vertical and horizontal spacing, so that the most commonly used QAM constellations have a number of points equal to a power of 4, such as 4-QAM, 16-QAM, 64-QAM, 256-QAM, and so on. In general, the number of points of the QAM modulation is  $4^m = 2^{2m}$ , where the number of the bits in each constellation symbol is  $M = 2m$  ( $m$  is an integer). In this paper, we use the 4-QAM modulation, thus the number of transmitted bits per symbol is  $M = 2$  ( $m = 1$ ), 16-QAM, thus  $M = 4$  ( $m = 2$ ), and 64-QAM, with  $M = 6$  ( $m = 3$ ). The points are represented in a complex plane having the in-phase component on the real axis and the quadrature component on the imaginary axis.

For a general QAM constellation the used alphabet symbols are chosen of the form:

$$\alpha_{\text{QAM}} = \left\{ \pm(2p-1) \pm (2q-1)j \right\}, \text{ where } p \in \left\{ 1, \dots, \frac{\sqrt{2^M}}{2} \right\} \text{ and } q \in \left\{ 1, \dots, \frac{\sqrt{2^M}}{2} \right\}. \quad (5)$$

For 16-QAM, the symbols used are chosen from the set

$$\alpha_{16\text{-QAM}} = \left\{ \pm 1 \pm j, \pm 1 \pm 3j, \pm 3 \pm j, \pm 3 \pm 3j \right\}$$

and the average energy is  $E_{16\text{-QAM}} = 10$ .

Fig. 3 presents the signal constellation for 16-QAM modulation using the Gray coded bit-mapping.

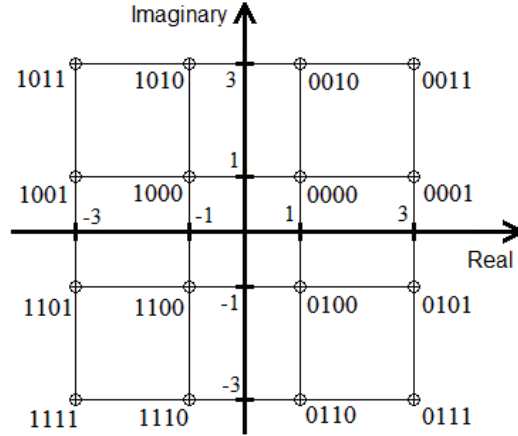


Fig. 3 – Bit mapping for 16-QAM signal constellation.

64-QAM is often used in digital cable television and cable modem applications. In the United States, 64-QAM is one of the mandated modulation schemes for digital cable. In the UK, 64-QAM is used for digital terrestrial television.

For 64-QAM, the symbols used are chosen from the set

$$\alpha_{64\text{-QAM}} = \left\{ \begin{array}{l} \pm 1 \pm j, \pm 1 \pm 3j, \pm 1 \pm 5j, \pm 1 \pm 7j, \pm 3 \pm j, \pm 3 \pm 3j, \pm 3 \pm 5j, \pm 3 \pm 7j, \\ \pm 5 \pm j, \pm 5 \pm 3j, \pm 5 \pm 5j, \pm 5 \pm 7j, \pm 7 \pm j, \pm 7 \pm 3j, \pm 7 \pm 5j, \pm 7 \pm 7j \end{array} \right\}$$

and the average energy is  $E_{16\text{-QAM}} = 42$ .

Fig. 4 presents the signal constellation for 64-QAM modulation using the Gray coded bit-mapping.

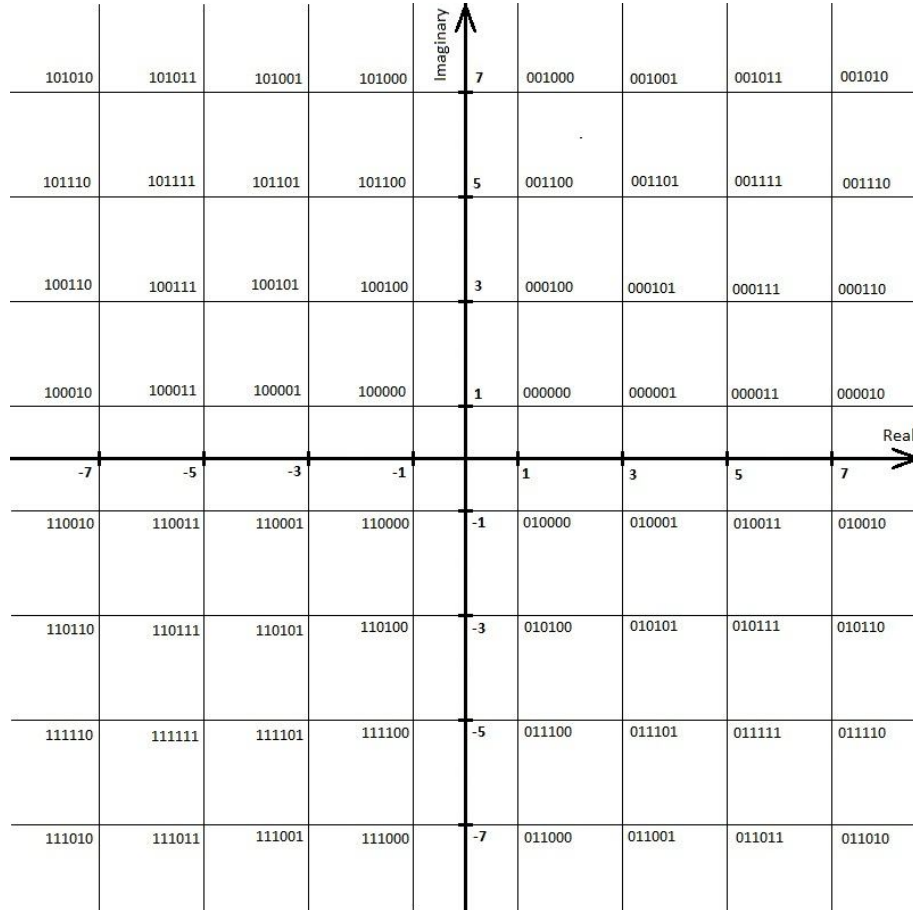


Fig. 4 – Bit mapping for 64–QAM signal constellation.

Knowing this and having a 1/2 global rate, the spectral efficiency is 32 bits/s/Hz for 16–QAM modulation and 48 bits/s/Hz for 64–QAM modulation, in the case of 16 transmit antennas.

#### 4. Soft Estimates for 16–QAM and 64–QAM

In (Trifina *et al.*, 2011) the associations between the three and four bit sequences from the constellation and the complex values of symbols are given for 8–PSK and 16–PSK modulation, respectively. Here, we present the four and six bit sequences from the constellation and the complex values of symbols for 16–QAM and 64–QAM modulation.

From the above explanations, it is reasonably intuitive to guess that the scaling factor of  $1/\sqrt{10}$  and  $1/\sqrt{42}$  will be used in 16–QAM and 64–QAM constellations, respectively, for normalizing the average transmit power.

For 16-QAM the associations between the constellation bit sequences and the complex values of the symbols are:

$$(0000), (1000) \rightarrow \text{Amp}(\pm 1 + j)$$

$$(0001), (1001) \rightarrow \text{Amp}(\pm 3 + j)$$

$$(0010), (1010) \rightarrow \text{Amp}(\pm 1 + 3j)$$

$$(0011), (1011) \rightarrow \text{Amp}(\pm 3 + 3j)$$

$$(0100), (1100) \rightarrow \text{Amp}(\pm 1 - j)$$

$$(0101), (1101) \rightarrow \text{Amp}(\pm 3 - j)$$

$$(0110), (1110) \rightarrow \text{Amp}(\pm 1 - 3j)$$

$$(0111), (1111) \rightarrow \text{Amp}(\pm 3 - 3j)$$

where:  $\text{Amp} = 1/\sqrt{10}$ .

We assume that  $c_{2i-1,t}$ ,  $c_{2i,t}$ ,  $c_{2i+1,t}$ ,  $c_{2i+2,t}$  are the four bits in the sequence associated to the symbol.

$c_{2i-1,t}$  is the first bit and gives the sign of the real part of the complex number representing the soft estimates. If this bit is 0, then the sign of the real part is positive, and if the bit is equal to 1, the sign of the real part is negative. The second bit  $c_{2i,t}$ , gives the sign of the imaginary part of the complex number. If the bit is 0, then the sign of the imaginary part is positive, and if the bit is 1, the sign of the imaginary part is negative. The sign of the real and the imaginary parts of the soft estimates is estimated by the quantity  $1 - 2P(c_{2i-1,t} = 1)$  and  $1 - 2P(c_{2i,t} = 1)$ , respectively. If one of these bits is 1, the probability of that bit to be 1 is equal to 1 and if the bit is 0, then, the probability of that bit to be 1 is equal to 0. For example, if  $c_{2i-1,t} = 0$ , then  $P(c_{2i-1,t} = 1) = 0$  and the sign of real part is given by the sign of the expression  $1 - 2P(c_{2i-1,t} = 1) = 1$ . Also, if  $c_{2i,t} = 1$ , then  $P(c_{2i,t} = 1) = 1$  and the sign of the imaginary part is given by the sign of  $1 - 2P(c_{2i,t} = 1) = -1$ .

$c_{2i+1,t}$  is the third bit and gives the amplitude of the imaginary part of the soft estimates. If the bit is 0, the amplitude of the imaginary part is equal to 1. Otherwise, if the bit is 1, the amplitude is 3. In the same way, the fourth bit,  $c_{2i+2,t}$ , gives the amplitude of the real part of the soft estimates. If the bit is 0, the amplitude of the real part is equal to 1 and if the bit is 1, the amplitude is 3. The amplitude of the imaginary part and the amplitude of the real part are estimated by the quantities  $1 + 2P(c_{2i+1,t} = 1)$  and  $1 + 2P(c_{2i+2,t} = 1)$ , respectively. For example, if  $c_{2i+1,t} = 1$ , then  $P(c_{2i+1,t} = 1) = 1$ , meaning that the

amplitude of the imaginary part is  $1 + 2P(c_{2i+1,t} = 1) = 3$ . Also, if  $c_{2i+2,t} = 0$  then  $P(c_{2i+2,t} = 1) = 0$  and the amplitude of the real part is  $1 + 2P(c_{2i+2,t} = 1) = 1$ .

In this example, for  $c_{2i-1,t} c_{2i,t} c_{2i+1,t} c_{2i+2,t} = 0110$ , the complex value is  $1 \cdot 1 + (-1) \cdot 3j = 1 - 3j$ .

According to the above considerations and using eq. (3), the soft estimates for 16-QAM are given in the next formula:

$$\begin{aligned} \hat{x}_{i,t} &= \text{Amp} \left[ \left( 1 - 2P(c_{2i-1,t} = 1) \right) \left( 1 + 2P(c_{2i+2,t} = 1) \right) + j \left( 1 - 2P(c_{2i,t} = 1) \right) \left( 1 + 2P(c_{2i+1,t} = 1) \right) \right] \\ &= \text{Amp} \left[ - \left( \tanh \left( \frac{L(c_{2i-1,t})}{2} \right) \right) \left( \tanh \left( \frac{L(c_{2i+2,t})}{2} \right) + 2 \right) - j \left( \tanh \left( \frac{L(c_{2i,t})}{2} \right) \right) \left( \tanh \left( \frac{L(c_{2i+1,t})}{2} \right) + 2 \right) \right] \quad (6) \end{aligned}$$

For 64-QAM, the associations between the constellation bit sequences and the complex values of the symbols are:

(000000), (100000) → Amp(±1 + j)	(010000), (110000) → Amp(±1 - j)
(000001), (100001) → Amp(±3 + j)	(010001), (110001) → Amp(±3 - j)
(000010), (100010) → Amp(±7 + j)	(010010), (110010) → Amp(±7 - j)
(000011), (100011) → Amp(±5 + j)	(010011), (110011) → Amp(±5 - j)
(000100), (100100) → Amp(±1 + 3j)	(010100), (110100) → Amp(±1 - 3j)
(000101), (100101) → Amp(±3 + 3j)	(010101), (110101) → Amp(±3 - 3j)
(000110), (100110) → Amp(±7 + 3j)	(010110), (110110) → Amp(±7 - 3j)
(000111), (100111) → Amp(±5 + 3j)	(010111), (110111) → Amp(±5 - 3j)
(001000), (101000) → Amp(±1 + 7j)	(011000), (111000) → Amp(±1 - 7j)
(001001), (101001) → Amp(±3 + 7j)	(011001), (111001) → Amp(±3 - 7j)
(001010), (101010) → Amp(±7 + 7j)	(011010), (111010) → Amp(±7 - 7j)
(001011), (101011) → Amp(±5 + 7j)	(011011), (111011) → Amp(±5 - 7j)
(001100), (101100) → Amp(±1 + 5j)	(011100), (111100) → Amp(±1 - 5j)
(001101), (101101) → Amp(±3 + 5j)	(011101), (111101) → Amp(±3 - 5j)
(001110), (101110) → Amp(±7 + 5j)	(011110), (111110) → Amp(±7 - 5j)
(001111), (101111) → Amp(±5 + 5j)	(010111), (111111) → Amp(±5 - 5j)

where:  $\text{Amp} = 1/\sqrt{42}$ .

We assume that  $c_{2i-1,t}$ ,  $c_{2i,t}$ ,  $c_{2i+1,t}$ ,  $c_{2i+2,t}$ ,  $c_{2i+3,t}$  and  $c_{2i+4,t}$  are the six bits in the sequence associated to the symbol.

$c_{2i-1,t}$  is the first bit and gives the sign of the real part of the complex number. If this bit is 0, the sign of the real part is positive and if the bit is 1, the sign of the real part is negative. The second bit  $c_{2i,t}$ , gives the sign of the

imaginary part of the complex number. If this bit is 0, the sign of the imaginary part is positive, otherwise, if the bit is 1, the sign of the imaginary part is negative. The sign of the real part is estimated by the sign of the quantity  $1 - 2P(c_{2i-1,t} = 1)$  and the sign of the imaginary part is estimated by the sign of the expression  $1 - 2P(c_{2i,t} = 1)$ . If one of these bits is 1, the probability of that bit to be 1 is equal to 1 and if the bit is 0, the probability of that bit to be 1 is equal to 0. For example, if  $c_{2i-1,t} = 1$ , then  $P(c_{2i-1,t} = 1) = 1$  and  $1 - 2P(c_{2i-1,t} = 1) = -1$ , thus the sign of the real part is negative. Through the same analogy, if  $c_{2i,t} = 1$ , the sign of the imaginary part is also negative.

The amplitude of the real and imaginary part is given by a group of two bits. Thus, the third and fourth bit,  $c_{2i+1,t}$  and  $c_{2i+2,t}$ , give the amplitude of the imaginary part, and the fifth bit together with the sixth bit,  $c_{2i+3,t}$  and  $c_{2i+4,t}$ , give the amplitude of the real part. We denote the first and the second bit, respectively, from a group of two bits, by  $b_1$  and  $b_2$ , respectively. Then, the amplitude value is given by the next formula:

$$A = 1 + 2 \left[ 2^1 P(b_1 = 1) + 2^0 P(b_1 \oplus b_2 = 1) \right]. \quad (7)$$

The XOR of two bits is 1 only when one of them is 0 and the other one is 1. Thus, the probability that XOR of the two bits is 1 is given by:

$$\begin{aligned} P(b_1 \oplus b_2 = 1) &= P(b_1 = 0 \cap b_2 = 1) + P(b_1 = 1 \cap b_2 = 0) = \\ &= P(b_1 = 0)P(b_2 = 1) + P(b_1 = 1)P(b_2 = 0) = \\ &= (1 - P(b_1 = 1))P(b_2 = 1) + P(b_1 = 1)(1 - P(b_2 = 1)) = \\ &= P(b_1 = 1) + P(b_2 = 1) - 2P(b_1 = 1)P(b_2 = 1). \end{aligned} \quad (8)$$

In (8) we assumed that the two bits,  $b_1$  and  $b_2$ , are independent. This assumption is valid, taking into account that the codeword at the turbo encoder output is interleaved by a random interleaver with large enough length. With (8), the amplitude in (7) becomes:

$$\begin{aligned} A &= 1 + 2 \left[ 2^1 P(b_1 = 1) + 2^0 (P(b_1 = 1) + P(b_2 = 1) - 2P(b_1 = 1)P(b_2 = 1)) \right] = \\ 4 \quad &= 1 + 4P(b_1 = 1) + 2P(b_1 = 1) + 2P(b_2 = 1) - 4P(b_1 = 1)P(b_2 = 1) = \\ &= 1 + 6P(b_1 = 1) + 2P(b_2 = 1) - 4P(b_1 = 1)P(b_2 = 1). \end{aligned} \quad (9)$$

As it was mentioned above, the third and fourth bit,  $c_{2i+1,t}$  and  $c_{2i+2,t}$ , give the amplitude of the imaginary part. If  $c_{2i+1,t}$  is 0 and  $c_{2i+2,t}$  is 0, then the amplitude of the imaginary part is equal to 1, but if  $c_{2i+1,t}$  is equal to 0 and  $c_{2i+2,t}$  to 1, the amplitude of the imaginary part is equal to 3. Also, if  $c_{2i+1,t}$  is 1 and  $c_{2i+2,t}$  is 0, then the amplitude of the imaginary part is equal to 7 and if  $c_{2i+1,t}$



is 1 and  $c_{2i+2,t}$  is 1, the amplitude of the imaginary part is equal to 5. The amplitude of the imaginary part is estimated by the quantity  $1 + 6 \cdot P(c_{2i+1,t} = 1) + 2 \cdot P(c_{2i+2,t} = 1) - 4 \cdot P(c_{2i+1,t} = 1) \cdot P(c_{2i+2,t} = 1)$ . For example, for  $c_{2i+1,t} = 1$  and  $c_{2i+2,t} = 0$ , the probabilities  $P(c_{2i+1,t} = 1) = 1$  and  $P(c_{2i+2,t} = 1) = 0$ , so that the amplitude of the imaginary part is  $1 + 6 \cdot 1 + 2 \cdot 0 - 4 \cdot 1 \cdot 0 = 7$ .

In the same way, the fifth bit together with the sixth bit,  $c_{2i+3,t}$  and  $c_{2i+4,t}$ , give the amplitude of the real part. If  $c_{2i+3,t}$  is equal to 0 and  $c_{2i+4,t}$  to 0, then the amplitude of the real part is equal to 1 and if  $c_{2i+3,t}$  is 0 and  $c_{2i+4,t}$  is 1 then the amplitude of the real part is equal to 3. If  $c_{2i+3,t}$  is equal to 1 and  $c_{2i+4,t}$  is 0, then the amplitude of the real part is equal to 7 and if  $c_{2i+3,t}$  is 1 and  $c_{2i+4,t}$  is 1, then the amplitude of the real part is equal to 5. The amplitude of the real part is estimated by the quantity  $1 + 6P(c_{2i+3,t} = 1) + 2P(c_{2i+4,t} = 1) - 4P(c_{2i+3,t} = 1)P(c_{2i+4,t} = 1)$ . For example, for  $c_{2i+3,t} = 0$  and  $c_{2i+4,t} = 1$ , the probabilities  $P(c_{2i+3,t} = 1) = 0$  and  $P(c_{2i+4,t} = 1) = 1$ , so that the amplitude of the real part is  $1 + 6 \cdot 0 + 2 \cdot 1 - 4 \cdot 0 \cdot 1 = 3$ .

For the given example, for  $c_{2i-1,t} c_{2i,t} c_{2i+1,t} c_{2i+2,t} c_{2i+3,t} c_{2i+4,t} = 111001$ , the complex number is  $(-1) \cdot 3 + (-1) \cdot 5j = -3 - 7j$ .

According to the above considerations and using equation (8), the soft estimates for 64 - QAM are given in the next formula:

$$\begin{aligned} \hat{x}_{i,t} &= \text{Amp} \left\{ \left( 1 - 2P(c_{2i-1,t} = 1) \right) \left[ 1 + 6P(c_{2i+3,t} = 1) + 2P(c_{2i+4,t} = 1) - 4P(c_{2i+3,t} = 1)P(c_{2i+4,t} = 1) \right] + \right. \\ &\quad \left. + j \left( 1 - 2 \cdot P(c_{2i,t} = 1) \right) \left[ 1 + 6P(c_{2i+1,t} = 1) + 2P(c_{2i+2,t} = 1) - 4P(c_{2i+1,t} = 1)P(c_{2i+2,t} = 1) \right] \right\} = \\ &= \text{Amp} \left\{ -\tanh \left( \frac{L(c_{2i-1,t})}{2} \right) \left[ 4 + \tanh \left( \frac{L(c_{2i+3,t})}{2} \right) \left( 2 - \tanh \left( \frac{L(c_{2i+4,t})}{2} \right) \right) \right] + \right. \\ &\quad \left. - j \tanh \left( \frac{L(c_{2i,t})}{2} \right) \left[ 4 + \tanh \left( \frac{L(c_{2i+1,t})}{2} \right) \left( 2 - \tanh \left( \frac{L(c_{2i+2,t})}{2} \right) \right) \right] \right\}. \quad (10) \end{aligned}$$

## 5. Simulation Results

In this paper, the simulations were performed for 4-QAM ( $M = 2$ ), 16-QAM ( $M = 4$ ) and 64-QAM ( $M = 6$ ), considering the same scenario as in (Rotopanescu *et al.*, 2012). The turbo encoder uses a random interleaver of length  $\text{Len} = 2,080$  for 4-QAM and 16-QAM modulations and of length 2,112 for 64-QAM modulation.

The global rate of the turbo code is  $1/2$  (with alternative puncturing of

parity bits), the forward and feedback generator polynomials are (5, 7) in octal form, and the interleaver between the turbo encoder and the serial to parallel converter is a random one. We considered a number of 16 transmit and 16 receive antennas. The space-time codeword is a matrix with 16 rows (the number of transmit antennas) and 130 columns for 4-QAM, 65 columns for 16-QAM, and 44 columns for 64-QAM. The number of distinct blocks with constant fading,  $F$ , is equal to 1.

In all three cases, the turbo decoder uses the Max-Log-APP algorithm, described in (Trifina *et al.*, 2011), and performs maximum 10 iterations. The stop criterion is genie stopper.

To cancel spatial interference, there was used a number of  $k = 0$ ,  $k = 1$  and  $k = 4$  iterations. In (Trifina *et al.*, 2011), an analysis was performed to evaluate the influence of the extrinsic information scaling coefficient denoted by “ $s$ ” on the BER/FER performance of the system with QPSK modulation. It ranges from 0.6 to 1 with the step 0.05. As in (Rotopanescu *et al.*, 2012), we consider the same extrinsic information scaling coefficient that performs the best FER and BER performance. For  $k = 0$ , we have considered the scaling coefficient  $s = 0.9$ , for  $k = 1$ ,  $s = 0.8$  and for  $k = 4$ ,  $s = 0.75$ .

The Monte Carlo simulation results are given in Figs. 5,...,7 for 4-QAM, 16-QAM, 64-QAM, respectively, using  $k = 0, 1$  and 4. These figures show the performance of the MMSE receiver through FER, versus signal-to-noise ratio per bit ( $E_b/N_0$ ).

For every modulation, we observe from simulations that the coding gain increases proportionally with the number of double iterations  $k$ , as we will analyze further. Increasing the iteration number of the MMSE iterative decoder up to 4, leads to improved performances. In (Biglieri *et al.*, 2005) it was shown that further increasing the number of iterations does not lead to additional performance improvement.

Also, we observe from simulation results that for  $k = 4$ , the double iterations introduce relatively more errors compared to  $k = 1$ , and the supplementary coding gain obtained for  $k = 4$  is smaller than the one achieved when  $k = 1$ .

From simulation results, we observe the performance degradation. We can see that it is a great difference in FER performances between all three modulations. There is a coding gain up to 6.44 dB for 4-QAM compared to 16-QAM and up to 9.68 dB for 16-QAM compared to 64-QAM.

For the same FER value, equal to  $2 \times 10^{-4}$ , we analyze all three modulations for different number of iterations.

From Table 1 we can see that for 4-QAM the supplementary coding gain is 3.31 dB for  $k = 1$  compared to  $k = 0$  and 0.05 dB for  $k = 1$  compared to  $k = 4$ . The coding gain is better for  $k = 1$  compared to  $k = 4$  at FER =  $2 \times 10^{-4}$  since the error floor phenomenon is more pronounced for  $k = 4$  than for  $k = 1$ . However at FER =  $2 \times 10^{-3}$ , a 0.4 dB supplementary coding gain is achieved for  $k = 4$ , compared to the performance when  $k = 1$ , as it was shown in

(Rotopanescu *et al.*, 2012) for QPSK modulation (which is the same with 4-QAM).

For 16-QAM, the coding gain is 3.61 dB for  $k = 1$  compared to  $k = 0$  and 0.32 dB for  $k = 4$  compared to  $k = 1$ . Thus, the total coding gain achieved for 16-QAM is 3.93 dB.

For 64-QAM, for the proposed FER value, the resulting coding gain is 3.30 dB for  $k = 1$  compared to  $k = 0$  and 0.55 dB for  $k = 4$  compared to  $k = 1$ , and the total coding gain for 64-QAM is 3.85 dB.

**Table 1**

SNR Values (dB), for a Given Value  $FER = 2 \times 10^{-4}$

SNR, [dB]	$k = 0$	$k = 1$	$k = 4$
4-QAM	-2.29	-5.60	-5.55
16-QAM	4.15	0.54	0.22
64-QAM	13.52	10.22	9.67

Because 4-QAM modulation is the same as QPSK, the FER performances for this modulation and different number of iterations are the same as by Rotopanescu *et al.*, (2012), Fig. 6. Therefore, we do not give them separately in this paper.

In Fig. 8 we analyze the FER performances for different number of iterations and 16-PSK and 16-QAM modulations. The results expressed in SNR are given in Table 2. The used number of iterations is  $k = 0$ ,  $k = 1$  and  $k = 4$ , in order to see the performance difference between them.

**Table 2**

SNR Values (dB), for a Given Value  $FER = 2 \times 10^{-4}$

SNR, [dB]	$k = 0$	$k = 1$	$k = 4$
16-PSK	7.10	3.42	2.95
16-QAM	4.15	0.54	0.22

Assuming a value  $FER = 2 \times 10^{-4}$ , for  $k = 0$ , the obtained coding gain is 2.95 dB for 16-QAM compared to 16-PSK. When  $k$  increases the supplementary coding gain slightly decreases. Thus, for the same FER value and  $k = 1$  the coding gain is 2.88 dB for 16-QAM compared to 16-PSK. Also, for  $k = 4$ , for the proposed FER value the supplementary coding gain is 2.73 dB for 16-QAM compared to 16-PSK.

Also, comparing the results obtained for the same modulations but different number of iterations, we see that for 16-PSK the supplementary coding gain is 3.68 dB for  $k = 1$  compared to  $k = 0$  and 0.47 dB for  $k = 4$  compared to  $k = 1$ . As it was shown above, for 16-QAM the obtained coding gain is 3.61 dB for  $k = 1$  compared to  $k = 0$  and 0.32 dB for  $k = 4$  compared to  $k = 1$ .

In Fig. 9 we analyze the FER performances for 64-QAM and for

different number of iterations. The results are given in the last line of Table 1. We consider the same value,  $\text{FER} = 2 \times 10^{-4}$ . The coding gain obtained for  $k = 1$  compared to  $k = 0$  is 3.30 dB and is larger than that obtained for  $k = 4$  compared to  $k = 1$ , which is 0.55 dB. This means that the supplementary coding gain obtained for  $k = 4$  is smaller than the one achieved when  $k = 1$ , explained by the fact that for  $k = 4$ , the double iterations introduce relatively more errors compared to  $k = 1$ .

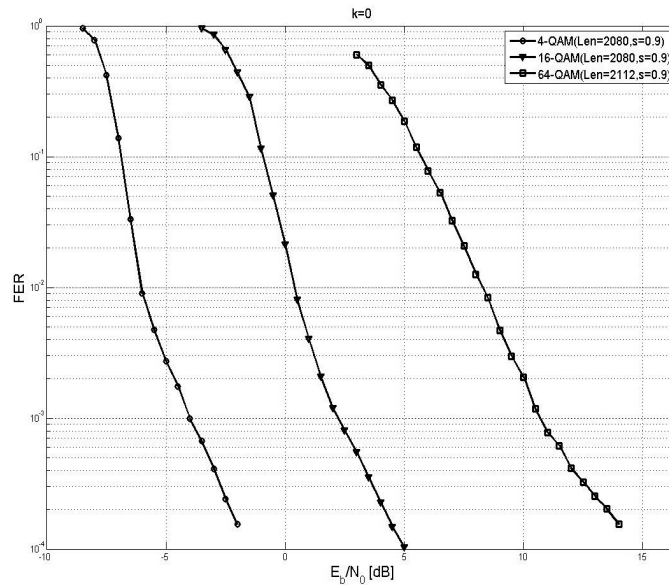


Fig. 5 – FER performances for 4-QAM, 16-QAM and 64-QAM when  $k = 0$ .

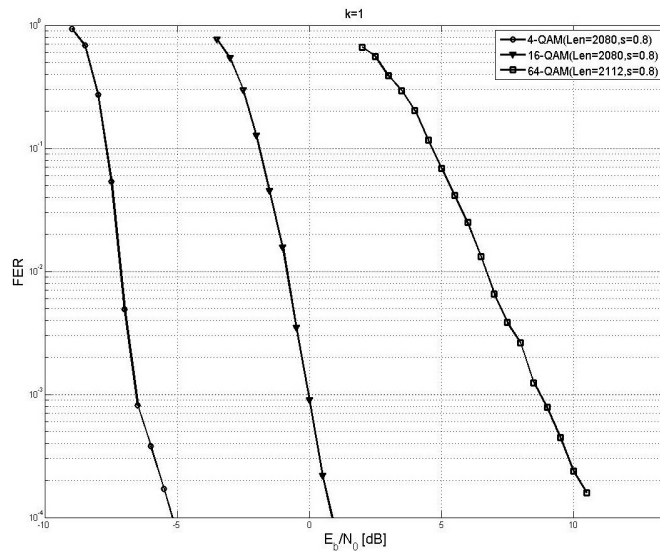


Fig. 6 – FER performances for 4-QAM, 16-QAM and 64-QAM when  $k = 1$ .

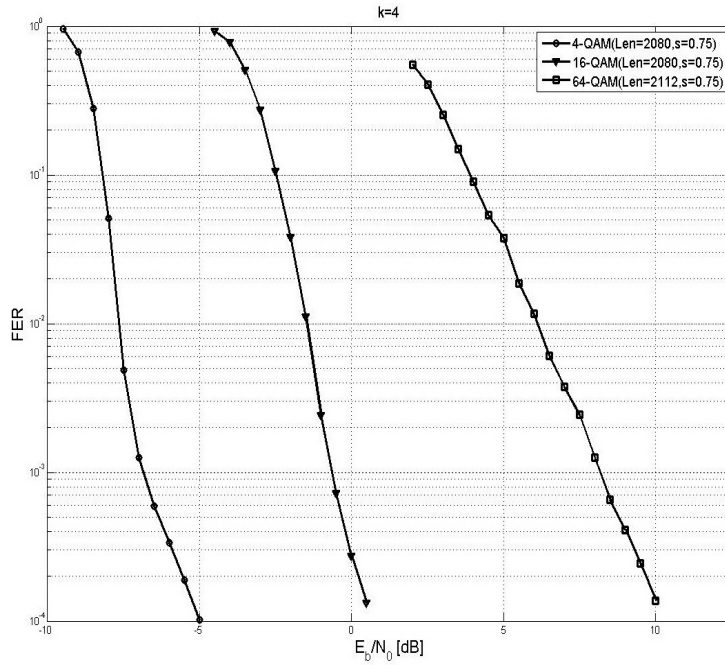


Fig. 7 – FER performances for 4-QAM, 16-QAM and 64-QAM when  $k = 4$ .

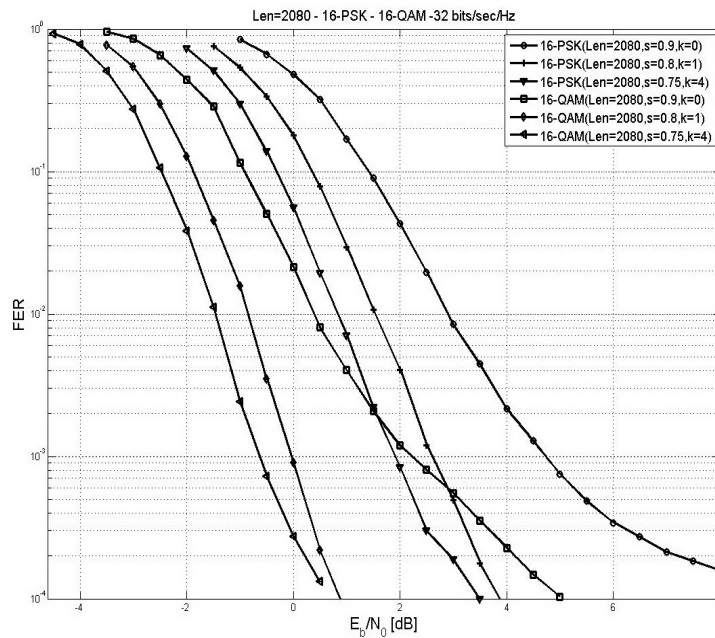


Fig. 8 – FER performances for 16-PSK and 16-QAM for  $k = 0, 1, 4$ .

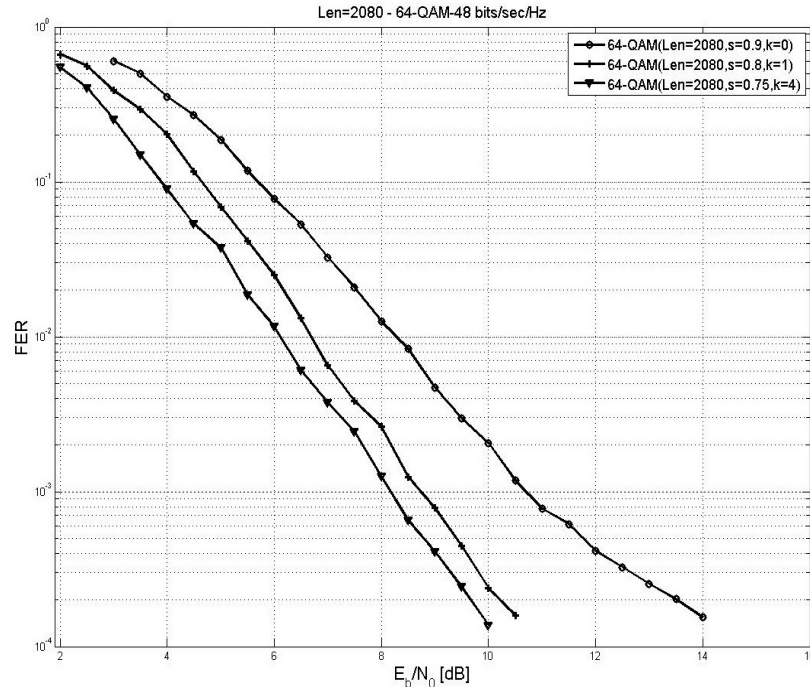


Fig. 9 – FER performances for 64–QAM for  $k = 0, 1, 4$ .

## 6. Conclusions

In this paper we determined and explained the soft estimates needed for a doubly iterative decoder for space-time turbo codes on a quasi-static fading channel, considering 16–QAM and 64–QAM modulations, respectively. Transmitting more bits per symbol is possible if the order of modulation is increased. In this way, much higher data rates and better spectral efficiency are obtained for transmission communication systems. Hence, for 16 transmit antennas, the spectral efficiency is increased, from 16 bits/s/Hz for 4–QAM modulation to 32 bits/s/Hz for 16–QAM and to 48 bits/s/Hz for 64–QAM modulation. A disadvantage is that the increased order of QAM modulation introduces a degradation of the FER performances.

The simulations were performed for a Max-Log-APP turbo decoding algorithm in a system using a spatial interference canceling interface (iterative MMSE receiver). We used a doubly-iterative decoding process, scaling both the extrinsic information of the turbo decoder and the information at the input of the interference canceling block. The number of iterations are  $k = 0$ ,  $k = 1$  or  $k = 4$  used to cancel spatial interference. The scaling coefficient used for performing the simulations is  $s = 0.9$  for  $k = 0$ ,  $s = 0.8$  for  $k = 1$ , and  $s = 0.75$  for  $k = 4$ . Increasing the number of iterations the FER performances are improved. So, for 4–QAM a supplementary coding gain up to 3.31 dB can be achieved, for

16-QAM up to 3.93 dB and for 64-QAM up to 3.85 dB. The degradation of the FER performances is shown in simulation results, as we can see a large difference in coding gain between modulations, up to 6.44 dB for 4-QAM compared to 16-QAM and up to 9.68 dB for 16-QAM compared to 64-QAM.

#### REFERENCES

- Benedetto S., Divsalar D., Montorsy G., Pollara F., *A Soft-Input Soft-Output APP Module for Iterative Decoding of Concatenated Codes*, IEEE Communications Letters, **1**, 1, 22-24 (1997)
- Biglieri E., Nardio A., Taricco G., *Suboptimum Receiver Interfaces And Space-Time Codes*. IEEE Transactions on Communications, IEEE Transactions on Signal Processing, **53**, 5, 773-779 (2003).
- Biglieri E., Nardio A., Taricco G., *Doubly Iterative Decoding of Space-Time Turbo Codes with a Large Number of Antennas*, IEEE Trans. on Communications, **53**, 5, 773-779 (2005).
- Caire G., Taricco G., Biglieri E., *Bit-Interleaved Coded Modulation*, IEEE Trans. on Information Theory, **44**, 3, 927-946 (1998).
- Rotopănescu A.-M., Trifina L., Tărniceriu D., *Soft Estimates for Doubly Iterative Decoding with 8-PSK and 16-PSK Modulations*, Frequenz, **66**, 3-4, 101-107 (2012).
- Trifina L., Tărniceriu D., Rotopănescu A.-M., *Influence of Extrinsic Information Scaling Coefficient on Doubly-Iterative Decoding Algorithm for Space-Time Turbo Codes with Large Number of Antennas*, Advances in Electrical and Computer Engineering, **11**, 1, 85-90 (2011).
- Stefanov A., Duman T. M., *Turbo-Coded Modulation for Systems with Transmit and Receive Antenna Diversity over Block Fading Channels: System Model, Decoding Approaches, and Practical Considerations*, IEEE J. on Selected Areas in Communications, **19**, 5, 958-968 (2001).

#### ESTIMAȚII SOFT PENTRU DECODARE DUBLU ITERATIVĂ PENTRU MODULAȚIA 16 – QAM ȘI 64 – QAM

(Rezumat)

În scopul creșterii eficienței spectrale, o modulație codată cu intercalarea biților poate fi combinată cu o modulație de ordin ridicat cum ar fi Phase Shift Keying (PSK) și Quadrature Amplitude Modulation (QAM), îmbunătățind astfel performanța sistemului. În acest articol, derivăm și explicăm estimații soft a decodului dublu iterativ folosind codurile turbo spațio – temporale și un număr mare de antene de transmisie și recepție pentru modulația 16-QAM și 64-QAM utilizată într-un sistem de comunicații mobile.

