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ON THE STATOR CURRENT OF AN INDUCTION MACHINE ASYMMETRICAL SUPPLIED

BY

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Abstract. Supplying an induction machine with an unbalanced voltage system leads to an asymmetrical system of stator currents. In addition, the stator current hodograph is no longer circular but elliptical. It is natural the currents dissymmetry factor to depend on the supply voltage dissymmetry factor. But the results obtained under different machine operating conditions indicate a significant influence of the rotor speed on the operation if asymmetrically powered. The paper aims to study the influence of the operating conditions on the asymmetric regime of the stator currents. The approach is based on some theoretical aspects confirmed and completed by the results obtained by simulation.

Key words: symmetrical components; induction machine; dissymmetry factor; PSpice.

1. Introduction

In 1918 Charles Legeyt Fortescue published the work *Methode of Symmetrical Coordinates Applied to the Solution of Poliphase Networks* (Fortescue, 1918). In this paper Fortescue resumed the idea of L.G. Stokvis in the case of electrical machines, to decompose the magnetic field into a positive rotating part, a negative rotating part and a pulsating field. He extends this idea to unbalanced three phase circuits. He demonstrated that any set of unbalanced

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three-phase quantities could be expressed as the sum of three symmetrical sets of balanced phasors.

In this paper we consider the case of an induction motor, supplied by an asymmetrical system voltage. The range of interest for angular speed ω_r is $[0, \omega_1)$ where ω_1 is the synchronous speed, so the slip s is in the range of (0, 1]. The supplying voltage system is characterized by the dissymmetry factor $\varepsilon_n = U_{sn}/U_{sp}$. The machine parameters (resistors, inductances, mutual inductances) have the same values for each phase since the machine is symmetric by construction.

One of the interest quantities of the induction machine, the stator current system was considered. It can be easily monitored and visualized to determine the machine's energetic parameters or to diagnose the induction machine (Ong, 1997).

It has been studied how angular rotor speed influences (modifies) the dissymmetry factor of the stator current. This is extremely important when diagnosing the machine. If monitoring the stator current leads to an asymmetrical current system, it must be determined whether the asymmetry is due only to the asymmetric supply or to a machine fault.

2. Theoretical Considerations

In the case of a symmetrical construction induction machine we are tempted to consider the induction motor connected to a three-phase supply grid as a three-phase balanced load.

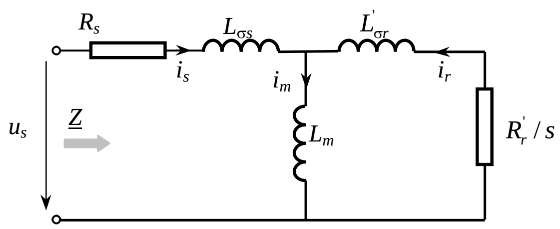

Fig. 1 – Per-phase equivalent circuit of an induction motor.

We consider the star connection with the neutral connected. If the neutral is not connected the problem is fundamentally changed. A single-phase equivalent circuit is usually used to calculate the phase voltage and current. The *a*-phase is usually chosen. The per-phase equivalent circuit is shown in Fig. 1. Viewed from the supply terminals it represents impedance – $\underline{Z}$ (Fortescue, 1918).

For a symmetrical healthy machine equal impedances on each phase are provided by construction: $\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}$. But a careful analysis reveals that the impedance value on each phase depends on the angular speed (or slip): $\underline{Z} = \underline{Z}(\omega_r) = \underline{Z}(s)$. The impedances become parametric elements and the generalized unbalanced load theory cannot be used (Popa, 2010; Cociu & Cociu, 2014).

In the case of a symmetric voltage supply, we have only the positive phase sequence characterized by the angular speed $\omega_1 = \omega/p$, where ω is the angular frequency of the supply voltage and $2p$ number of poles. The impedance Z expression includes $\omega_r \in [0, \omega_1)$ respectively $s \in (0, 1]$. If the supply is asymmetric, for the positive sequence $\omega_{r+} = \omega_r$ (or $s_+ = s$) is used. For the negative phase sequence characterized by the angular speed $-\omega_1$, the value $\omega_{r-} = -\omega_r$ respectively $s_- = (\omega_1 + \omega_r)/\omega_1 = 2 - s$ must be used . For an induction motor the range of interest for the slip is:

$$s \in (0,1] \;\Rightarrow\; \begin{matrix} s_+ \in (0,1] \\ s_- \in [1,2) \end{matrix} \tag{1}$$

We will use the following notation:

$$\begin{aligned} \underline{Z}_s &= R_s + \mathrm{j}X_{\sigma s} \\ \underline{Z}_m &= \mathrm{j}X_m \\ \underline{Z}_r^{'} &= R_r^{'}/s + \mathrm{j}X_{\sigma r} = \underline{Z}_r^{'}(s) \end{aligned} \tag{2}$$

From Fig. 1 it results:

$$\underline{Z} = \underline{Z}_s + \underline{Z}_m \cdot \underline{Z}_r^{'}/(\underline{Z}_m + \underline{Z}_r^{'}) = \frac{\underline{Z}_m \cdot \underline{Z}_s + \underline{Z}_m \cdot \underline{Z}_r^{'} + \underline{Z}_s \cdot \underline{Z}_r^{'}}{\underline{Z}_m + \underline{Z}_r^{'}} \tag{3}$$

$$\underline{Z} = \frac{\underline{Z}_s + \underline{Z}_r^{'}(1 + \underline{Z}_s/\underline{Z}_m)}{1 + \underline{Z}_r^{'}/\underline{Z}_m} \tag{4}$$

Using the classical notation:

$$\underline{c} = 1 + \underline{Z}_s/\underline{Z}_m \tag{5}$$

we get:

$$\underline{Z} = \frac{\underline{Z}_s + \underline{c}\cdot\underline{Z}_r^{'}}{1 + \underline{Z}_r^{'}/\underline{Z}_m} \tag{6}$$

The stator current expression becomes:

$$\underline{I}_s = \frac{\underline{U}_s}{\underline{Z}} = \underline{U}_s \frac{1 + \underline{Z}_r^{'}/\underline{Z}_m}{\underline{Z}_s + \underline{c}\cdot\underline{Z}_r^{'}} \tag{7}$$

3. Simulation Considerations

As shown in (Cociu & Cociu, 2017) Clarke transformation does not affect the asymmetrical systems. The corresponding two-phase system has the same properties as the three-phase one. For this reason, although the theoretical considerations are made for the three-phase machine, PSpice simulation made to confirm the theoretical results, uses the two-phase model. If the three-phase supply is characterized by dissymmetry factor $\varepsilon_n = U_{sn} / U_{sp}$ the same value is found for the two-phase system obtained by using Clarke transformation.

To confirm the rightness of the currents expressions the machine behavior has been simulated and numerical values have been assigned for the parameters. It was used a three-phase induction motor rated as follows:

$$\begin{aligned} &P_n = 5\ \text{kW}; \quad U_{1\text{ln}} = 400\ \text{V}; \quad f_1 = 50\ Hz; \\ &R_s = 1.2\ \Omega; \quad R_r^{'} = 1.2\ \Omega; \quad L_m = 120\ \text{mH}; \\ &L_{\sigma s} = 10\ \text{mH}; \quad L_{\sigma r}^{'} = 10\ \text{mH}; \quad J = 40\text{g} \cdot \text{m}^2; \\ &\text{Y}\, conn.; \quad p = 1; \quad F_\alpha = 2 \cdot 10^{-3}\ \text{N} \cdot \text{m} \cdot \text{s} / \text{rad}. \end{aligned}$$

It was considered the neutral connected. The simulation results were based on the two-phase model implemented in PSpice. (Justus 1993; Cociu & Cociu, 2017) but were also confirmed on the three-phase model.

In the case of symmetric supply, corresponding to the rated of the machine, the following values were used:

$$\begin{aligned} \sqrt{2} U_{sd} &= 400\ \text{V} \\ \sqrt{2} U_{sq} &= 400\ \text{V} \end{aligned} \tag{8}$$

and for asymmetric supply:

$$\begin{aligned} \sqrt{2} U_{sd} &= 500\ \text{V} \quad ; \quad \underline{U}_{sd} = 500/\sqrt{2} \\ \sqrt{2} U_{sq} &= 300\ \text{V} \quad ; \quad \underline{U}_{sq} = -\text{j}\, 300/\sqrt{2} \end{aligned} . \tag{9}$$

Symmetrical components are:

$$\begin{aligned} \underline{U}_{sp} &= \underline{U}_{sdp} = (\underline{U}_{sd} + \text{j} \cdot \underline{U}_{sq})/2 = (500/\sqrt{2} + 300/\sqrt{2})/2 = 400/\sqrt{2} \\ \underline{U}_{sn} &= \underline{U}_{sdn} = (\underline{U}_{sd} - \text{j} \cdot \underline{U}_{sq})/2 = (500/\sqrt{2} - 300/\sqrt{2})/2 = 100/\sqrt{2} \end{aligned} \tag{10}$$

Dissymmetry factor is:

$$\varepsilon_{n\,us} = \frac{U_{sn}}{U_{sp}} = \frac{100}{400} = 0.25 . \tag{11}$$

Fig. 2 shows the stator supply voltage hodograph. Since the three-phase (two-phase) supply system is not symmetrical, the hodograph is not a circle but an ellipse. The major axis of the ellipse is horizontal, corresponding to the phase of the supply voltages. Starting from the ellipse coordinates the dissymmetry factor can be calculated:

$$\varepsilon_{n\,us} = \frac{U_{sn}}{U_{sp}} = \frac{(R_M - R_m)/2}{(R_M + R_m)/2} = \frac{100}{400} = 0.25. \tag{12}$$

Three representative situations were considered: starting (s = 1), running at rated speed ($s = s_n$) and synchronism (s = 0).

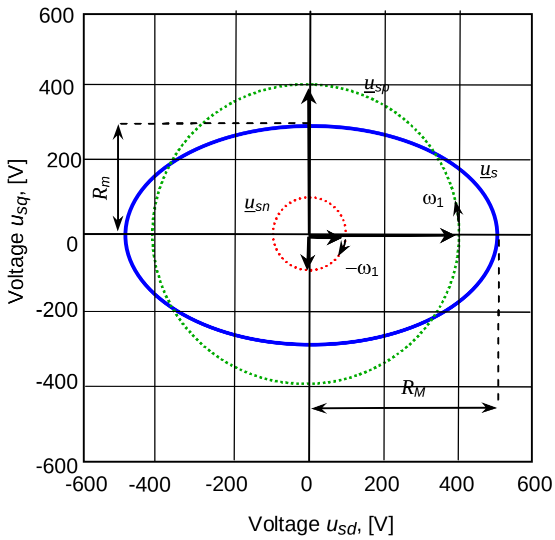

Fig. 2 – Supply voltage hodograph.

3. Asymmetrical Stator Current

Consider the tree-phase induction machine supplied by an asymmetric system voltage which can be decomposed in symmetrical components. The resulting stator current is an asymmetric system as well, having the components:

$$\underline{I}_{s\,p} = \frac{\underline{U}_{s\,p}}{\underline{Z}(s_+)} = \underline{U}_{s\,p} \frac{1 + \underline{Z}_r^{'}(s_+)/\underline{Z}_m}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_+)} \tag{13}$$

$$\underline{I}_{s\,n} = \frac{\underline{U}_{s\,n}}{\underline{Z}(s_-)} = \underline{U}_{s\,n} \frac{1 + \underline{Z}_r^{'}(s_-)/\underline{Z}_m}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_-)} \tag{14}$$

Dissymmetry factor of the stator current can be calculated:

$$\varepsilon_{n\,is} = \frac{I_{s\,n}}{I_{s\,p}} = \frac{U_{s\,n}}{U_{s\,p}} \cdot \left| \frac{1 + \underline{Z}_r^{'}(s_-)/\underline{Z}_m}{1 + \underline{Z}_r^{'}(s_+)/\underline{Z}_m} \cdot \frac{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_+)}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_-)} \right|. \tag{15}$$

$$\varepsilon_{n\,is} = \varepsilon_{n\,us} \cdot k_{\varepsilon is}(s)\,. \tag{16}$$

As we can see, the current dissymmetry factor depends on the voltage dissymmetry factor but also on a relatively complicated expression depending on the impedances of the machine. We define this quantity *dissymmetry factor gain* (*DFG*):

$$k_{\varepsilon is}(s) = \left| \frac{\underline{Z}_m + \underline{Z}_r^{'}(s_-)}{\underline{Z}_m + \underline{Z}_r^{'}(s_+)} \cdot \frac{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_+)}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(s_-)} \right|. \tag{17}$$

The expression could be processed but we could not get a significant simplification for the whole range $s \in (0, 1]$. The only speed-dependent element is the equivalent rotor resistance $R_r^{'}/s$. It occurs in the impedance expression $\underline{Z}_r^{'}$ and is found in four different positions in the expression of *DFG*. For this reason, it was preferred to calculate the numerical value in the three cases considered and to compare the result to the one obtained by simulation.

$$k_{\varepsilon is}(s=1) = 1\,. \tag{18}$$

$$k_{\varepsilon is}(s=s_n) = \left| \frac{\underline{Z}_m + \underline{Z}_r^{'}(1.953)}{\underline{Z}_m + \underline{Z}_r^{'}(0.047)} \cdot \frac{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(0.047)}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(1.953)} \right| = 3.59\,. \tag{19}$$

$$k_{\varepsilon is}(s=0) = \left| \underline{c}\,\frac{\underline{Z}_m + \underline{Z}_r^{'}(2)}{\underline{Z}_s + \underline{c} \cdot \underline{Z}_r^{'}(2)} \right| = \left| \underline{c}\,\frac{R_r^{'}/2 + \mathrm{j}X_r}{R_s + R_r^{'}/2 + \mathrm{j}(X_{\sigma s} + \underline{c} \cdot X_{\sigma r})} \right| = 6.41\,. \tag{20}$$

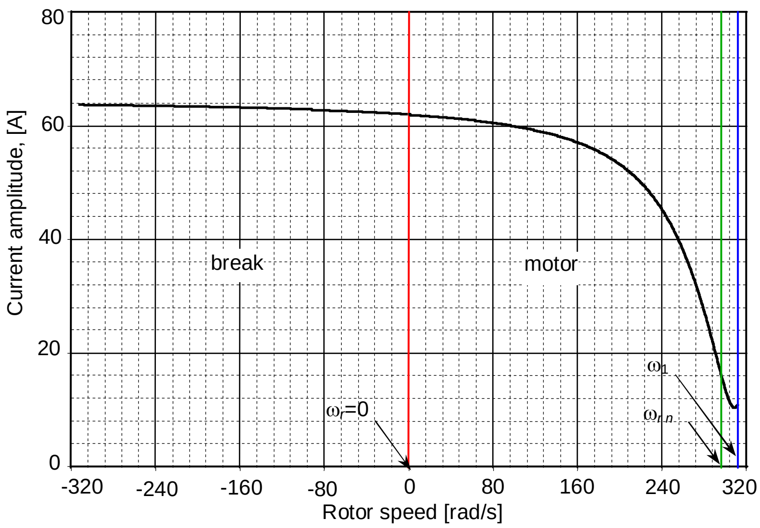

Fig. 3 – Stator current versus speed.

The results obtained by simulation, although valid for a particular case, have a high degree of generality. Fig. 3 shows variation of the stator current versus speed, for both the motor ($\omega_r \in (0, \omega_1)$) and brake operation ($\omega_r \in (-\omega_1, 0)$).

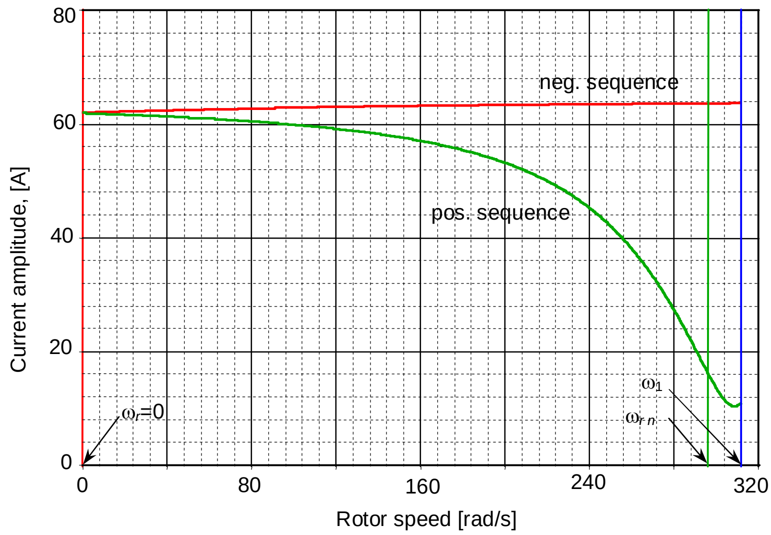

Fig. 4 – Symmetrical components of stator current versus speed.

In motor operation, the current decreases continuously with speed due to the equivalent rotor resistance $R_r^{'} / s$. In braking operation the decrease is very small, the current being approximately constant. The range of positive speed is used in the calculation of the positive sequence and the negative speed is used in the negative one. Fig. 4 shows the currents corresponding to the two sequences versus speed.

In starting operation, $\omega_r = 0$, then $s_+ = s_- = 1$, $\underline{Z}(s_+) = \underline{Z}(s_-)$ and the stator currents corresponding to the two sequences are equal. As the speed increases, the current corresponding to the negative sequence increases slightly while the current corresponding to the positive one strongly decreases as in Fig. 4. This explains the growth of *DFG* with rotor speed, as shown in Fig. 5.

At rated load, the *DFG* value is roughly equal to the ratio of the starting current to the rated current. The usual values are 3,...,5. At no-load operation, the ratio of currents can be even double (Lyshevsky, 1999). So *DFG* can reach 6,...,10 at no-load or synchronism operation.

This result indicates a more strongly asymmetry of the stator currents than the supply voltage one. The faster the speed (closer to the synchronous speed), the more important the difference. *DFG* is a measure of the increase in the dissymmetry factor. The numerical values obtained by using (18), (19) and (20) correspond to those obtained by simulation (Fig. 5, points A, B and C).

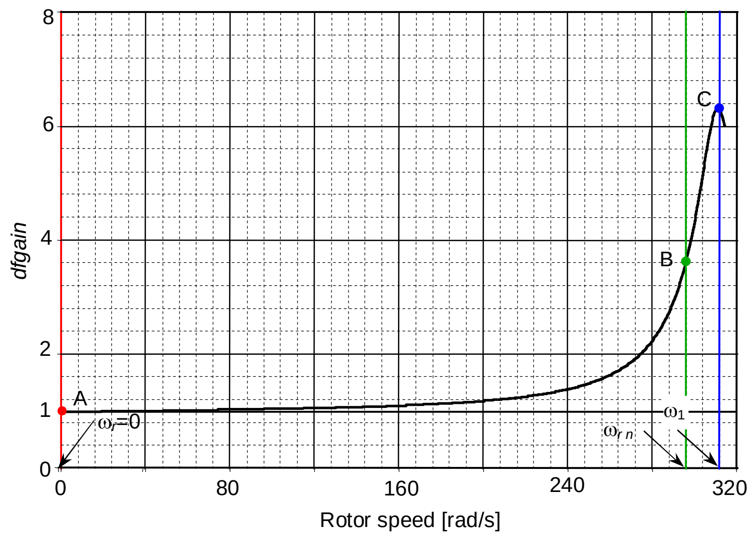

Fig. 5 − Dissymmetry factor gain versus speed.

Fig. 6 shows the variation of the current in a window of 100 ms in the two cases. It can be noted strong current oscillations corresponding to the current hodograph in Fig. 8. Based on the maximum and minimum values (Fig. 7), the dissymmetry factor can be calculated:

$$\omega_r = 0 \quad \rightarrow \quad \varepsilon_{n\,is} = \frac{I_{sn}}{I_{sp}} = \frac{(I_M - I_m)/2}{(I_M + I_m)/2} = \frac{38.4}{61.5} = 0.25\,. \tag{21}$$

$$\omega_r = \omega_{rn} \quad \rightarrow \quad \varepsilon_{n\,is} = \frac{I_{sn}}{I_{sp}} = \frac{(I_M - I_m)/2}{(I_M + I_m)/2} = \frac{16.5}{19.5} = 0.85\,. \tag{22}$$

Fig. 8 presents the stator current hodograph in the three particular cases considered. In starting operation, the ellipse dimensions indicate the high supply currents. The eccentricity of the ellipse is identical to that of the supply voltage in Fig. 2. At starting the current has the same dissymmetry factor as the supply voltage factor. The machine behaves like a balanced receiver. As the speed increases, the dimensions of the ellipse decrease and the eccentricity increases. At rated operation the dissymmetry factor of stator current has a much higher value than the supply voltage factor. Even though at no load operation there is a slight improvement in the ellipse shape, its eccentricity is greater than that of the supply voltage.

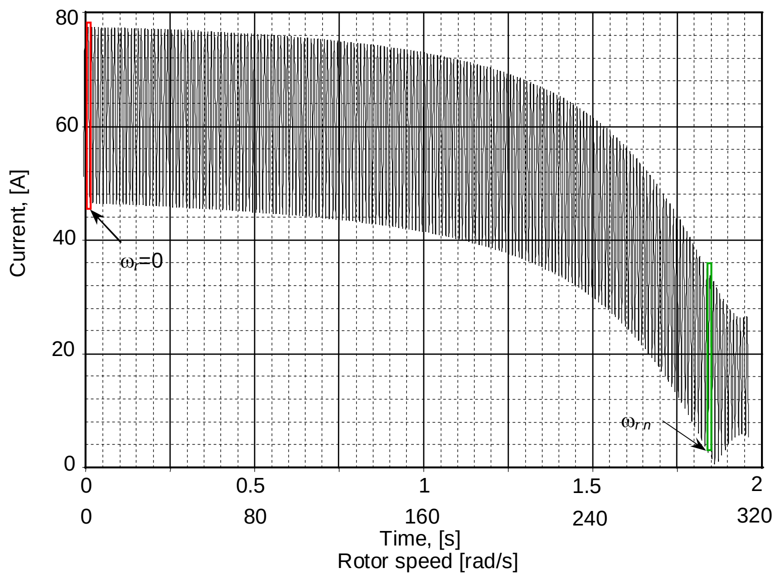

Fig. 6 − Stator current versus time.

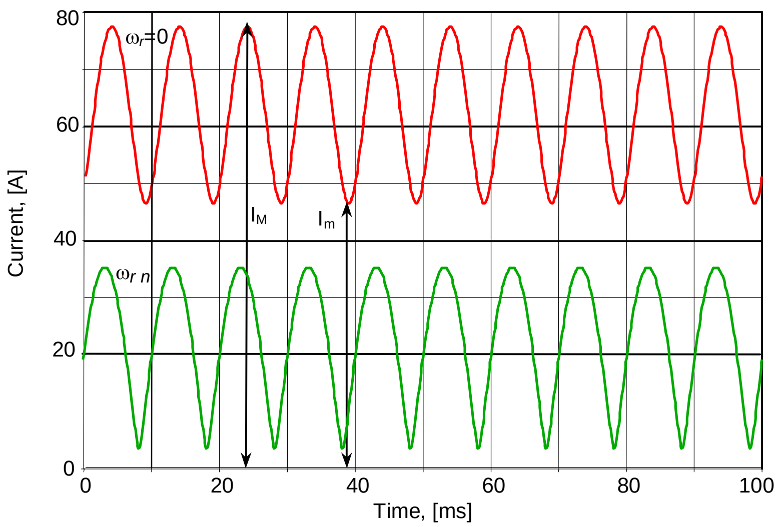

Fig. 7 − Stator current versus time.

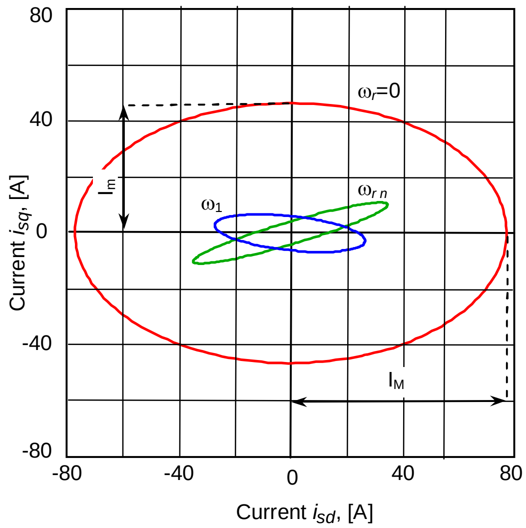

Fig. 8 – Stator current hodograph

4. Conclusions

The case of an induction motor with the star connection and the neutral connected was considered. The machine parameters (resistors, inductances, mutual inductances) have the same values for each phase since the machine is symmetric by construction. Supplying an induction machine by an unbalanced voltage system leads to an asymmetrical system of stator currents.

For a symmetrical healthy machine equal impedances on each phase are provided. An analysis reveals that the impedance value on each phase depends on the angular speed (or slip): $\underline{Z} = \underline{Z}(\omega_r) = \underline{Z}(s)$. The impedances become parametric elements and the generalized unbalanced load theory cannot be used.

If the supply is asymmetric, for the positive sequence $\omega_{r+} = \omega_r$ (or $s_+ = s$) is used. For the negative phase sequence characterized by the angular speed $-\omega_1$, the value $\omega_{r-} = -\omega_r$ respectively $s_- = (\omega_1 + \omega_r)/\omega_1 = 2 - s$ must be used.

The current dissymmetry factor depends on the voltage dissymmetry factor but also on a relatively complicated expression depending on the impedances of the machine defined as *DFG* (dissymmetry factor gain). In starting operation, $\omega_r = 0$, the two sequences are equal. As the speed increases, the current corresponding to the negative sequence increases slightly while the current corresponding to the positive one strongly decreases. This explains the growth of *DFG* with rotor speed. So *DFG* can reach the value 6,...,10 at no-load or synchronism operation.

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CONSIDERAȚII PRIVIND CURENȚII STATORICI AI MAŞINII DE INDUCȚIE ALIMENTATĂ ASIMETRIC

(Rezumat)

Alimentarea unei maşini de inducție cu un sistem nesimetric de tensiuni conduce la un sistem nesimetric de curenți statorici absorbiți de la rețea. Deasemenea hodograful curenților statorici nu mai este circular şi devine eliptic. Este normal ca factorul de disimetrie al curenților să depindă de factorul de disimetrie al tensiunii de alimentare. Dar rezultatele obținute în diferite condiții de funcționare indică o influență notabilă a turației rotorului asupra regimului maşinii de inducție alimentată nesimetric. Lucrarea îşi propune să studieze influența regimului de funcționare asupra regimului asimetric al curenților statorici absorbiți de maşină de la rețea. Abordarea problemei s-a facut pornind de la aspectele teoretice confirmate şi completate de rezultatele obținute prin simulare.