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PARK TRANSFORMATION FOR AN ASYMMETRIC SYSTEM

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Abstract. In 2000 the method of symmetrical components (Fortescue) and the Park Transformation was voted the works with the greatest impact in the last century on electrical and energetic engineering. In the study of a.c. electrical machines the Park Transformation is a classical tool that allows the use of the d-q model. This paper aims to analyze to what extent and with what costs the Park Transformation can be used in the transformation of an asymmetrical threephase system. Three different cases have been identified in which transformation changes differently the three-phase system. The final conclusion is that in a general case Park Transformation changes to some extent the characteristics of the three-phase system and the results obtained by using this transformation must be regarded critically from case to case.

Key words: symmetrical components; Park transformation; PSpice; Simulink; electrical machines.

1. Introduction

In 2000 the organizers of the North American Power Symposium choose to recognize accomplishments of the past by assessing the written contribution to electric power engineering. Over 50 nominations were received as high impact paper of the years 1900-1999 (Heydt *et al.*, 2000). The list was reduced to 39 nominations. Finally the first two places were voted as:

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- 1. C. Fortescue, Methode of Symmetrical Coordinates Applied to the Solution of Poliphase Networks, 1918.
- 2. R. Park, Two Reaction Theory of Synchronous Machines, 1929.

Analyzing the rotating magnetic field in electrical machines, L. G. Stokvis has the idea to decompose the field into a positive rotating part, a negative rotating part and a pulsating field. In 1918 Charles Legeyt Fortescue extended the concept to voltages and currents. Fortescue expanded the method to unbalanced three phase circuits and proposed the term of *symmetrical coordinates*. He demonstrated that any set of unbalanced three-phase quantities could be expressed as the sum of three symmetrical sets of balanced phasors (Fortescue, 1918). Using this tool, unbalanced system conditions, like those caused by common fault types may be visualized and analyzed. The method of symmetrical components is usually used in power systems to simplify the analysis of unbalanced three-phase systems. In the most common case of three-phase system, the resulting symmetrical components are referred to as positive (or direct), negative (or inverse) and zero (or homopolar).

A study on the possibilities of using symmetrical components method in the analysis of induction machine behavior is presented in (Cociu & Cociu, 2014). Starting from the observation that induction machine is generally described by a system of nonlinear equations the authors aim to determine the circumstances in which this method can be used.

Park presented an extension to the work of Blondel, Dreyfus and Nickle. The salient aspect of this work is a novel transformation that transforms linear differential equations with time varying coefficients to linear differential equations with time invariant coefficients.

Starting from a three-phase sinusoidal signal, the Park transformation allows determining the direct and quadratic components, and also, if necessary, zero sequence component in a two-axis (rotating) reference frame (Park, 1929). In the electrical machines study, both the change in the number of sinusoidal quantities from three to two and the rotation of the reference frame are very useful. The transformation is used in the study and analysis of both synchronous and asynchronous electrical machines.

To perform the analysis of the three-phase induction machine supplied from a non-sinusoidal, unbalanced three-phase voltage or current source in (Cociu & Cociu, 2015) is presented a bidirectional separator circuit that can be used in Park transformation implementation.

Typically, when using the Park transformation, a symmetrical sine wave system is used as input. But applying this transformation is not restricted to the symmetrical character. Basically it can be applied to any three-phase system. Actually the Stokvis-Fortescue method in an asymmetric system is a Park transformation of all positive, negative and zero symmetrical components.

This paper aims to study whether using the Park transformation changes the characteristics of an asymmetrical system and if so to what extent and what characteristics are affected. The original system is decomposed in symmetrical components to make easier the calculus of the transformed quantities. Consider a three-phase (asymmetric) system of sinusoidal quantities of the same angular frequency ω and some value for the amplitude and phase, generally different for each quantity:

$$y_{a}(t) = \sqrt{2}Y_{a}\sin(\omega t + \gamma_{a});$$

$$y_{b}(t) = \sqrt{2}Y_{b}\sin(\omega t + \gamma_{b});$$

$$y_{c}(t) = \sqrt{2}Y_{c}\sin(\omega t + \gamma_{c}),$$

(1)

and their corresponding complex phasors:

$$\underline{Y}_{a} = Y_{a} e^{jY_{a}};$$

$$\underline{Y}_{b} = Y_{b} e^{jY_{b}};$$

$$\underline{Y}_{c} = Y_{c} e^{jY_{c}}.$$
(2)

According to symmetrical components method, these can be expressed as:

$$\underline{Y}_{a} = \underline{Y}_{ap} + \underline{Y}_{an} + \underline{Y}_{0};$$

$$\underline{Y}_{b} = \underline{Y}_{bp} + \underline{Y}_{bn} + \underline{Y}_{0};$$

$$\underline{Y}_{c} = \underline{Y}_{cp} + \underline{Y}_{cn} + \underline{Y}_{0}.$$
(3)

The system is uniquely determined by the vectors:

$$\underline{\underline{Y}}_{p} = (\underline{\underline{Y}}_{a} + \underline{\underline{a}}\underline{\underline{Y}}_{b} + \underline{\underline{a}}^{2}\underline{\underline{Y}}_{c}) / 3 = \underline{\underline{Y}}_{ap};$$

$$\underline{\underline{Y}}_{n} = (\underline{\underline{Y}}_{a} + \underline{\underline{a}}^{2}\underline{\underline{Y}}_{b} + \underline{\underline{a}}\underline{\underline{Y}}_{c}) / 3 = \underline{\underline{Y}}_{an};$$

$$\underline{\underline{Y}}_{0} = (\underline{\underline{Y}}_{a} + \underline{\underline{Y}}_{b} + \underline{\underline{Y}}_{c}) / 3,$$
(4)

where: \underline{a} is a rotation phasor operator which rotates a phasor counterclockwise by 120 degree.

Now consider a two-phase (asymmetric) system of sinusoidal quantities of the same angular frequency ω and some value for the amplitude and phase, generally different for each quantity:

$$y_{d}(t) = \sqrt{2}V_{d}\sin(\omega t + \gamma_{d});$$

$$y_{q}(t) = \sqrt{2}V_{q}\sin(\omega t + \gamma_{q}),$$
(5)

and their corresponding complex phasors:

$$\underline{Y}_{d} = Y_{x} e^{j\gamma_{d}};$$

$$\underline{Y}_{q} = Y_{y} e^{j\gamma_{q}}.$$
(6)

According to symmetrical components method, these can be expressed as:

$$\underline{\underline{Y}}_{d} = \underline{\underline{Y}}_{dp} + \underline{\underline{Y}}_{dn};$$

$$\underline{\underline{Y}}_{q} = \underline{\underline{Y}}_{qp} + \underline{\underline{Y}}_{qn}.$$
(7)

The system is uniquely determined by the vectors:

$$\underline{\underline{Y}}_{p2ph} = (\underline{\underline{Y}}_d + \underline{j}\underline{\underline{Y}}_q) / 2 = \underline{\underline{Y}}_{dp};$$

$$\underline{\underline{Y}}_{n2ph} = (\underline{\underline{Y}}_d - \underline{j}\underline{\underline{Y}}_q) / 2 = \underline{\underline{Y}}_{dn}.$$
(8)

In the two-phase system there is no homopolar component. It has only a marginal effect in the study of electrical machines and it is studied outside the two-phase model.

2. Clark Transformation of an Unbalanced System

At the beginning consider the particular case of a phase transformation (from three to two components) without modifying the reference system, that is a Park transformation of zero angle (Clarke transformation). The transformation matrix is:

$$P[0] = \gamma \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$
(9)

If coefficient $\gamma = \sqrt{2/3}$ the transformation is power invariant. The amplitude of the two phase components are $\sqrt{3/2}$ times greater. Using $\gamma = 2/3$ the transformation is not power invariant but keep the same amplitude.

$$\begin{bmatrix} \underline{Y}_{d} \\ \underline{Y}_{q} \\ \underline{Y}_{0} \end{bmatrix} = \gamma \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} \underline{Y}_{ap} + \underline{Y}_{an} + \underline{Y}_{0} \\ \underline{Y}_{bp} + \underline{Y}_{bn} + \underline{Y}_{0} \\ \underline{Y}_{cp} + \underline{Y}_{cn} + \underline{Y}_{0} \end{bmatrix}.$$
(10)

Using (10) the direct component expression results as:

$$\underline{Y}_{d} = \gamma \cdot \left(\underline{Y}_{a} - \frac{1}{2} \underline{Y}_{b} - \frac{1}{2} \underline{Y}_{c} \right) =$$

$$= \gamma \cdot \left[\underline{Y}_{ap} + \underline{Y}_{an} + \underline{Y}_{0} - \frac{1}{2} \left(\underline{Y}_{bp} + \underline{Y}_{bn} + \underline{Y}_{0} \right) - \frac{1}{2} \left(\underline{Y}_{cp} + \underline{Y}_{cn} + \underline{Y}_{0} \right) \right] = (11)$$

$$= \gamma \cdot \left[\underline{Y}_{ap} - \frac{1}{2} \underline{Y}_{bp} - \frac{1}{2} + \underline{Y}_{cp} + \underline{Y}_{an} - \frac{1}{2} \underline{Y}_{bn} - \frac{1}{2} + \underline{Y}_{cn} \right]$$

$$\underline{Y}_{d} = \gamma \cdot \left[\underline{Y}_{ap} \left(1 - \frac{a^{2}}{2} - \frac{a}{2} \right) + \underline{Y}_{an} \left(1 - \frac{a}{2} - \frac{a^{2}}{2} \right) \right].$$

$$(12)$$

$$\underline{a} = e^{j2\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}; \quad \underline{a}^2 = e^{j4\pi/3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}.$$
 (13)

$$\underline{Y}_{d} = \gamma \cdot \frac{3}{2} \left(\underline{Y}_{ap} + \underline{Y}_{an} \right). \tag{14}$$

The quadratic component is:

$$\underline{Y}_{q} = \gamma \cdot \left(\frac{\sqrt{3}}{2} \underline{Y}_{b} - \frac{\sqrt{3}}{2} \underline{Y}_{c}\right) = \gamma \cdot \frac{\sqrt{3}}{2} (\underline{Y}_{b} - \underline{Y}_{c}) =$$

$$= \gamma \cdot \frac{\sqrt{3}}{2} (\underline{Y}_{bp} + \underline{Y}_{bn} + \underline{Y}_{0} - \underline{Y}_{cp} - \underline{Y}_{cn} - \underline{Y}_{0}) = \gamma \cdot \frac{\sqrt{3}}{2} (\underline{Y}_{bp} - \underline{Y}_{cp} + \underline{Y}_{bn} - \underline{Y}_{cn})$$

$$\underline{Y}_{y} = \gamma \cdot \frac{\sqrt{3}}{2} [\underline{Y}_{ap} (\underline{a}^{2} - \underline{a}) + \underline{Y}_{an} (a - \underline{a}^{2})].$$
(15)
(16)

$$\underline{Y}_{q} = \gamma \cdot \frac{3}{2} j \Big(-\underline{Y}_{ap} + \underline{Y}_{an} \Big). \tag{17}$$

The positive and negative sequences of the two-phase system can be determined by using (8):

$$\underline{Y}_{p\,2\,ph} = \underline{Y}_{dp} = (\underline{Y}_{d} + \mathbf{j} \cdot \underline{Y}_{q})/2 = \gamma \cdot \frac{3}{4} \left(\underline{Y}_{ap} + \underline{Y}_{an} - \mathbf{j}^{2} \underline{Y}_{ap} + \mathbf{j}^{2} \underline{Y}_{an} \right)$$
(18)

$$\underline{Y}_{p\,2\,ph} = \underline{Y}_{dp} = \gamma \cdot \frac{3}{2} \cdot \underline{Y}_{ap} \tag{19}$$

$$\underline{Y}_{n\,2ph} = \underline{Y}_{dn} = (\underline{Y}_d - \mathbf{j} \cdot \underline{Y}_q)/2 = \gamma \cdot \frac{3}{4} \left(\underline{Y}_{ap} + \underline{Y}_{an} + \mathbf{j}^2 \underline{Y}_{ap} - \mathbf{j}^2 \underline{Y}_{an} \right)$$
(20)

$$\underline{Y}_{n\,2\,ph} = \underline{Y}_{np} = \gamma \cdot \frac{3}{2} \cdot \underline{Y}_{an} \tag{21}$$

The ratio of negative to positive sequence, which is called dissymmetry factor, can be calculated:

$$\varepsilon_{n\,2\,ph} = \frac{\underline{Y}_{n\,2\,ph}}{\underline{Y}_{p\,2\,ph}} = \frac{\underline{Y}_{an}}{\underline{Y}_{ap}} = \frac{\underline{Y}_{n\,3\,ph}}{\underline{Y}_{p\,3\,ph}} = \varepsilon_{n\,3\,ph}$$
(22)

Fig. 1 shows the results obtained when transforming a three-phase system characterized by: $Y_p = 1$; $Y_n = 0.5$; $\gamma_p = \pi/6$ rad; $\gamma_n = \pi/9$ rad.

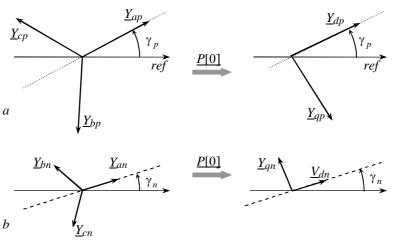


Fig. 1 – Clark transformation of symmetrical components: a – positive sequence ; b – negative sequence.

It can be noticed that for the Clarke transformation (Park of $\theta = 0$) the two symmetrical, positive and negative sequences, go through the same type of transformation and keep the amplitude and phase. Their combination yields a two-phase asymmetric system with the same properties. So, Clarke transformation does not make changes to the asymmetrical systems.

3. Park Transformation of an Unbalanced System

In Park transformation in addition with the phase transformation, the reference system rotates with an angle θ , generally different from zero. Therefore the three-phase system transformation gives: ٦

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$$\begin{bmatrix} y_x(t) \\ y_y(t) \\ y_0(t) \end{bmatrix} = \gamma \cdot \begin{bmatrix} \cos\theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin\theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} y_a(t) \\ y_b(t) \\ y_c(t) \end{bmatrix}.$$
(23)

Symmetrical components are uniquely defined by:

$$y_{p\,2ph} = \gamma \cdot \frac{3}{2} Y_{pm} \sin\left(\omega t + \gamma_p - \theta\right), \tag{29}$$

$$y_{n\,2\,ph} = \gamma \cdot \frac{3}{2} Y_{nm} \sin(\omega t + \gamma_n + \theta). \tag{30}$$

Fig. 2 shows the results obtained when transforming a three-phase system characterized by:

 $Y_p = 1$; $Y_n = 0.5$; $\omega = 100\pi \text{ rad/s}$; $\gamma_p = \pi/6 \text{ rad}$; $\gamma_n = \pi/9 \text{ rad}$; $\theta = \pi/6 \text{ rad}$.

Analyzing (29) and (30) it is found that in all situations the amplitudes of the symmetrical components Y_{pm} and Y_{nm} are transferred unchanged (depending on coefficient γ) from the three-phase system into the two-phase system. As with Clarke transformation, the components ratio (dissymmetry coefficient) remains the same. However, we can identify two situations:

A. $\theta = const.$

Although the amplitudes of the symmetrical components do not change, differences between the phases occur. Whereas the phase of the positive sequence is reduced with the rotation angle θ (29), the phase of the negative sequence increases with the same value (30). The phase shift between the two symmetrical components changes. This fact is shown in Fig. 2 (phasor domain) and in Fig. 3 (time domain) and leads to the modification of the sinusoids hodograph with significant effects in transients study. The asymmetric two-phase system obtained through the Park transformation is similar but not identical to the original three-phase asymmetric system. Even though inverse Park transformation leads to the original system there are phase differences, in the two-phase system, from the three-phase system.

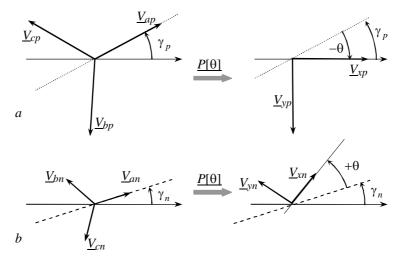
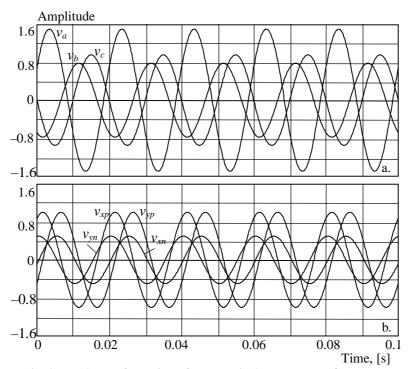
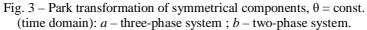


Fig. 2 – Park transformation of symmetrical components, $\theta = \text{const.:}$ *a* – positive sequence ; *b* – negative sequence.





B. $\theta = \omega_{rot} \cdot t$ From (29) and (30) results:

$$y_{p\,2\,ph} = \gamma \cdot \frac{3}{2} Y_{pm} \sin\left[\left(\omega - \omega_{rot}\right) \cdot t + \gamma_{p}\right]$$
(31)

$$y_{n\,2\,ph} = \gamma \cdot \frac{3}{2} Y_{nm} \sin\left[\left(\omega + \omega_{rot}\right) \cdot t + \gamma_{p}\right]$$
(32)

The two symmetrical components which initially had the same angular frequency ω change differently. The positive sequence reduces its angular frequency to the value $\omega_p = \omega - \omega_{rot}$ whereas the negative sequence increases its frequency to the value $\omega_p = \omega + \omega_{rot}$. The Park transformation shifts the frequency spectrum of the signal by the value $\pm \omega_{rot}$.

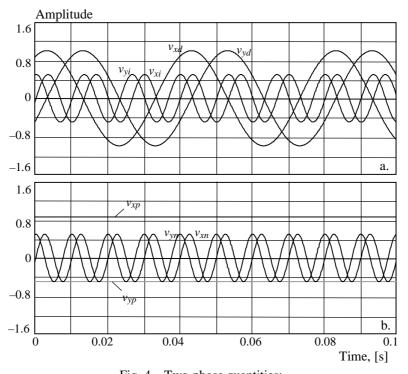


Fig. 4 – Two-phase quantities: a – $\omega_{rot} = \omega/2$; b – $\omega_{rot} = \omega$.

The asymmetric system is characterized by the fact that both components have the same angular frequency. Since they modify in a different way we can say that the Park transformation converts an asymmetric system into a non-sinusoidal system. The waveform of two-phase signals resulted through Park transformation for $\theta = \omega/2 \cdot t$ and $\theta = \omega \cdot t$ is illustrated in Fig.4. In both situations the frequency of the transformed signals is changed even though the amplitudes are unchanged.

All the practical results that validated the final expressions were obtained by using PSpice software (Cociu *et al.*, 2012), (Justus, 1993) and Simulink.

4. Conclusions

Although in the case of Park transformation no restrictive conditions are imposed on the three-phase sinusoidal signals, in the case of asymmetric systems major changes may occur.

1° In Clarke transformation ($\theta = 0$) the symmetrical components are transformed in the same way; both amplitude and phase angle do not change.

The two-phase asymmetric system has the same properties as the three-phase one. Clarke transformation does not make changes to the asymmetrical systems.

 2° In Park transformation for $\theta = \text{const.}$ the symmetrical components preserve the amplitude by transformation but the phases are modified. The asymmetric two-phase system obtained through the Park transformation is similar but not identical to the three-phase asymmetric system.

3° In Park transformation for $\theta = \omega_{rot} \cdot t$ the positive sequence reduces its angular frequency to the value $\omega_p = \omega - \omega_{rot}$ whereas the negative sequence increases its frequency to the value $\omega_p = \omega + \omega_{rot}$. The Park transformation shifts the frequency spectrum of the signal by the value $\pm \omega_{rot}$. It converts an asymmetric system into a non-sinusoidal one.

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TRANSFORMATA PARK APLICATĂ UNUI SISTEM ASIMETRIC

(Rezumat)

În anul 2000 metoda componentelor simetrice (Fortescue) și transformarea Park au fost votate lucrările cu cel mai mare impact din secolul trecut asupra ingineriei energetice și electrice. În studiul mașinilor electrice de c.a. transformata Park este o unealtă clasică ce permite utilizarea modelului bifazat.

Lucrarea de față își propune să analizeze în ce măsură și cu ce costuri transformata Park poate fi utilizată în transformarea unui sistem trifazat nesimetric. Au fost identificate trei cazuri distincte în care transformarea modifica în mod diferit sistemul trifazat.

Concluzia finală este că, în caz general, transformarea Park modifică într-o anumită măsură caracteristicile sistemului trifazat iar rezultatele obținute prin utilizarea acestei transformări trebuie, de la caz la caz, privite în mod critic.