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# SYMMETRICAL COMPONENTS EVALUATION IN REALTIME 

## BY

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#### Abstract

Using Symmetrical components methodology, unbalanced system conditions, like those caused by common fault types may be visualized and analyzed. In order to calculate direct and inverse components, the phasors must be multiplied by the phasor rotation operator $\underline{a}=e^{j 2 \pi / 3}$. The classical mathematical expressions associated with this issue are well-suited especially in phasic calculus. However, in the time domain, the authors do not offer practical solutions. The paper presents concrete ways to solve the problem of rotating a sinusoidal size in real-time using PSpice and Matlab Simulink software. Based on these, the real and the inverse components of an unbalanced three-phase system are determined in real time.


Key words: symmetrical components; PSpice; Simulink; electrical machines.

## 1. Introduction

Symmetrical components is a methodology discovered by Charles Legeyt Fortescue. He demonstrated that any set of unbalanced three-phase quantities could be expressed as the sum of three symmetrical sets of balanced phasors (Fortescue, 1918). Using this tool, unbalanced system conditions, like

[^0]those caused by common fault types may be visualized and analyzed. The method of symmetrical components is usually used in power systems to simplify the analysis of unbalanced three-phase systems. In the most common case of three-phase system, the resulting symmetrical components are referred as direct (or positive), inverse (or negative) and homopolar (or zero). The resulting transform equations are mutually linearly independent.

The basic idea is that an asymmetrical set of three phasors can be expressed as a linear combination of three symmetrical sets of phasors by means of a complex linear transformation.

The first application of the symmetrical components to the electrical machines was described by Lyon (1954). In addition, other authors have developed studies on the use of symmetrical components in the study of electric machines (White \& Woodson, 1959; Harris et al., 1970; Paap, 2000).

A study on the possibility of using symmetrical components method in the analysis of induction machine behavior is presented in (Cociu \& Cociu, 2014). Starting from the observation that induction machine is described by a system generally of nonlinear equations, the authors try to determine the circumstances in which this method can be used in studying the behavior of the induction machine.

Consider a three-phase (asymmetric) system of sinusoidal quantities with the same pulse $\omega$, the amplitude and the initial phase having an arbitrary value, generally different for each voltage:

$$
\begin{align*}
& v_{a}(t)=\sqrt{2} V_{a} \sin \left(\omega t+\gamma_{a}\right) \\
& v_{b}(t)=\sqrt{2} V_{b} \sin \left(\omega t+\gamma_{b}\right)  \tag{1}\\
& v_{c}(t)=\sqrt{2} V_{c} \sin \left(\omega t+\gamma_{c}\right)
\end{align*}
$$

These quantities can be mapped to their corresponding complex phasors:

$$
\begin{align*}
& \underline{\mathrm{V}}_{a}=V_{a} e^{\mathrm{j} \gamma_{a}} \\
& \underline{\mathrm{~V}}_{b}=V_{b} e^{\mathrm{j} \gamma_{b}}  \tag{2}\\
& \underline{\mathrm{~V}}_{c}=V_{c} e^{\mathrm{j} \gamma_{c}}
\end{align*}
$$

According to symmetrical components method, they can be expressed as:

$$
\begin{align*}
& \underline{\mathrm{V}}_{a}=\underline{\mathrm{V}}_{a d}+\underline{\mathrm{V}}_{a i}+\underline{\mathrm{V}}_{0} \\
& \underline{\mathrm{~V}}_{b}=\underline{\mathrm{V}}_{b d}+\underline{\mathrm{V}}_{b i}+\underline{\mathrm{V}}_{0}  \tag{3}\\
& \underline{\mathrm{~V}}_{c}=\underline{\mathrm{V}}_{c d}+\underline{\mathrm{V}}_{c i}+\underline{\mathrm{V}}_{0}
\end{align*}
$$

The system is uniquely determined by the origin vectors:

$$
\begin{gather*}
\underline{\mathrm{V}}_{d}=\left(\underline{\mathrm{V}}_{a}+\underline{\mathrm{a}} \cdot \underline{\mathrm{~V}}_{b}+\underline{\mathrm{a}}^{2} \cdot \underline{\mathrm{~V}}_{c}\right) / 3=\underline{\mathrm{V}}_{a d}  \tag{4}\\
\underline{\mathrm{~V}}_{i}=\left(\underline{\mathrm{V}}_{a}+\underline{\mathrm{a}^{2}} \cdot \underline{\mathrm{~V}}_{b}+\underline{\mathrm{a}}^{\mathrm{V}} \underline{\mathrm{~V}}_{c}\right) / 3=\underline{\mathrm{V}}_{a i} \\
\underline{\mathrm{~V}}_{0}=\left(\underline{\mathrm{V}}_{a}+\underline{\mathrm{V}}_{b}+\underline{\mathrm{V}}_{c}\right) / 3 \tag{5}
\end{gather*}
$$

where: $\underline{a}$ is a phasor rotation operator which rotates a phasor vector counterclockwise by 120 degree.

From eq. (4), for the determination of the direct and inverse phase component of the three voltages we must multiply with $\underline{a}$ and $\underline{a}^{2}$. In the time domain it corresponds to phasing sinusoidal quantities $v_{x}(t)$ with $2 \pi / 3$ or $4 \pi / 3$ radians respectively. The same problem applied to two-dimensional sizes requires shifting.

## 2. Theoretical Part

The paper aims is to solve in real time the problem of determining a sinusoidal quantities with the angle $\theta$, fig. 1 .


Fig. 1 - Phase modifying of a sinusoidal quantity.
Consider a sinusoidal quantity (voltage) with amplitude value equal to 1 , pulse $\omega$, and initial phase $\gamma$ :

$$
\begin{equation*}
v_{x}(t)=\sin (\omega t+\gamma) \tag{6}
\end{equation*}
$$

The quantity to be determined has the expression:

$$
\begin{equation*}
v_{y}(t)=\sin (\omega t+\gamma+\theta) \tag{7}
\end{equation*}
$$

Applying trigonometric formulas:

$$
\begin{equation*}
v_{y}(t)=\sin (\omega t+\gamma+\theta)=\sin (\omega t+\gamma) \cos (\theta)+\sin (\theta) \cos (\omega t+\gamma) \tag{8}
\end{equation*}
$$

On the other hand,

$$
\begin{equation*}
\cos (\omega t+\gamma)=\frac{1}{\omega} \cdot \frac{\mathrm{~d}[\sin (\omega t+\gamma)]}{\mathrm{d} t}=\frac{1}{\omega} \cdot \frac{\mathrm{~d}\left[v_{x}(t)\right]}{\mathrm{d} t}, \tag{9}
\end{equation*}
$$

resulting:

$$
\begin{equation*}
v_{y}(t)=v_{x} \cos (\theta)+\sin (\theta) \frac{1}{\omega} \cdot \frac{\mathrm{~d}\left(v_{x}\right)}{\mathrm{d} t} . \tag{10}
\end{equation*}
$$

Eq. (10) allows the sinusoidal quantities to be determined for the angle $\theta$ starting from any sinusoidal size. Note that, pulse $\omega$ must be known. Otherwise it should be determined. When a machine is supplied from the network, the fundamental frequency is constant over time or over large time intervals. In the worst case, the frequency can be measured for a large interval of time and its value used as an input constant. This gives $\omega(t)=$ const $=\omega$. (Cociu \& Cociu, 2017).

## 3. Pspice Implementation of the Derivative

The starting point is to evaluate the possibility of determining the sinusoidal quantities using eq. (10). As it turns out, it is necessary to determine the derivative of a quantities variable in time. Consider an ordinary first order differential equation:

$$
\begin{equation*}
y(t)=a \frac{\mathrm{~d} x}{\mathrm{~d} t} \tag{11}
\end{equation*}
$$

which in Simulink does not raise problems to compute because there is a specialized block for this mathematical operation (Fig. 2).


Fig. 2 - Derivative block in Simulink.

PSpice integration of differential equations is usually performed by means of generic integrators that are obtained by implementing electric circuits comprising controlled voltage $-E$ (or current $-G$ ) sources, capacitors and inductors (Justus, 1993). The sources allow describing the forcing terms, the inductors and capacitors accomplish the derivative terms. The result is obviously a voltage or current whose value is numerically equal to the desired quantity.


Fig. 3 - Generic integrators.
As shown in Fig. $3 a$, the relationship among currents and voltages is:

$$
\begin{equation*}
i_{y}(t)=C \frac{\mathrm{~d} u_{x}(t)}{\mathrm{d} t} \leftrightarrow y(t)=a \frac{\mathrm{~d} x(t)}{\mathrm{d} t} \tag{12}
\end{equation*}
$$

Identifying terms gives correspondence between the variables and parameters.

$$
\begin{align*}
& y  \tag{13}\\
& x
\end{align*} \leftrightarrow i_{y} \quad, \quad a \quad 4 \quad u_{x} \quad, \quad a<c
$$

For the circuit in Fig. $3 b$ :

$$
\begin{align*}
u_{y}(t)= & L \frac{\mathrm{~d} i_{x}(t)}{\mathrm{d} t} \leftrightarrow \quad y(t)=a \frac{\mathrm{~d} x(t)}{\mathrm{d} t}  \tag{14}\\
y & \leftrightarrow u_{y}  \tag{15}\\
x & \leftrightarrow \quad i_{x}
\end{align*}, a \leftrightarrow L
$$

Based on both the correspondences, of variables and parameters, above mentioned, in order to solve eq. (8) two possible integrator configurations can be used, as shown in Fig. 3.

The PSpice code for the two integrators to be used in practice is comparatively given below:
case a.
.param w=10
Ex $0 x$ Value $=\left\{\sin \left(w^{*}\right.\right.$ time $\left.)\right\}$
Cx $0\{1 / \mathrm{w}\}$
Rx $0 \times 1 \mathrm{meg}$
.probe
.tran 0.11000 .1 m
.end
case b.
.param w=10
Gx 0 y Value $=\left\{\sin \left(w^{*}\right.\right.$ time $\left.)\right\}$
Lj y $0\{\mathrm{Ljj}\}$
Ry g y $\{1 / \mathrm{w}\}$
.probe
.tran 0.11000 .1 m
.end

The case $a$ is superior from both points of view, of integration time and data output file size. This is due to the circuit topology description in PSpice where, even if the two electrical circuits contain the same number of nodes, PSpice description of case $a$ comprises only two nodes where as the description in case $b$ has three nodes.

This method of solving differential equations has been widely used for modeling and simulating the motion mass equation (Cociu \& Cociu, 2011), (Cociu \& Cociu, 2014).

In Fig. 4 are presented the results obtained using an input signal $v_{x}(t)=$ $=324 \sin (\omega t-2 \pi / 3)$ and $\theta=3 \pi / 4$.


Fig. 4 - Input and output signal waveform;
$\mathrm{a}-v_{x}(t) ; \mathrm{b}-\mathrm{d} / \mathrm{d} t\left[v_{x}(t)\right] / \omega ; \mathrm{c}-v_{y}(t)$

## 4. Symmetrical Components Evaluation in Real-Time Using Pspice and Simulink

The calculation of symmetric components $v_{d}, v_{i}, v_{0}$ of an unbalanced three-phase voltage system $v_{a}, v_{b}, v_{c}$ is performed based on the block diagram in Fig. 5.


Fig. 5 - Unbalanced three-phase voltage system block diagram.

The implementation of component blocks in PSpice leads to the textbased source program presented below:

Symmetrical components calculus
** Parameters
.param w=\{100*pi $\}$
.param Vam=324, ga=0
.param $\mathrm{Vbm}=324, \mathrm{gb}=\left\{-2^{*} \mathrm{pi} / 3\right\}$
.param Vcm $=162, \mathrm{gc}=\{+2 * \mathrm{pi} / 3\}$
.param a1=(2*pi/3)
.param $\mathrm{a} 2=(-2 * \mathrm{pi} / 3)$
** Unbalanced three-phase voltage system
Ea a 0 Value $=\left\{\right.$ Vam $* \sin \left(w^{*}\right.$ time + ga $\left.)\right\}$
Eb b 0 Value $=\left\{\mathrm{Vbm} * \sin \left(\mathrm{w}^{*}\right.\right.$ time +gb$\left.)\right\}$
Ec c 0 Value $=\left\{\mathrm{Vcm} * \sin \left(\mathrm{w}^{*}\right.\right.$ time +gc$\left.)\right\}$
Ra a 01
Rbb0 1
Rc c 01
**Homopolar component calculus " 0 "
Eh h 0 Value $=\{(\mathrm{V}(\mathrm{a})+\mathrm{V}(\mathrm{b})+\mathrm{V}(\mathrm{c})) / 3\}$
Rh h 01
** Rotation: a1 and a2 *
Eba1 ba1 0 Value $=\{\mathrm{V}(\mathrm{b}) * \cos (\mathrm{a} 1)+\sin (\mathrm{a} 1) * \mathrm{~V}(\mathrm{bder})\}$
Eba2 ba2 0 Value $=\left\{\mathrm{V}(\mathrm{b})^{*} \cos (\mathrm{a} 2)+\sin (\mathrm{a} 2)^{*} \mathrm{~V}(\mathrm{bder})\right\}$
Eca1 ca1 0 Value $=\{\mathrm{V}(\mathrm{c}) * \cos (\mathrm{a} 1)+\sin (\mathrm{a} 1) * \mathrm{~V}(\mathrm{cder})\}$

Eca2 ca2 0 Value $=\left\{\mathrm{V}(\mathrm{c})^{*} \cos (\mathrm{a} 2)+\sin (\mathrm{a} 2) * \mathrm{~V}(\mathrm{cder})\right\}$
Gb 0 bder Value $=\{\mathrm{V}(\mathrm{b})\}$
Gc 0 cder Value $=\{\mathrm{V}(\mathrm{c})\}$
Lb bder $0\{1 / \mathrm{w}\}$
Lc cder $0\{1 / w\}$
** Direct component calculus "d"*
Ead ad 0 Value $=\{(\mathrm{V}(\mathrm{a})+\mathrm{V}(\mathrm{ba} 1)+\mathrm{V}(\mathrm{ca} 2)) / 3\}$
Rad ad 01
** indirect component calculus "i"*
Eai ai 0 Value $=\{(\mathrm{V}(\mathrm{a})+\mathrm{V}(\mathrm{ba} 2)+\mathrm{V}(\mathrm{ca} 1)) / 3\}$
Rai ai 01
**
.probe
.tran 0.010 .10 .1 m 0.1 m uic
.end
In Fig. 6 is an unbalanced three-phase voltage system characterized by $V_{a}=324 \mathrm{~V}$ with $\gamma_{a}=0 \mathrm{rad}, V_{b}=324 \mathrm{~V}$ with $\gamma_{b}=-2 \pi / 3 \mathrm{rad}, V_{c}=162 \mathrm{~V}, \gamma_{c}=$ $=2 \pi / 3 \mathrm{rad}$ and the results obtained following the determination of the symmetrical components.


Fig. 6 - Unbalanced three-phase voltage system curves.


Fig. 7 - Block diagram implemented in Simulink.


Fig. 8 - Unbalanced three-phase voltage system curves.

The calculation of symmetric components $v_{d}, v_{i}, v_{0}$ of an unbalanced three-phase voltage system $v_{a}, v_{b}, v_{c}$ is also performed based on the block in Simulink (Fig. 7). In Fig. 8 is an unbalanced three-phase voltage system characterized by $V_{a}=324 \mathrm{~V}$ with $\gamma_{a}=0 \mathrm{rad}, V_{b}=324 \mathrm{~V}$ with $\gamma_{b}=-2 \pi / 3 \mathrm{rad}$, $V_{c}=324 \mathrm{~V}, \gamma_{c}==\pi / 2 \mathrm{rad}$ and the results obtained following the determination of the symmetrical components.

## 5. Conclusions

For the determination of the direct and inverse phase component of the three voltages are multiply with $\underline{a}$ and $\underline{a}^{2}$. In the time domain it returns to phasing with $2 \pi / 3$ or $4 \pi / 3$ radians toward sinusoidal quantities $v_{x}(t)$. The sinusoidal quantities are determined by the angle $\theta$ starting from any sinusoidal size with pulse $\omega$ known by modelling in PSpice to solve differential equations resulting from trigonometric equations and with Matlab Simulink the mathematical operation are simulated.

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## EVALUAREA COMPONENTELOR SIMETRICE ÎN TIMP REAL

(Rezumat)

Folosind metodologia componentelor simetrice, pot fi vizualizate și analizate condițiile de sistem dezechilibrat, cum ar fi cele cauzate de defecțiuni comune. Pentru a calcula componentele directe și inverse, fazorii trebuie înmulțiți cu operatorul de rotație a fazorului $a=\mathrm{e}^{\mathrm{j} 2 \pi / 3}$. Expresiile matematice clasice asociate cu această problemă sunt potrivite în special în calculul fazorilor. Cu toate acestea, în domeniul timpului, autorii nu oferă soluții practice. Lucrarea prezintă modalități concrete de a rezolva problema rotirii unei mărimi sinusoidale în timp real folosind software-ul PSpice și Matlab Simulink. Pe baza acestora sunt determinate în timp real componentele reale și inverse ale unui sistem trifazat dezechilibrat.


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