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TRANSLINEAR CIRCUITS FOR SYNTHESIS OF PROGRAMMABLE NONLINEAR FUNCTIONS

BY

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Abstract. Two methods to synthesize piecewise programmable nonlinear functions are presented. The synthesized functions are implemented using current mode elementary circuits that exhibit piecewise characteristics. Two versions of elementary circuits are introduced, a bipolar transistors based circuit and a MOS version, respectively, that can be combined by means of arithmetic operators (addition/subtraction) or composition operator to generate complex piecewise nonlinear functions. HSpice simulations confirm the expected behaviour of the proposed circuits.

Key words: piecewise programmable nonlinear functions; current mode circuits; subthreshold MOS circuits; bipolar circuits.

1. Introduction

Analogue circuits that operate based on nonlinear functions are used in a broad range of applications as intelligent systems (Andreou et. all., 1991), (Madrenas *et al.*, 1996; Wilamovski *et al.*, 1999), signal processing (Popa, 2012), instrumentation or linearization of sensor characteristics (Benammar, 2005), etc.

Various techniques have been proposed over the years to synthesize nonlinear functions. Some of them are based on the diode-linear resistor

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combination (Abuelma'atti, 1981), or current mirrors (Wilamowski *et al*., 2000) to generate piecewise nonlinear functions. Others exploit the square-law characteristic of the MOS transistors (Liu, 1996) or are based on the translinear principle to build current mode function synthesizers (Abuelma'atti, 1999).

One of the main challenges raised by the circuits reported in the literature consists on the ability to adjust the shape of the synthesized function, especially their slopes.

The present paper describes a method, based on current mode translinear circuits, that allows a fully programmable synthesized function. According to this, the height and the slopes of the synthesized function are independently tuned by means of electrical currents. Two proposed synthesis techniques are described in Section **2**. Section **3** introduces two elementary current mode circuits that exhibits piecewise programmable characteristics, one being built using bipolar transistors and the other being based on MOS transistors that operate in the subthreshold region. Section **4** shows how these elementary circuits can be combined using the synthesis techniques described in Section **2** to generate fully programmable piecewise nonlinear functions. The response of each circuit is analysed based on HSpice simulations, which confirm their expected behaviour. Some conclusions are drawn in Section **5**.

2. Two Nonlinear Function Synthesis Techniques

The proposed technique allows the synthesis of positive programmable shape linear functions. It uses two basic functions, chosen so as to have opposite variations in their values. Thus, considering an input variable, denoted by x, and two output variables, denoted by y_1 and y_2 respectively, the variations of the output variables are defined according to the basic functions. The shape of these functions is controlled by means of three free parameters, denoted by H, L, and R respectively, where H is always considered positive and sets the hight of the functions and L and R, with $L < R$, set their range $S = R - L$. The basic functions are defined according to the equations:

$$y_1 = f\left(x,H,L,R\right) = \begin{cases} H & \text{if} \quad x \le L \\ -m\cdot\left(x-R\right) & \text{if} \quad L < x < R \\ 0 & \text{if} \quad R \le x \end{cases} \tag{1.a}$$

$$y_2 = f\left(x,H,L,R\right) = \begin{cases} 0 & \text{if} \quad x \le L \\ +m\cdot\left(x-L\right) & \text{if} \quad L < x < R \\ H & \text{if} \quad R \le x \end{cases} \tag{1.b}$$

where m represents the slope of the functions, defined as below:

$$m = \frac{H}{R-L} = \frac{H}{S} \tag{1.c}$$

Some variants of the basic functions, for two different pairs of free parameters are depicted in Fig. 1. As can be seen, the heights and slopes of these functions can be entirely programmed by means of their parameters.

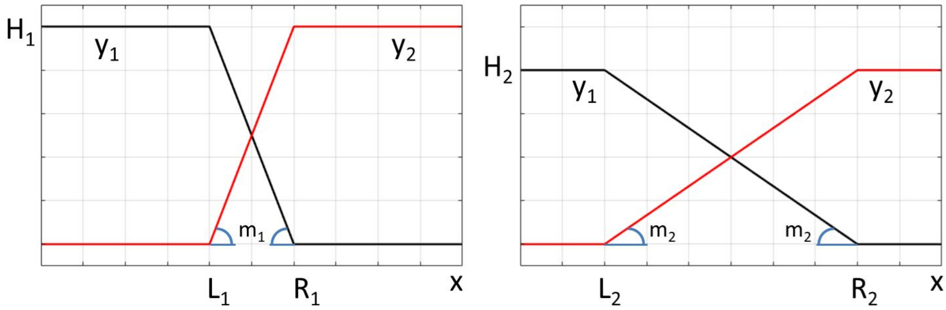

Fig. 1 – The graphs of the proposed basic functions.

The proposed synthesis technique considers two basic functions $f_1(x, H_1, L_1, R_1)$ and $f_2(x, H_2, L_2, R_2)$, combined by subtraction or composition operators, respectively. The choice of these operators is motivated by the simplicity of their hardware implementation, especially when current mode analog circuits are used.

To show the proposed technique, two pairs of basic functions are considered:

$$y_{11} = \begin{cases} H_1 & if \quad x \le L_1 \\ -m_1 \cdot (x - R_1) & if \quad L_1 < x < R_1 \\ 0 & if \quad R_1 \le x \end{cases} \qquad y_{21} = \begin{cases} 0 & if \quad x \le L_1 \\ m_1 \cdot (x - L_1) & if \quad L_1 < x < R_1 \\ H_1 & if \quad R_1 \le x \end{cases}$$

$$y_{12} = \begin{cases} H_2 & if \quad x \le L_2 \\ -m_2 \cdot (x - R_2) & if \quad L_2 < x < R_2 \\ 0 & if \quad R_2 \le x \end{cases} \qquad y_{22} = \begin{cases} 0 & if \quad x \le L_2 \\ m_2 \cdot (x - L_2) & if \quad L_2 < x < R_2 \\ H_2 & if \quad R_2 \le x \end{cases} \tag{3}$$

where

$$m_1 = \frac{H_1}{R_1 - L_1} = \frac{H_1}{S_1} \quad and \quad m_2 = \frac{H_2}{R_2 - L_2} = \frac{H_2}{S_2} \tag{4}$$

2.1. Synthesis by Subtraction

This technique synthesizes a new function by subtracting two basic functions. When it is taken into consideration for a hardware implementation, current mode analog circuits can be effective solutions, where the subtraction is merely wired, based on the Kirkhoff current law.

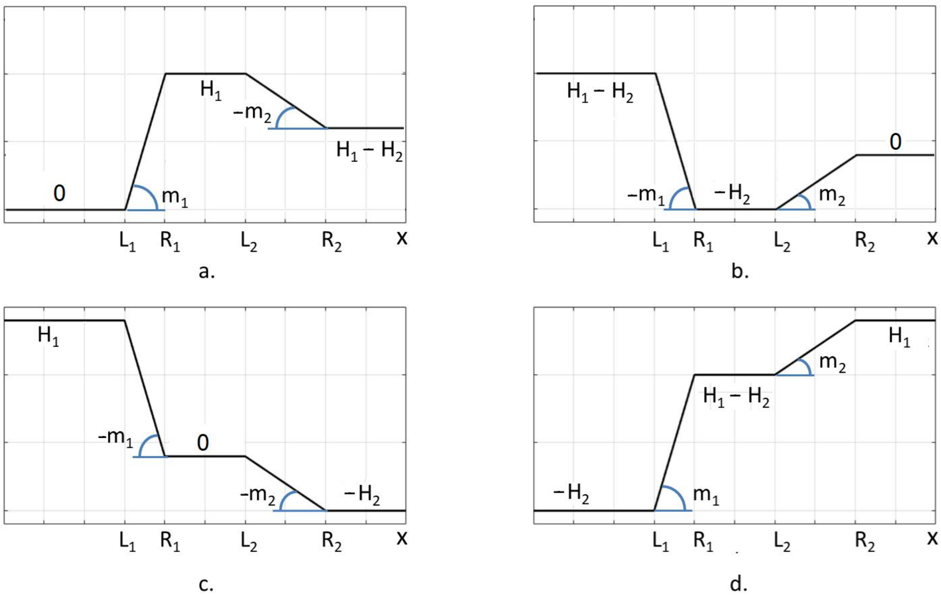

Fig. 2 – Various functions synthesized by subtraction technique:
a. $y_{21} - y_{22}$; b. $y_{11} - y_{12}$; c. $y_{11} - y_{22}$; d. $y_{21} - y_{12}$.

Using this technique, if two similar basic functions are subtracted, trapezoidal - shaped or u-shaped functions can be synthesized, depending on choosing the minuend and the subtrahend in the computed difference.

For example, if $L_1 < R_1 < L_2 < R_2$, considering $y_{21} - y_{22}$, the trapezoidal – shaped function depicted in Fig. 2a and described by equation (5) is synthesized. On the other hand, taking $y_{11} - y_{12}$, the u-shaped function depicted in Fig. 2b is generated.

$$y_{21} - y_{22} = \begin{cases} 0 & \text{if} \quad x \le L_1 \\ m_1 \cdot (x - L_1) & \text{if} \quad L_1 < x < R_1 \\ H_1 & \text{if} \quad R_1 \le x \le L_2 \\ -m_2 \cdot (x - R_2) & \text{if} \quad L_2 < x < R_2 \\ H_1 - H_2 & \text{if} \quad R_2 \le x \end{cases} \quad (5)$$

If two opposite basic functions are subtracted, then monotonic decreasing or increasing functions are generated. Some examples are shown in Figs. 2c and 2d, respectively. With this technique, if the application requires, the inner steady levels of the synthesized functions can be removed if the condition $R_1 = L_2$ is adopted.

2.2. Synthesis by Composition

The key idea of this technique is to combine opposite basic functions and to set the height of one basic function equal to the opposite function. By this condition, the height of the first basic function follows the variation of the second one, yielding a programmable shape nonlinear function. For example, if $L_1 < R_1 \le L_2 < R_2$, imposing $H_1 = y_{12}$ leads to a variation of y_{21} defined as below:

$$y_{21} = \begin{cases} 0 & \text{if} \quad x \le L_1 \\ m_1 \cdot (x - L_1) & \text{if} \quad L_1 < x < R_1 \\ H_2 & \text{if} \quad R_1 \le x \le L_2 \\ -m_2 \cdot (x - R_2) & \text{if} \quad L_2 < x < R_2 \\ 0 & \text{if} \quad R_2 \le x \end{cases} \tag{6}$$

Similar results can be found at y_{12} if $L_1 < R_1 \le L_2 < R_2$ and $H_2 = y_{21}$, at y_{11} if $L_2 < < R_2 \le L_1 < R_1$ and $H_1 = y_{22}$ or at y_{22} if $L_2 < R_2 \le L_1 < R_1$ and $H_2 = y_{11}$, respectively. When this technique is implemented in hardware, because it avoids additional arithmetic operators, as addition or subtraction, to synthesize a new function, it could be more effective than the previous one. Fig. 3 shows various functions synthesised by composition technique.

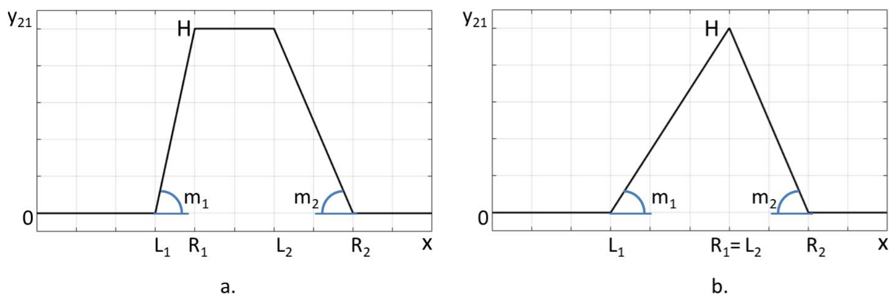

Fig. 3 – Various functions synthesized by composition technique:
a. $y_{21} \circ y_{12}$, $R_1 < L_2$; b. $y_{21} \circ y_{12}$, $R_1 = L_2$.

3. The Implementation of the Basic Functions by Translinear Circuits

This section presents two different elementary circuits, able to implement the previously introduced basic functions, used to synthesize programmable nonlinear functions. The first circuit employs bipolar transistors while the second one uses MOS transistors working in the weak inversion (subthreshold) region. Both circuits use similar techniques to implement the basic functions, namely they use a translinear loop to set their behaviour inside the range *S*, where the values of the basic functions vary according to a programmable slope and rectifier elements to set their behaviour outside the range *S*, where the values of the basic functions are constant, respectively.

Due to te fact that both circuits operate in current mode, all variables and parameters considered in the previous section are represented by means of electrical currents. The correspondence between these and the notations introduced in Section **2** are as follows: $x \rightarrow i_X$, $y_1 \rightarrow i_1$, $y_2 \rightarrow i_2$, $H \rightarrow I_H$, $L \rightarrow I_L$, $R \rightarrow I_R$.

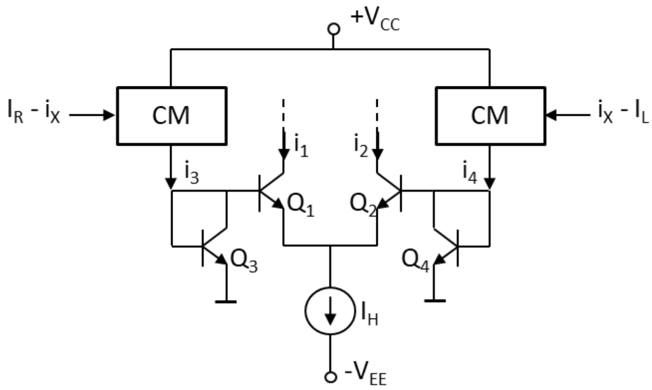

Fig. 4 – The bipolar version of the elementary circuit.

Fig. 4 shows the bipolar version of the elementary circuit. It uses a translinear loop, consisted of $Q_1 \div Q_4$ transistors and two current mirrors, denoted by *CM*, respectively. The diode connected transistors Q_3 and Q_4 control the operating regime of the emitter connected transistor Q_1 and Q_2. In turn, the operating regime of Q_3 and Q_4 transistors depends on the output currents of the currents mirrors, that finally control the variation of the current in the collector of Q_1 and Q_2 transistors, namely i_1 and i_2.

Since the current mirrors accept only positive currents, they act as current rectifiers for the difference currents $I_R - i_X$ and $i_X - I_L$, respectively, what allow them setting the threshold conditions marked in the equations (1). According to the previous notations, the output currents of the current mirrors are:

$$i_3 = \begin{cases} I_R - i_X & \text{if} \quad i_X < I_R \\ 0 & \text{if} \quad I_R \le i_X \end{cases} \qquad i_4 = \begin{cases} 0 & \text{if} \quad i_X \le I_L \\ i_X - I_R & \text{if} \quad I_L < i_X \end{cases} \qquad (7)$$

Even if a diode connected transistor is able to rectify the electrical current, connecting Q_3 and Q_4 to the base terminals of Q_1 and Q_2 transistors impedes their use as current rectifiers. This motivates the presence of the current mirrors in the circuit.

Depending on the output currents of the current mirrors, one of the emitter coupled transistors Q_1, Q_2 could be blocked while the other is active, or both can be active simultaneously, respectively. In the former case, the entire bias current I_H is supplied to the active transistor.

Thus, if $i_X \leq I_L$, since $I_L < I_R$, according to the equation (7) the left side current mirror outputs a nonzero current feeding Q_3 which yields Q_1 to work in the forward active regime. On the other hand, from (7) the right side current mirror outputs a null current, removing the electrical current in the diode connected transistor Q_4, yielding Q_2 to be blocked. Consequently, the entire bias current I_H flows through Q_1, whereas Q_2 has a null current:

$$i_1 = I_H \qquad i_2 = 0 \tag{8}$$

A similar behaviour, but in an opposite fashion, is obtained if the input current i_X passes above I_R threshold, $I_R \leq i_X$. In this case the state of the transistors in the translinear loop is reversed. Consequently,

$$i_1 = 0 \qquad i_2 = I_H \tag{9}$$

Finally, if the input current i_X has a value between both thresholds, $I_L < < i_X < I_R$ all transistors in the translinear loop are active. In this case, if the Early voltage is neglected, v_{BE} transistor voltages of the translinear loop depend on their collector currents by the logarithmic expressions $v_{BE} = V_T \cdot \ln(i_C/V_T)$, where I_S is the saturation current and V_T is the thermal voltage. If the base currents in Q_1 and Q_2 are neglected, considering all transistors matched, Kirkhoff voltage law along the translinear loop gives

$$V_T \cdot \ln\frac{i_1}{I_S} + V_T \cdot \ln\frac{i_4}{I_S} = V_T \cdot \ln\frac{i_2}{I_S} + V_T \cdot \ln\frac{i_3}{I_S}$$

that leads to

$$\frac{i_1}{i_2} = \frac{i_3}{i_4}$$

Taking into account that $i_1 + i_2 = I_H$ and, for the aforementioned condition, $i_3 = I_R - i_X$ and $i_4 = i_X - I_L$, using some simple mathematical properties of ratios, the output currents i_1 and i_2 can be expressed as:

$$i_1 = -m \cdot (i_X - I_R) \qquad i_2 = m \cdot (i_X - I_L) \tag{10.a}$$

where: m is the slope wherewith the output currents change their values, which can be easily tuned by means of the free parameters I_L and I_R, according to the equation:

$$m = \frac{I_H}{I_R - I_L} \tag{10.b}$$

Finally, considering the entire range of the input current i_X, the output currents can be expressed as below:

$$i_1 = \begin{cases} I_H & \text{if} \quad i_X \le I_L \\ -m \cdot (i_X - I_R) & \text{if} \quad I_L < i_X < I_R \\ 0 & \text{if} \quad I_R \le i_X \end{cases} \tag{11.a}$$

$$i_2 = \begin{cases} 0 & \textit{if} \quad i_X \le I_L \\ +m \cdot (i_X - I_L) & \textit{if} \quad I_L < i_X < I_R \\ I_H & \textit{if} \quad I_R \le i_X \end{cases} \tag{11.b}$$

where: m is given by (10.b). As can be seen, the equations (11) are similar to the equations (1). Consequently, the output currents i_1 and i_2 implement the basic functions y_1 and y_2, respectively, introduced in the previous section.

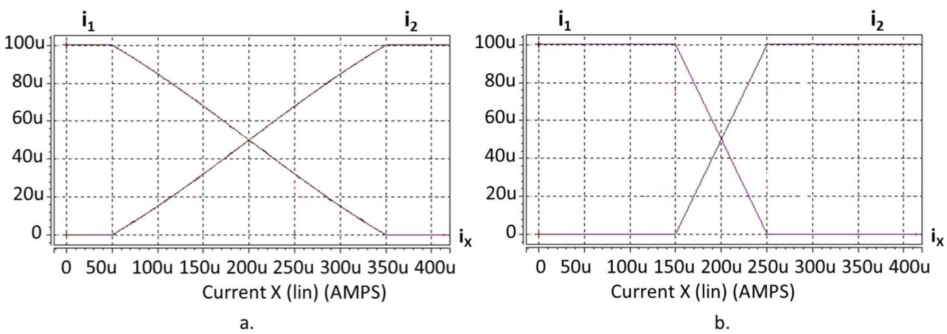

Fig. 5 – The transfer characteristics of the bipolar version of the elementary circuit: I_H = 100µA a. I_L = 50µA, I_R = 350µA; b. I_L = 150µA, I_R = 250µA.

The functionality of the bipolar version of the elementary circuit was checked by simulations in HSpice environment. Fig. 5 shows the transfer characteristics of the circuit obtained for different sets of parameters, where, to point out the accuracy in the slope programmability, I_H parameter was kept constant.

Fig. 6 presents the MOS alternative of the elementary circuit, where the level of all electrical currents is chosen such as all transistors operate in the week inversion region. The MOS based solution is more compact than the bipolar one. As can be seen, unlike the bipolar alternative, this solution requires only a single power supply. Another benefit of this solution is based on the fact that the gate currents in M_1 and M_2 are zero, what allows that M_3 and M_4 transistors to operate as rectifier elements for the difference currents $I_R - i_X$ and $i_X - I_L$, respectively, causing the presence of the current mirrors to be no longer required.

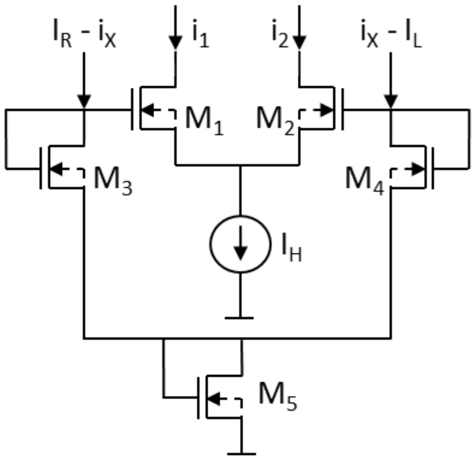

Fig. 6 – The MOS version of the basic circuit.

Similarly to the bipolar solution, the MOS elementary circuit consists of a translinear loop $M_1 \div M_4$. The additional transistor M_5 is required to raise the electrical potential in the common source node S, to keep the operation of $M_1 \div M_4$ in the saturation region with only a single power supply.

If $V_{BS} = 0$ for all transistors of the translinear loop, if all these operate in the weak inversion region, their drain current depends on the corresponding V_{GS} voltage according to the equation:

$$I_D = I_S \exp\left(\frac{k \cdot V_{GS}}{V_T}\right) \tag{12}$$

where: I_S is a current proportional to W/L ratio, mobility μ and other silicon physical properties, V_T is the thermal voltage and k is a constant between 0.7 ÷1 that represents the back-gate coefficient describing the effectiveness of the gate voltage change on the surface potential.

If all transistors are matched and work in the week inversion region, the operation of this circuit is similar to the one described for the bipolar circuit. Consequently, the variation of its output currents is described by means of equations (11), where the slope can be programmed according to the equation (10.b). To check this, a set of simulations was performed in HSpice environment and the obtained results are depicted in the Fig. 7. As can be seen, this solution also allows a high level of accuracy in the slope programmability.

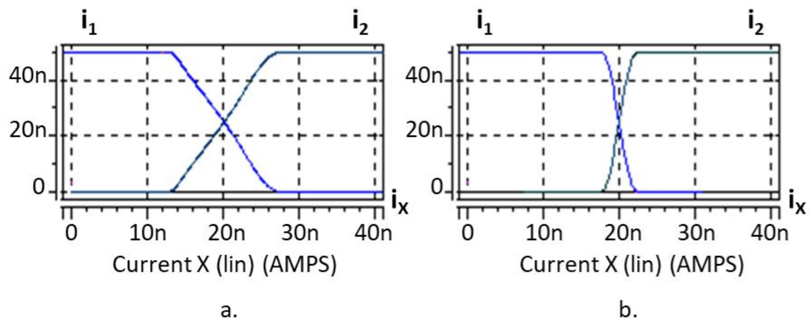

Fig. 7 – The transfer characteristics of the MOS version of the elementary circuit: I_H = 50nA a. I_L = 13nA, I_R = 27nA; b. I_L = 18nA, I_R = 22nA.

4. Results

Various piecewise programmable nonlinear functions can be synthesized using the circuits introduced in Section **3**. For example, Fig. 8 shows how two bipolar elementary circuits can be combined by subtraction to generate nonlinear trapezoidal – shaped functions. For the sake of simplicity, the bipolar elementary circuits were depicted as box elements, where the positive supply terminal was omitted. The subtraction operation between the output currents of the bipolar elementary circuits is implemented based on a current mirror, applying the Kirkhoff current law in its output node. When subtraction is used to synthesize nonlinear functions these are always represented by means of electrical voltages.

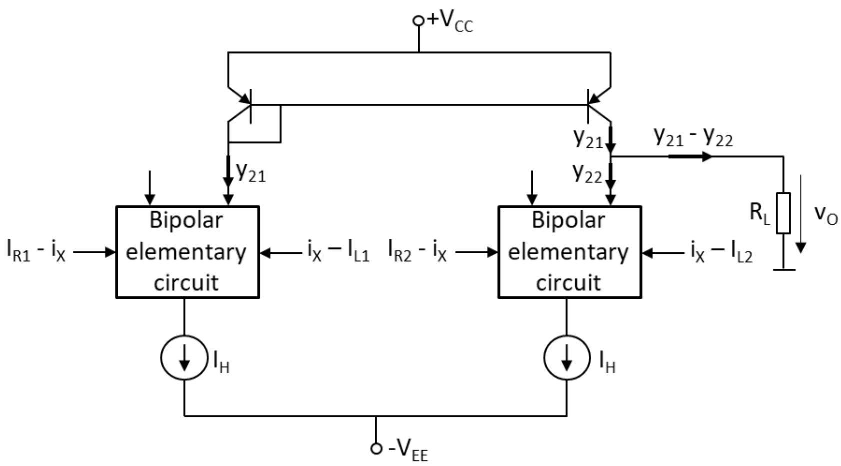

Fig. 8 – Example of using bipolar elementary circuit for nonlinear functions synthesis.

The circuit presented in Fig. 8 was simulated in HSpice and the obtained results, presented in Fig. 9, confirm the expected behavior.

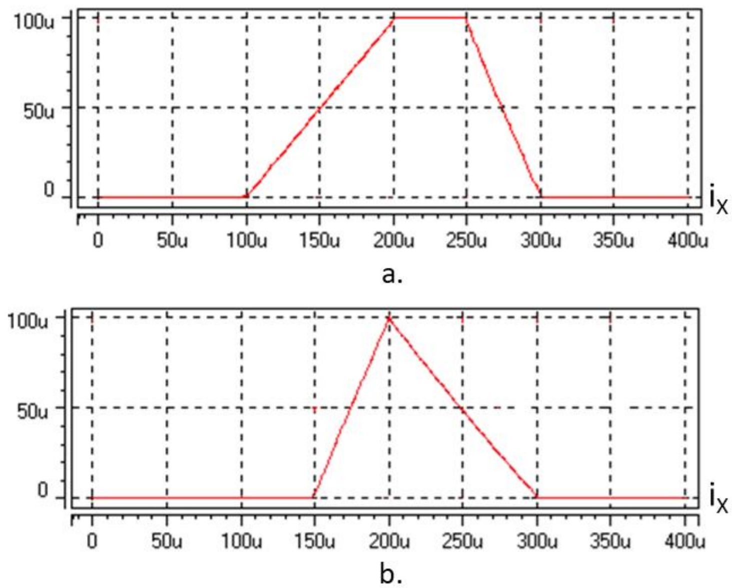

Fig. 9 – Nonlinear functions synthesized using the subtraction technique:
a. Trapezoidal-shaped nonlinear function: I_H =100μA, I_{L1} = 100μA, I_{R1} = 200μA,
b. I_{L2} = 250μA, I_{R2} = 300μA; b. Triangular-shaped nonlinear function: I_H =100μA,
I_{L1} = 150μA, I_{R1} = 200μA, I_{L2} = 200μA, I_{R2} = 300μA.

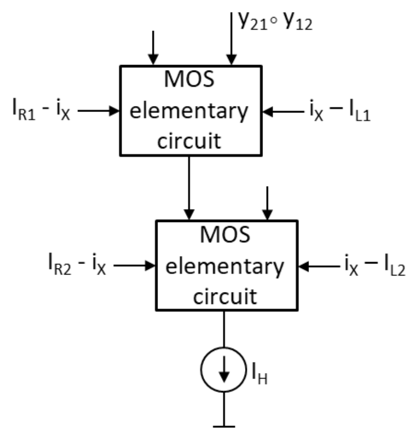

Fig. 10 – Example of using MOS elementary circuit
for nonlinear functions synthesis.

A more compact solution to synthesize nonlinear piecewise functions can be obtained using composition technique. Fig. 10 presents how two MOS elementary circuits can be combined to generate nonlinear functions by composition method. Again, for the sake of simplicity, the MOS elementary circuits were depicted as box elements. The circuit was simulated in HSpice and the obtained results are depicted in Fig. 11.

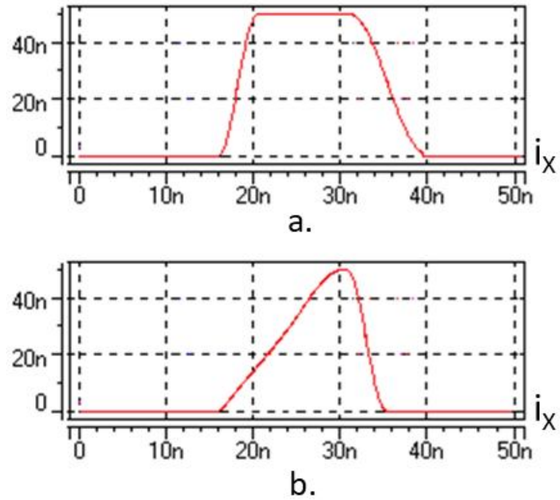

Fig. 11 – Nonlinear functions synthesized using the composition technique:
a. Trapezoidal-shaped nonlinear function: I_H =50nA, I_{L1} = 16nA, I_{R1} = 20nA, I_{L2} = 32nA, I_{R2} = 40nA; b. Triangular-shaped nonlinear function: I_H =50nA, I_{L1} = 16nA, I_{R1} = 30nA, I_{L2} = 30nA, I_{R2} = 36nA.

5. Conclusion

The present paper presents two different methods to synthesize programmable piecewise nonlinear functions. These techniques are implemented based on elementary current mode circuits that exhibit piecewise transfer characteristics. Two versions of elementary circuits are presented and verified by HSpice simulations, a bipolar based circuit and a MOS based version, respectively. These circuits are combined to generate complex programmable nonlinear functions, and HSpice simulations confirm the expected behavior.

REFERENCES

Abuelma’atti, M.T., *Synthesis of a Concave Monotonically Increasing Function Using the Diode-Equation Model*, Internat. J. of Electronics, *51*, 57-62 (1981).

Abuelma'atti M.T., Abed S.M., *A Translinear Circuit for Analogue Function Synthesis Based on a Taylor Series*, **56**, 1341-1348 (1999).

Andreou A.G, Boahen K.A., Pouliquen P.O., Parasovic A., Jenkins R.E., *Current Mode Subthreshold MOS Circuits for Analog VLSI Neural Systems*, IEEE Transactions on Neural Networks, **2**, *2*, 205-213 (1991).

Benammar M., *Precise, Wide-Range Approximation to a Sine Function Suitable for Analog Implementation in Sensors and Instrumentation Applications*, IEEE Transactions on Circuits and Systems-I: Regular Papers, **52**, 262-270 (2005).

Liu S.I., Chang C.C., *A CMOS Square-Law Vector Summation Circuit*, IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing, **43**, 520-524 (1996).

Madrenas J., Verleysen M., Thissen P., Voz J.L, *A CMOS Analog Circuit for Gaussian Functions*, IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing, **43**, *I*, 70-74 (1996).

Popa C., *High-Accuracy Function Synthesizer Circuit with Applications in Signal Processing*, EURASIP J. on Advances in Signal Processing (2012).

Wilamowski B.M., Ferre-Pikal E.S., Kaynak O., *Low Power, Current Mode CMOS Circuits for Synthesis of Arbitrary Nonlinear Functions*, 9th NASA Symposium on VLSI Design, 2000.

Wilamowski B.M., Jaeger R.C., Kaynak M.O., *Neuro-Fuzzy Architecture for CMOS Implementation*, IEEE Transactions on Industrial Electronics, **46**, *6*, 1132-1136 (1999).

CIRCUITE TRANSLINEARE PENTRU SINTEZA FUNCȚIILOR NELINIARE PROGRAMABILE

(Rezumat)

Sunt prezentate două metode utilizate pentru sinteza funcțiilor neliniare. Metodele respective sunt implementate electronic pe baza a două circuite elementare, în care informațiile sunt reprezentate prin intermediul curenților electrici. Pentru circuitele propuse, în conținutul lucrării, sunt prezentate două versiuni de circuite electronice şi anume unul în care sunt utilizate tranzistoare bipolare şi unul în care sunt utilizate tranzistoare MOS. Aceste circuite pot fi combinate prin intermediul unor operatori aritmetici (adunare/scădere) sau prin operatorul de compoziție, în scopul generării unor funcții neliniare complexe. Simulări realizate în mediul HSpice confirmă comportamentul aşteptat pentru circuitele propuse.