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ANALYSIS OF THE POWER TRANSMITTED BY TRANSVERSE ELECTROMAGNETIC WAVES IMPINGING ON THE BOUNDARY BETWEEN CHARGED, LOSSY DIELECTRICS

BY

CAMELIA PETRESCU*

Technical University “Gheorghe Asachi” of Iaşi,
Faculty of Electrical Engineering

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Abstract. The paper presents a study of the refraction of transverse electromagnetic (TEM) waves incident on the plane boundary between charged and lossy dielectrics. Using the expressions for the transmission angle, derived in a previous paper, and the expressions for wave impedances, the active and reactive powers transmitted by the wave are established, and a numerical simulation is conducted in order to assess the influence of the incidence angle and media parameters on the efficiency of active power transmission.

Key words: charged dielectrics; refraction; efficiency of power transmission; transmission angle.

1. Introduction

Refraction and reflection of TEM waves with oblique incidence are well documented in electromagnetic field textbooks and supported by scientific papers for the case when the two media, that of the incident and that of the refracted wave, are both ideal dielectrics, or one is an ideal dielectric and the other one is a perfect conductor. These two cases pose no theoretical difficulty.

*Corresponding author: *e-mail*: campet@tuiasi.ro

When at least one of the two media is lossy (real dielectric, non-ideal conductor) a theoretical paradox occurs, leading to a complex refraction angle. This paradox is mentioned by Miner (1996), and further addressed by de Roo & Tai (2003), Petrescu (2005), McMichael (2010).

The parameters of the TEM waves (complex propagation constant, wave impedance) for lossy dielectrics and non-ideal conductors are well established in electromagnetic field theory. In Petrescu (2005, 2016) a modified expression for the complex propagation constant in the second medium was proposed in order to overcome the aforementioned paradox.

Furthermore (de Roo & Tai, 2003) it is demonstrated that in the case of lossy media an elliptically polarized wave occurs in the second medium. The authors establish the expressions for the Poynting vector of the refracted wave and argue that the direction of $\underline{\mathbf{S}}$ for the transmitted wave is more meaningful in terms of the direction of propagation.

The study of TEM waves with oblique incidence is further addressed by McMichael (2010), in connection with an application called ground penetrating radar (GPR), in this case medium 1 being air, and medium 2 soil with varying degrees of humidity. The author does not attempt to reduce the paradox of a complex transmission angle, but focuses on the determination of the pseudo−Brewster angle for which the reflection coefficient is close to zero.

In Ji & Varadan (2015), the conditions for negative or positive phase and energy refraction are analysed. The authors emphasize the fact that Snell's law does not apply for lossy media and that the refraction angles must be calculated separately for the phase angle and for the energy (Poynting vector). They conclude that the occurrence of negative phase refraction (negative phase constant) does not depend on the type of wave polarization, and that negative energy refraction (refracted wave in medium 2 on the same "side" as the incident wave) may be obtained if the second medium has negative permittivity (in the case of parallel polarization), and, in the case of perpendicular polarization, medium 2 must have a negative permeability.

Tretyakov and Maslovski analyse in their paper (Tretyakov & Maslovski, 2003) the reflection and refraction of parallel and perpendicular linearly polarized waves impinging on an array of metal patches disposed on a lossy dielectric layer, intended to work as a wave absorber. The dependence of the absolute value of the reflection coefficient on frequency and incidence angle is studied. The authors conclude that in the case of parallel (TM) polarized waves the structure attains a minimum reflection coefficient at approximately the same frequency for all incidence angles.

This paper continues the study presented by Petrescu (2016), focussing on the active power transmitted by the TEM wave that propagates through lossy, possibly charged dielectrics and weak conductors. The problem analysed in this paper presents both a theoretical and a practical importance for TEM waves propagating from air (possibly ionized) to water or soil, with applications in underwater and underground communications.

2. Problem Formulation

Let us consider two semi-infinite domains separated by a plane surface. Domain 1 is a lossy dielectric with the complex permittivity

$$\underline{\varepsilon}_1 = \varepsilon' - \mathrm{j}\,\varepsilon''. \tag{1}$$

A space charge with the volume density $\rho_V = Nq$, where N is the number of charges per unit volume and q is the elementary charge, is present in the dielectric. As demonstrated by Petrescu (2016), in a sinusoidal varying electric field, the presence of spatial charges modifies the expression of the complex permittivity which becomes:

$$\varepsilon_{1\ \mathrm{eff}} = \varepsilon_1 - \frac{\rho_v q}{\omega^2 m}, \tag{2}$$

where: ω is the angular frequency and m – the mass of the elementary charge q.

The pde Helmholtz equation for complex magnetic field intensity is:

$$\Delta\underline{\mathbf{H}} + \underline{\gamma}_1^2\,\underline{\mathbf{H}} = 0 \tag{3}$$

and allows the determination of the complex propagation constant $\underline{\gamma}_1$:

$$\underline{\gamma}_1 = \alpha_1 + \mathrm{j}\beta_1 = \mathrm{j}\,\omega\sqrt{\mu\left(\varepsilon' - \frac{\rho_v q}{\omega^2 m} - \mathrm{j}\,\varepsilon''\right)}. \tag{4}$$

Thus the attenuation α_1 and the phase constant β_1 for the losssy dielectric with a spatial charge distribution have, respectively, the expressions:

$$\alpha_1 = \omega\sqrt{\frac{\mu\left[\left(-\varepsilon' + \frac{\rho_v q}{\omega^2 m}\right) + \sqrt{\left(\varepsilon' - \frac{\rho_v q}{\omega^2 m}\right)^2 + \left(\varepsilon''\right)^2}\right]}{2}}, \tag{5}$$

$$\beta_1 = \omega\sqrt{\frac{\mu\left[\left(\varepsilon' - \frac{\rho_v q}{\omega^2 m}\right) + \sqrt{\left(\varepsilon' - \frac{\rho_v q}{\omega^2 m}\right)^2 + \left(\varepsilon''\right)^2}\right]}{2}}. \tag{6}$$

It is to be noted that an ideal, charge free dielectric is characterized by

$$\alpha = 0\ ,\quad \beta = \omega\sqrt{\mu\varepsilon}\ , \tag{7}$$

while for an ideal dielectric with spatial charge these parameters are:

$$\alpha = 0\quad ,\quad \beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \frac{\rho_v q}{\omega^2 \varepsilon m}}\ , \tag{8}$$

indicating a dispersive medium.

The wave impedance for the lossy dielectric with space charges is:

$$\underline{\varsigma}_1 = \sqrt{\frac{\mu}{\underline{\varepsilon}_{\text{eff}}}} = \sqrt{\frac{\mu}{\varepsilon' - \frac{\rho_v q}{\omega^2 m} - \mathrm{j}\varepsilon''}} = R_1 + \mathrm{j}X_1 \tag{9}$$

Domain 2 is a weak conductor (eg. sea water or soil with different degrees of humidity) and has the material constants ε_2, μ_0, σ_2. The complex propagation constant is (Sadiku, 1995)

$$\underline{\gamma}_2 = \alpha_2 + \mathrm{j}\beta_2 = \sqrt{\mathrm{j}\,\omega\mu\left(\sigma_2 + \mathrm{j}\,\omega\underline{\varepsilon}_2\right)} \tag{10}$$

and the wave impedance has the expression

$$\underline{\varsigma}_2 = \frac{\mathrm{j}\omega\mu}{\underline{\gamma}_2} = R_2 + \mathrm{j}X_2\,. \tag{11}$$

Consider a linearly polarized TEM wave propagating from medium 1 that suffers refraction and reflection on the boundary that separates the two media.

In the case of parallel, transverse magnetic (TM) polarization the field vectors, $\underline{\boldsymbol{E}}$ and $\underline{\boldsymbol{H}}$, and the Poynting vector for the refracted and reflected waves are represented in Fig. 1.

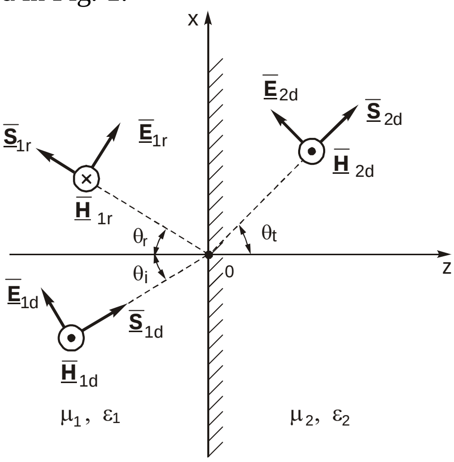


Fig. 1 – Refraction of parallel polarized TEM wave.

The expressions of $\underline{\boldsymbol{E}}$ and $\underline{\boldsymbol{H}}$ for the incident and refracted waves are:

$$\begin{cases} \underline{\mathbf{E}}_{1\mathrm{d}} = \underline{E}_{\mathrm{i}0}\,\mathrm{e}^{-\underline{\gamma}_1\left(x\sin\theta_i + z\cos\theta_i\right)}\left(\cos\theta_i\,\bar{\mathbf{i}} - \sin\theta_i\,\overline{\mathbf{k}}\right) \\ \underline{\mathbf{H}}_{1\mathrm{d}} = \dfrac{\underline{E}_{\mathrm{i}0}}{\underline{\varsigma}_1}\mathrm{e}^{-\underline{\gamma}_1\left(x\sin\theta_i + z\cos\theta_i\right)}\,\bar{\mathbf{j}} \end{cases} \tag{12}$$

$$\begin{cases} \underline{\mathbf{E}}_{2\mathrm{d}} = \underline{E}_{20}\,\mathrm{e}^{-\mathrm{j}\beta_2(x\sin\theta_t + z\cos\theta_t)}\left(\cos\theta_t\bar{\mathbf{i}} - \sin\theta_t\bar{\mathbf{k}}\right) \\ \underline{\mathbf{H}}_{2\mathrm{d}} = \dfrac{\underline{E}_{20}}{\underline{\varsigma}_2}\mathrm{e}^{-\mathrm{j}\beta_2(x\sin\theta_t + z\cos\theta_t)}\bar{\mathbf{j}} \end{cases}. \tag{13}$$

For a TEM wave with perpendicular, transverse electric (TE) polarization (Fig. 2) the field expressions are:

$$\begin{cases} \underline{\mathbf{E}}_{1\mathrm{d}} = \underline{E}_{\mathrm{i}0}\,\mathrm{e}^{-\underline{\gamma}_1(x\sin\theta_i + z\cos\theta_i)}\bar{\mathbf{j}} \\ \underline{\mathbf{H}}_{1\mathrm{d}} = \dfrac{\underline{E}_{\mathrm{i}0}}{\underline{\varsigma}_1}\mathrm{e}^{-\underline{\gamma}_1(x\sin\theta_i + z\cos\theta_i)}\left(-\cos\theta_i\bar{\mathbf{i}} + \sin\theta_i\bar{\mathbf{k}}\right) \end{cases} \tag{14}$$

$$\begin{cases} \underline{\mathbf{E}}_{2\mathrm{d}} = \underline{E}_{20}\,\mathrm{e}^{-\underline{\gamma}_2(x\sin\theta_t + z\cos\theta_t)}\bar{\mathbf{j}} \\ \underline{\mathbf{H}}_{2\mathrm{d}} = \dfrac{\underline{E}_{i0}}{\underline{\varsigma}_2}\mathrm{e}^{-\underline{\gamma}_2(x\sin\theta_t + z\cos\theta_t)}\left(-\cos\theta_t\bar{\mathbf{i}} + \sin\theta_t\bar{\mathbf{k}}\right) \end{cases} \tag{15}$$

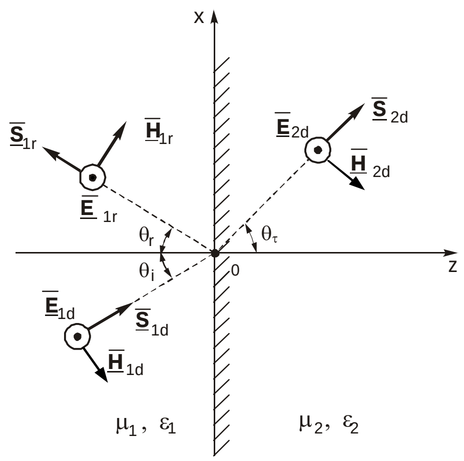


Fig. 2 – Refraction of perpendicular polarized TEM wave.

The complex Poynting vectors for the incident and refracted waves for both types of polarizations have the expressions:

$$\underline{\mathbf{S}}_{1\mathrm{d}} = \underline{\mathbf{E}}_{1\mathrm{d}} \times \underline{\mathbf{H}}_{1\mathrm{d}}^{*} = \frac{E_{i0}^{2}}{\underline{\zeta}_1^{*}}\mathrm{e}^{-2\alpha_1(x\sin\theta_i + z\cos\theta_i)}\left(\cos\theta_i\bar{\mathbf{k}} + \sin\theta_i\bar{\mathbf{i}}\right) \tag{16}$$

$$\underline{\mathbf{S}}_{2\mathrm{d}} = \underline{\mathbf{E}}_{2\mathrm{d}} \times \underline{\mathbf{H}}_{2\mathrm{d}}^{*} = \frac{E_{20}^{2}}{\underline{\zeta}_2^{*}}\mathrm{e}^{-2\alpha_2(x\sin\theta_t + z\cos\theta_t)}\left(\cos\theta_t\bar{\mathbf{k}} + \sin\theta_t\bar{\mathbf{i}}\right) \tag{17}$$

If the complex admittances of the two media are denoted by:

$$1/\underline{\zeta}_1 = G_1 - \mathrm{j}B_1, \qquad 1/\underline{\zeta}_2 = G_2 - \mathrm{j}B_2 \tag{18}$$

and taking into account that the complex apparent power is

$$\underline{S} = P + \mathrm{j}Q \tag{19}$$

the expressions of the active and reactive powers per unit surface, carried by the incident and by the refracted waves in directions *z* and *x* are:

$$\begin{aligned} P_{1\mathrm{d}z} &= G_1 E_{i0}^2 \,\mathrm{e}^{-2\alpha_1(x\sin\theta_i + z\cos\theta_i)} \cos\theta_i = G_1 A\cos\theta_i \\ P_{1\mathrm{d}x} &= G_1 E_{i0}^2 \,\mathrm{e}^{-2\alpha_1(x\sin\theta_i + z\cos\theta_i)} \sin\theta_i = G_1 A\sin\theta_i \end{aligned} \tag{20}$$

$$\begin{aligned} Q_{1\mathrm{d}z} &= B_1 E_{i0}^2 \,\mathrm{e}^{-2\alpha_1(x\sin\theta_i + z\cos\theta_i)} \cos\theta_i = B_1 A\cos\theta_i \\ Q_{1\mathrm{d}x} &= B_1 E_{i0}^2 \,\mathrm{e}^{-2\alpha_1(x\sin\theta_i + z\cos\theta_i)} \sin\theta_i = B_1 A\sin\theta_i \end{aligned} \tag{21}$$

$$\begin{aligned} P_{2\mathrm{d}z} &= G_2 E_{20}^2 \,\mathrm{e}^{-2\alpha_2(x\sin\theta_t + z\cos\theta_t)} \cos\theta_t = G_2 C\cos\theta_t \\ P_{2\mathrm{d}x} &= G_2 C\sin\theta_t \end{aligned} \tag{22}$$

$$\begin{aligned} Q_{2\mathrm{d}z} &= B_2 C\cos\theta_t \\ Q_{2\mathrm{d}x} &= B_2 C\sin\theta_t \end{aligned} \tag{23}$$

It is to be noted that in lossless dielectrics, with or without space charges, the TEM wave carries only active power since the susceptance is zero. In lossy dielectrics and weak conductors, since $B \neq 0$, the wave also carries a reactive power. In the case of normal incidence, when $\theta_i = \theta_t = 0$ the active and reactive powers in direction *Ox* are zero.

3. Determination of the Transmission Angle

In lossless dielectrics the transmission angle θ_t is determined using Snell's law

$$\theta_t = \arcsin\left(\frac{\beta_1}{\beta_2}\sin\theta_i\right). \tag{24}$$

In the case of lossy dielectrics and non−ideal conductors, by imposing the condition of continuity for tangential $\underline{\mathbf{H}}$ and tangential $\underline{\mathbf{E}}$ components at any point on the boundary between the two media, the relation between the incidence and the transmission angle results of the form:

$$\underline{\gamma}_1 \sin\theta_i = \underline{\gamma}_2 \sin\theta_t\,, \tag{25}$$

so that

$$\theta_t = \arcsin\left(\frac{\underline{\gamma}_1}{\underline{\gamma}_2}\sin\theta_i\right). \tag{26}$$

If either of the two media is lossy, the previous relation indicates a complex value of θ_t. Although attempts have been made to resolve this theoretical paradox, in this paper the case of dielectrics with small loss tangent and that of weak conductors is considered, which means that $\alpha_1 << \beta_1$ and $\alpha_2 << \beta_2$. In this hypothesis the transmission angle can be obtained using the approximate formula:

$$\theta_t = \arcsin\left(\left|\frac{\underline{\gamma}_1}{\underline{\gamma}_2}\right|\sin\theta_i\right), \tag{27}$$

which still takes into account the losses produced by conduction and dipole relaxation phenomena. Such an approximation is accepted in technical literature for comparison reasons in different case studies (McMichael, 2010).

4. Results and Discussions

Using the relations established in &.2, a study was conducted regarding active power transmission from air, ionized or not, to water with different degrees of salinity and to earth with different degrees of humidity.

The efficiency of active power transmission in direction Oz normal to the boundary, defined as:

$$\eta = \frac{P_{2\text{dz}}}{P_{1\text{dz}}} = \frac{G_2}{G_1}\left|\tau\right|^2\frac{\cos\theta_t}{\cos\theta_i} \tag{28}$$

was also calculated. In eq. (28) τ is the transmission coefficient defined as $\tau = E_{20}/E_{i0}$.

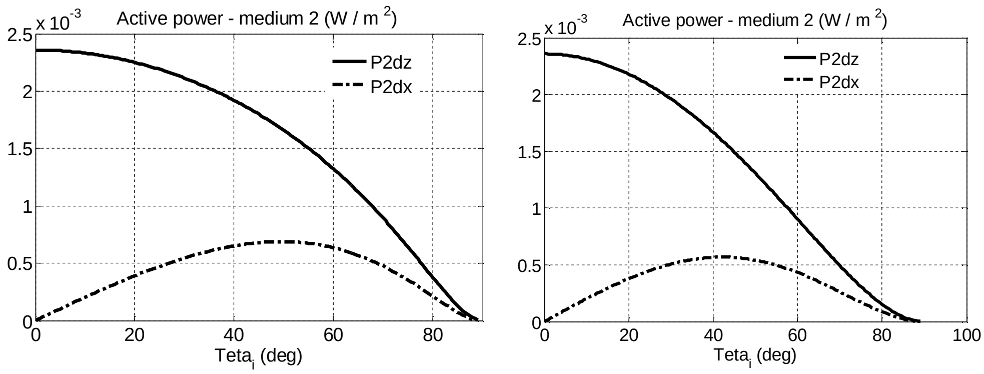
The numerical values used in simulations were: ε_{r1} = 1.006, ε_{r2} = 78.8 (for distilled water), $\underline{\varepsilon}_{r2} = 3 - 0.2j$, for soil with humidity 0.8%, $\varepsilon_{r2} = 19 - 17j$ for soil with relative humidity 30.3% (McMichael, 2010), σ_2 = 0 (distilled water) or σ_2 = 2.2 S/m (salt sea water), $q = 1.602 \times 10^{-12}$ C (elementary charge), $\rho_v = 200 \times 10^6 q$, $m = 1.672 \times 10^{-27}$ kg (proton mass); the frequency was f = = 2 MHz and f = 200 MHz. The cases outlined in the following paragraphs were considered.

a) Medium 1 – nonionized air, medium 2 – ideal dielectric with ε_{r2} = 4. Fig. 3 *a* and 3 *b* present the active power transmitted in direction *Oz*, $P_{2\text{dz}}$ and in direction *Ox*, $P_{2\text{dx}}$, as a function of the incidence angle θ_i, for parallel and perpendicular polarization. The plots show, as expected, that the active power is mainly transmitted in direction *Oz* and that it vanishes when θ_i = 90°.

Fig. 4 *a* and 4 *b* plot the active power in direction *Oz* in the two media, $P_{1\text{dz}}$, $P_{2\text{dz}}$ versus θ_i for parallel (TM) and perpendicular (TE) polarization, respectively. In the case of parallel polarization $P_{1\text{dz}}$ equals $P_{2\text{dz}}$ for an angle $\theta_{iB} \approx 60°$ which represents the Brewster angle (total transmission). Of course, in the case of perpendicular polarization, for media with equal permeabilities ($\mu_1 = = \mu_2 = \mu_0$) total transmission cannot be attained.

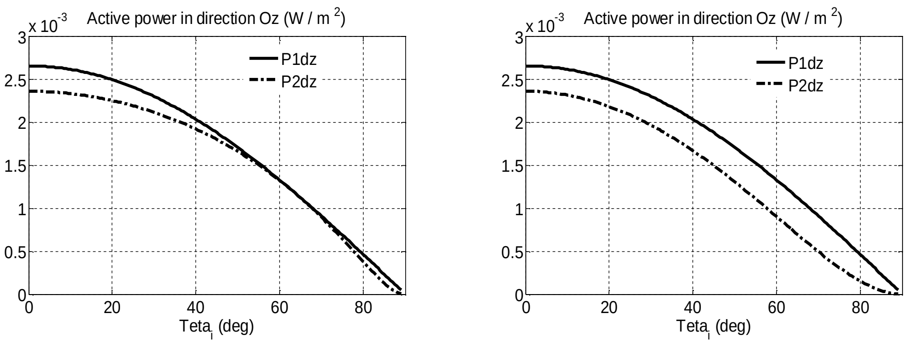
Finally, Fig. 5 *a* and 5 *b* present the efficiency η for the TM and TE waves. In the case of a TM wave the efficiency reaches unity value for $\theta_i = \theta_{iB}$ (total transmission), while for the TE wave η decreases with θ_i, as expected.

b) Medium 1 is ionized air and medium 2 is a dielectric with small losses $\underline{\varepsilon}_{r2}$ = 78.8 (1 – 0.04*j*) and small conductivity σ_2 = 2.2 S/m. Since the power transmission efficiency is very much influenced by the transmission coefficient τ, Fig. 6 plots the dependence $\tau(\theta_i)$. As may be observed, τ has only a small variation in the case of TM polarization and decreases for TE polarization, showing that parallel polarization is the preferred choice for power transmission to the second lossy medium.



a) parallel polarization *b*) perpendicular polarization

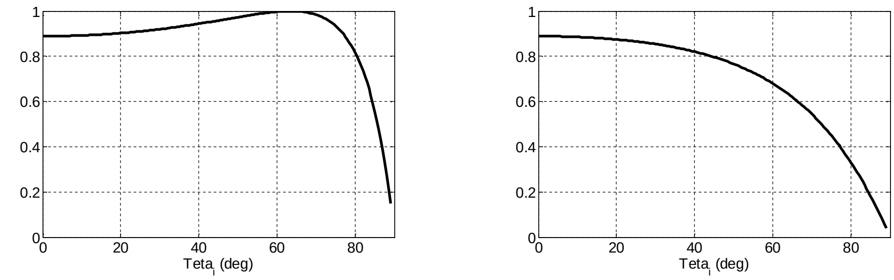
Fig. 3 – Active power transmitted in medium 2.



a) parallel polarization *b*) perpendicular polarization

Fig. 4 – Active power in direction *Oz* in the two media.

The efficiency of power transmission for the TM wave is plotted in Fig. 7, considering that medium 2 is distilled water, and afterwards water with a small conductivity σ_2 = 2.2 S/m. As may be seen the efficiency decreases dramatically when medium 2 is salt water compared to the distilled water case.



a) parallel polarization *b*) perpendicular polarization

Fig. 5 – Efficiency of power transmission in direction *Oz*.

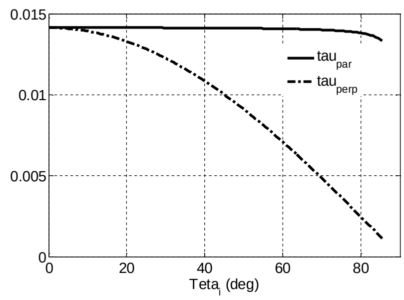


Fig. 6 – Transmission coefficient.

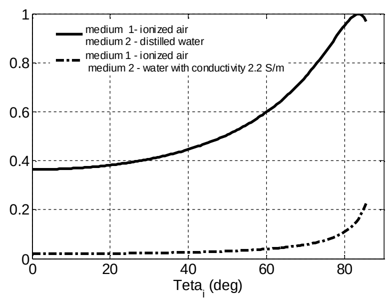


Fig. 7 − Efficiency of power transmission.

c) Medium 1 is ionized air and medium 2 is soil with two degrees of humidity: 0.8 % (dry soil) and 30.3% (wet soil) (Fig. 8).

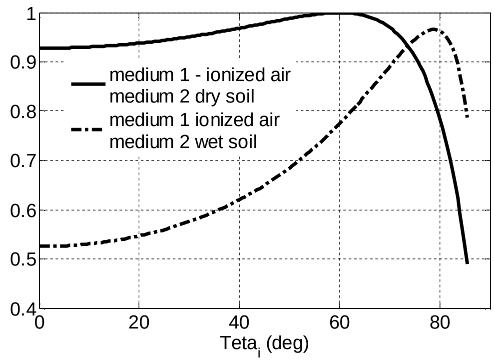


Fig. 8 − Efficiency of power transmission in direction *Oz*: air to dry soil and to wet soil.

The results show that soil humidity greatly decreases the efficiency of power transmission for the TM polarized wave. Total transmission (η = 1) can be attained when medium 2 is dry soil, but for a wet soil only a quasi−Brewster angle is obtained (max{η} < 1).

5. Conclusions

The analysis performed in this paper shows that parallel polarization must be used in order to obtain a higher efficiency of power transmission for lossless as well as for lossy media. Unity value of the efficiency can be obtained for the incidence angle $\theta_i = \theta_{iB}$ (Brewster angle) in the case of parallel (TM) polarization. The presence of losses (increased conductivity or loss angle of the second medium) greatly impairs power transmission.

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ANALIZA PUTERII TRANSMISE DE UNDA TEM REFRACTATĂ PE SUPRAFAŢA DE SEPARAŢIE DINTRE DOI DIELECTRICI CU PIERDERI, INCĂRCAŢI CU SARCINĂ ELECTRICĂ

(Rezumat)

Se prezintă un studiu al refracţiei undelor TEM la suprafaţa de separaţie dintre un dielectric cu pierderi, eventual încărcat cu sarcină electrică şi un mediu slab conductor. Utilizând expresiile pentru unghiul de refracţie (stabilite într-o lucrare anterioară), precum şi pe cele ale impedanţelor de undă ale celor două medii, se determină puterile active şi reactive pentru unda incidentă şi cea refractată, determinându-se influenţa unghiului de incidenţă şi a constantelor de mediu asupra randamentului de transmitere a puterii active.