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AN OPTIMAL MODEL FOLLOWING PROBLEM FOR A VOLTAGE CONTROLLED DRIVE SYSTEM

BY

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Abstract. The linear quadratic optimal model following problem with finite final time and free end point for a drive system is studied. A technique to achieve reduced energy consumption and an adequate behavior of the drive system by imposing the dynamic of the system near to a chosen model one is presented. The study takes into account the influence of the exogenous variables (e.g. initial state and load torque), using some previous results of the authors. For the load torque, a step variation in the transient time is considered. Such circumstances are often met in the electrical drive systems, when the no-load or small load torque is switched to a great one. The influence of the erroneous estimation of the load torque or of the switching moment is considered. The applicability of the proposed algorithm is validated by simulation tests.

Key words: electrical drive; different motors; variable load torque; imposed dynamics; linear quadratic optimal problem.

1. Introduction

Imposing a certain dynamic behaviour is one of the most interesting approaches to design an automatic system due to the possibility to force the

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desired performances. For example, one can impose for the system to have a dynamic behaviour similar to a chosen model. The model could exist in the control system or could be an implicit one, whose behaviour must be followed.

The paper deals with optimal implicit model following control of a voltage controlled drive system with variable load torque. Of course, a general algorithm for a variable torque can be established, but the implementation is significantly easier for a constant load torque. Therefore, a suboptimal solution can be obtained if the variation of the load torque in the transient period is approximated with a step function. Such situations are frequently met in the electrical drive systems, when the no-load or small load torque is followed by a great one. Examples for such operation are the rolling mills, or cutting processes.

A linear electrical drive system is described by the state equation

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w(t),\ x(t_0) = x^0 \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where: $x(t) = \begin{bmatrix} \omega(t) & i(t) \end{bmatrix}^T$ is the state vector (T denotes the transposition), $w(t) = \begin{bmatrix} w_a & 0 \end{bmatrix}^T m(t)$ is the disturbance vector (with $m(t)$ the load torque) and $\omega(t)$ is the rotor speed. The variables i and u are the rotor current and voltage in the case of a brushed d.c. motor. For synchronous and asynchronous motors, i and u correspond to the q current and voltage components and similar equations may be adopted with adequate assumptions. The matrices A, B, C are in the form

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\quad B = \begin{bmatrix} 0 \\ b_2 \end{bmatrix},\quad C = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

where: $a_{11}, a_{12}, a_{21}, a_{22}, b_2, w_a$ are constant which depend on the parameters of the motor and load.

The aim of the optimal control is to achieve reduced energy consumption and an acceptable behaviour of the system (Cavallaro *et al.,* 2002; Kurihara & Rahman, 2004; Lorenz, 2006; Sheta *et al.*, 2009; Takami, 2006; Xu, 2012; Yang & Zou, 2013). A way to reach these goals is to impose to the system a dynamic near to a chosen model one. For this one can introduce the term $\dot{y}(t) - Ly(t),\ L \in \Re^{m \times m}$ in the performance criterion (Tyler, 1964). A more convenient way is to use the term $\dot{x}(t) - Lx(t),\ L \in \Re^{n \times n}$ in the criterion, because this offers more possibilities in the choice of the dynamic behaviour of the model (Botan & Onea 1999; Li & Yu, 2009; Shibasaki *et al.*, 2015). Indeed, in the first variant, the order of the model is limited at the number of the outputs. However, the both indicated ways are properly only for the free response of the system, because the model do not includes the influence of the control $u(t)$.

Moreover, for many applications, such as the electrical drive system, it is essential to consider the influence of the disturbance (*e.g.* the load torque) on the system and also on the model. Consequently, it is useful to introduce a term depending on the errors between the model and the system in the form $\dot{x}(t) = Lx(t) + Bu(t) + w(t)$. Starting from the above remarks, the following quadratic criterion was adopted:

$$\begin{gathered} J = \frac{1}{2}(x(t_f) - x_d)^T S(x(t_f) - x_d) + \\ + \frac{1}{2}\int_{t_0}^{t_f} \Big\{ [\dot{x}(t) - Lx(t) - Bu(t) - w(t)]^T Q_1 \left[\dot{x}(t) - Lx(t) - Bu(t) - w(t)\right] + \\ + \left[x(t) - x_d\right]^T Q_2 \left[x(t) - x_d\right] + u^T(t)Pu(t) \Big\} \mathrm{d}t, \end{gathered} \tag{2}$$

where: x_d denotes the desired value of the state. The matrices from the criterion have appropriate dimensions and $S \geq 0$, $Q_1 \geq 0$, $P > 0$.

The optimal control problem refers to the system (1) and criterion (2). In order to ensure a small transient period, a finite final time was adopted in the performance criterion.

The solution of the problem is similar to that of the LQ optimisation problem with finite final time and the result is a time variant controller (Athans & Falb, 1966; Anderson & Moore, 1990). The paper proposes a different solution based on previous research of the authors (Botan *et al.*, 2008; Botan & Ostafi, 2012) and leads to an easier implementation.

2. Main Results

The hamiltonian for the above formulated model following problem is:

$$\begin{gathered} H = \frac{1}{2}[\dot{x}(t) - Lx(t) - Bu(t) - w(t)]^T Q_1[\dot{x}(t) - Lx(t) - Bu(t) - w(t)]^T + \\ + \frac{1}{2}[x(t) - x_d]^T Q_2[x(t) - x_d] + \frac{1}{2}u^T(t)Pu(t). \end{gathered} \tag{3}$$

The Hamilton equations lead to:

$$u^*(t) = -P^{-1}B^T\lambda(t), \tag{4}$$

$$-\dot{\lambda}(t) = Qx(t) - Q_2 x_d + A^T\lambda(t), \tag{5}$$

with the final condition

$$\lambda(t_f) = S[x(t_f) - x_d], \tag{6}$$

where:

$$Q = \hat{A}^T Q_1 \hat{A} + Q_2; \quad \hat{A} = A - L. \tag{7}$$

The classical solution is achieved imposing a linear dependence between the co-state and the state vectors as $\lambda(t) = \tilde{R}(t)x(t)$ and the solution to the problem is in the form (Athans & Falb, 1966; Anderson & Moore, 1990):

$$u(t) = -P^{-1}B^T \tilde{R}(t)x(t), \tag{8}$$

where: $\tilde{R}(t)$ is the time-variant solution to a Riccati differential equation. Since this matrix is time-variant, the optimal controller is also time-variant. This implies a difficult implementation. Moreover, the Riccati equation must be integrated in inverse time, starting from the final condition

$$\tilde{R}(t_f) = S. \tag{9}$$

This paper proposes another type of dependence instead of (8) (Botan *et al.*, 2007; Botan & Ostafi, 2012):

$$\lambda(t) = \tilde{R}(t)x(t) + v(t), \tag{10}$$

with $v(t) \in \Re^n$ and R a $n \times n$ symmetrical constant matrix.

From (1), (4), (5), and (10) one obtains:

$$(RNR - RA - A^T R - Q)x(t) - \dot{v}(t) + (RN - A^T)v(t) - Rw + Q_2 x_d = 0.$$

The previous relation must be true for any $x(t)$ and $v(t)$, hence:

$$RNR - RA - A^T R - Q = 0 \tag{11}$$

and

$$\dot{v}(t) = -F^T v(t) + Q_2 x - Rw(t), \tag{12}$$

where:

$$N = BP^{-1}B^T \text{ and } \quad F = A - NR. \tag{13}$$

The final condition leads to:

$$v(t_f) = (S - R)x(t_f) - Sx_d. \tag{14}$$

The optimal control variable $u^*(t)$ can be written as:

$$u^*(t) = u_f(t) + u_c(t), \tag{15}$$

where the feedback component can be computed with

$$u_f(t) = -P^{-1}B^T Rx(t) \tag{16}$$

and the corrective one is

$$u_c(t) = -P^{-1}B^T v(t) . \tag{17}$$

The matrix R is the solution to the Riccati algebraic equation (11) and optimal corrective vector $v(t)$ is the solution to the time-invariant linear differential equation (12), with the final condition (14). The feedback component is identical with the optimal control law obtained in the similar optimal problem, but with infinite final time. The corrective component ensures the identity between the control $u^*(t)$ given by (15) and the control obtained if the constant vector $\lambda(t)$ is adopted in the form (10). As one can see, the corrective component depends on the final condition $x(t_f)$, but the only known value at the beginning of the optimisation process is $x(t_0)$. Therefore, it is necessary to replace the final condition (14) by one depending on the initial condition. On this purpose the above differential equations for $x(t)$ and $v(t)$ are rewritten as:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = G \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} w(t) \\ z(t) \end{bmatrix}, \tag{18}$$

where:

$$G = \begin{bmatrix} F & -N \\ 0 & -F^T \end{bmatrix} \tag{19}$$

and

$$z(t) = Q_2 x - Rw(t) . \tag{20}$$

One can prove (Botan, 1992) that the transition matrix for G is

$$\Omega(t,t_f) = \begin{bmatrix} \Psi(t,t_f) & \Omega_{12}(t,t_f) \\ 0 & \Phi(t,t_f) \end{bmatrix}, \tag{21}$$

where: $\Psi(t,t_f)$ and $\Phi(t,t_f)$ are the transition matrices for F and $-F^T$ respectively, and

$$\Omega_{12}(t,t_f) = \int_t^{t_f} \Psi(t,\tau) N \Phi(\tau,t_f) \mathrm{d}\tau. \tag{22}$$

Taking into account the form (21) for the transition matrix, the state vector can be written as:

$$x(t) = M(t,t_f)x(t_f) - \Omega_{12}(t,t_f)Sx_d + \\ + \int_t^{t_f} [\Psi(t,t_f) - \Omega_{12}(t,t_f)R]w(\tau)\mathrm{d}\tau + \int_t^{t_f} \Omega_{12}(t,t_f)Q_2 x_d \mathrm{d}\tau, \tag{23}$$

where

$$M(t,t_f) = \Psi(t,t_f) + \Omega_{12}(t,t_f)(S-R). \tag{24}$$

From (23) and (14), the vector $v(t_f)$ can be expressed in terms of $x(t_0)$. Finally, the corrective vector $v(t)$ can be computed as:

$$v(t) = \Phi(t,t_0)\{V(t_0,t_f)[x(t_0) + \Omega_{12}(t_0,t_f)Sx_d - g_2(t_0,t_f)] - Sx_d\} + g_1(t,t_f) \tag{25}$$

where

$$g_1(t,t_f) = \int_{t_f}^{t} \Phi(t,\tau)z(\tau)\mathrm{d}\tau, \tag{26}$$

$$g_2(t_0,t_f) = \int_{t_f}^{t_0} [\Psi(t,\tau)w(\tau) + \Omega_{12}(t,\tau)z(\tau)]\mathrm{d}\tau, \tag{27}$$

$$V(t_0,t_f) = \Phi(t_0,t_f)(S-R)M^{-1}(t_0,t_f). \tag{28}$$

The solution can be achieved only if the vector $z(t)$ is known on the optimization interval $[t_0,t_f]$; it means that the shape of the disturbance $w(t)$ must known and its magnitude at the initial moment t_0 must be available (measured or estimated). For simplicity, in the sequel, the disturbance will be considered constant on the optimization interval and therefore the exogenous vector $z(t)$ is constant too. These constant vectors can be extracted from integrals and the computation is simplified. A part of remaining integrals have a constant value, which can be established beforehand and not in real time computation:

$$\int_{t_j}^{t_s} \Psi(t_{j,}\tau)\mathrm{d}\tau = [\Psi(t_j,t_s) - I](-F)^{-1}, \tag{29}$$

$$\int_{t_j}^{t_s} \Phi(t_j,\tau)\mathrm{d}\tau = [I - \Phi(t_j,t_s)]F^{-T}, \quad F^{-T} = (F^T)^{-1}. \tag{30}$$

The advantages which appear for constant disturbance can be extended for variant disturbances if they have a step function form or if an approximation with a piecewise constant function is used. In these cases, the estimation of the disturbance in the first sampling periods is necessary.

The cases when the load torque has a step variation during the transient period, from a small operation to another great value are frequently met in electrical drive systems; in many situations, it is possible to know beforehand the two values of the torque and the switching moment. Therefore, we shall presume that the load torque has two constant values: m_1 for $t \in [t_0, \theta]$ and m_2 for $t \in [\theta, t_f]$. Consequently, the disturbance vector w has the values w_1 and w_2 and the vector z given by (20) has the values z_1 and z_2 on these two intervals.

In this case, from (26) and (27) one obtains:

$$g_1(t,t_f) = \begin{cases} -\int_t^{\theta} \Phi(t,\tau)\mathrm{d}\tau\, z_1 - \int_{\theta}^{t_f} \Phi(t,\tau)\mathrm{d}\tau\, z_2, & t_0 \le t \le \theta \\ -\int_{\theta}^{t_f} \Phi(t,\tau)\mathrm{d}\tau\, z_2, & \theta < t \le t_f \end{cases} \tag{31}$$

$$g_2(t_0,t_f) = -\int_{t_0}^{\theta} [\Psi(t,\tau)w_1 + \Omega_{12}(t,\tau)z_1]\mathrm{d}\tau - \int_{\theta}^{t_f} [\Psi(t,\tau)w_2 + \Omega_{12}(t,\tau)z_2]\mathrm{d}\tau \tag{32}$$

The most previous relations can be computed off line and only two elements (the matrix $\Phi(.)$ and the vector $g_1(t,t_f)$) have to be computed in real time. Both elements can be iteratively obtained (Botan & Ostafi, 2012).

3. Simulation results

Different simulations were performed for an electrical drive system with

$$A = \begin{bmatrix} -0.03 & 18.6 \\ -3.5 & 19.38 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 6.25 \end{bmatrix}.$$

The matrices S, Q_1, Q_2, P, L from criterion (2) and the final state are chosen based on the following considerations (Botan & Onea, 1999):

– $x_d = [\omega_d \quad 0]^T$ because the interest is to penalise the big values of the current $i(t)$ and not $i_d - i(t)$.

– $S = \mathrm{diag}(s_1, 0)$ is chosen in order to select a desired weight of the final error $\omega(t_f) - \omega_d$;

– Q_1 and Q_2 are chosen in order to impose $Q = \mathrm{diag}(q_1, q_2) > 0$;

– $L \in R^{2x2}$ is chosen in order to obtain a certain imposed behaviour for the state. For instance, L was chosen in the same form as the matrix A, but with a smaller value for J_r.

The simulations were done for $\omega_d = 25\text{rad/s}$, $S = \mathrm{diag}(20,0)$, $Q_1 = \mathrm{diag}(0.5,1)$, $Q_2 = \begin{bmatrix} 1 & 1.25 \\ 1.25 & -693.8 \end{bmatrix}$, $P = p = 1$, $L = \begin{bmatrix} -0.1 & 56 \\ -3.5 & -19.4 \end{bmatrix}$, $t_0 = 0$, $t_f = 0.3s$. The sampling period is 2ms in all cases.

The control variable, angular speed and current variations are presented in all figures. The Fig. 1 shows the behaviour of the optimal drive system for constant load torque in the transient period ($m = 0.8\text{Nm}$). Fig. 2 presents the behaviour of the optimal system when the load torque switches from 0.4Nm to 1.5Nm at the moment $t = 0.15\text{s}$.

The influence of the erroneous estimation of the torque or of the commutation moment is indicated in Fig. 3, 4 and 5, respectively. The continuous curves are for correct estimation and the dashed curves are for erroneous estimation. In the first two cases, the step variation is produced at the moment $t = 0.15$s and the load torque has the erroneous estimated values smaller than the real ones (0.2 Nm and 1 Nm instead 0.4 Nm and 1.5 Nm) or the same mean value as the true ones (0.7 Nm and 1.2 Nm instead 0.4 Nm and 1.5 Nm).

The second situation is for a correct estimation of the torque (the values are 0.4Nm and 1.5 Nm), but the real moment t = 0.15 s of the commutation is estimated at t = 0.2 s.

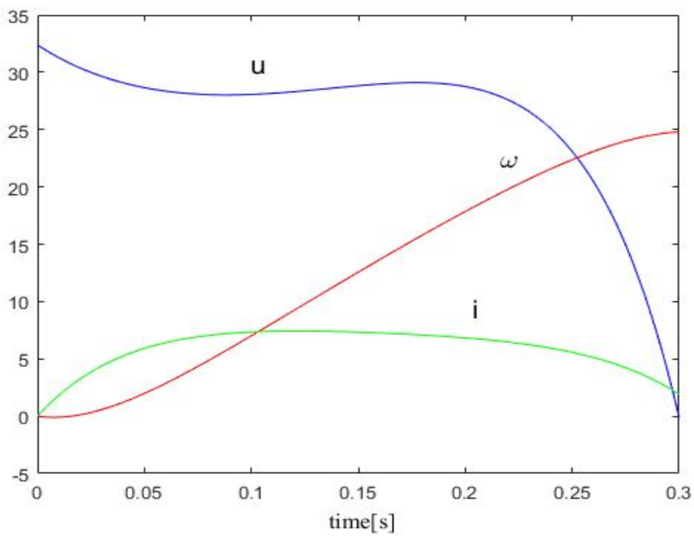

Fig.1 – Optimal system behaviour in the case of a constant load torque.

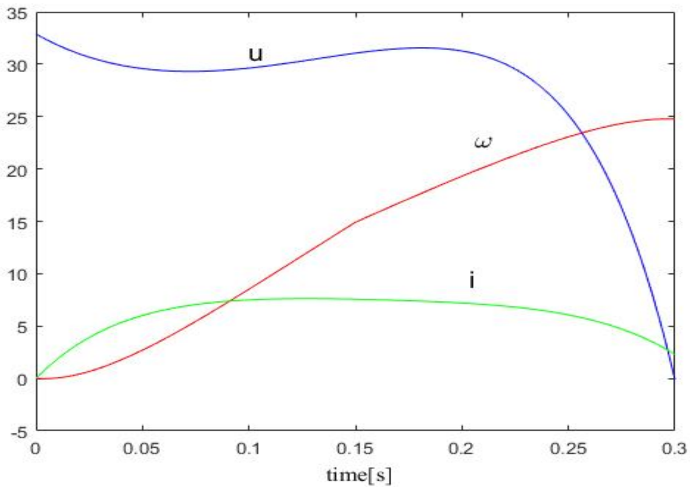

Fig. 2 –System behaviour for m = 0.4 Nm and m = 1.5 Nm; switching moment t = 0.15 s.

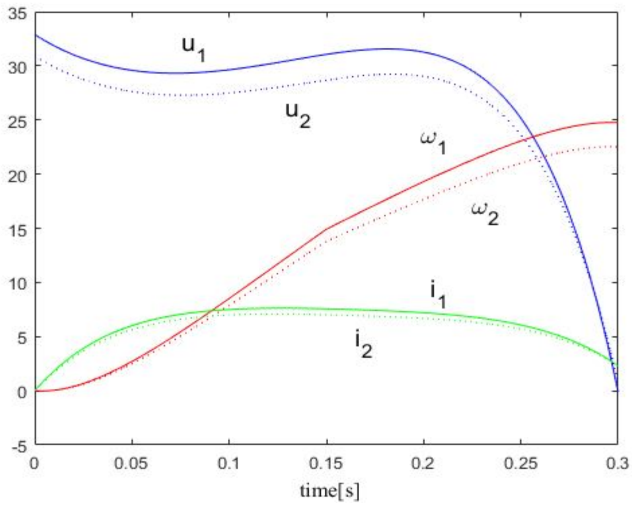

Fig. 3 – Erroneous estimation of the load torque – estimated values smaller than the true ones (true: 0.4; 1.5 Nm; estimated: 0.2; 1 Nm; switching moment at 0.15 s).

The performance criterion and the energy losses were computed in all simulated tests. Because the behaviour is non optimal in the case of erroneous estimations, the performance criterion increases in comparison with a correct one; the differences are not high for reasonable error estimation. The energy losses can decrease for erroneous estimation, because the optimization criterion takes also into account other aspects, like different errors of the state variables.

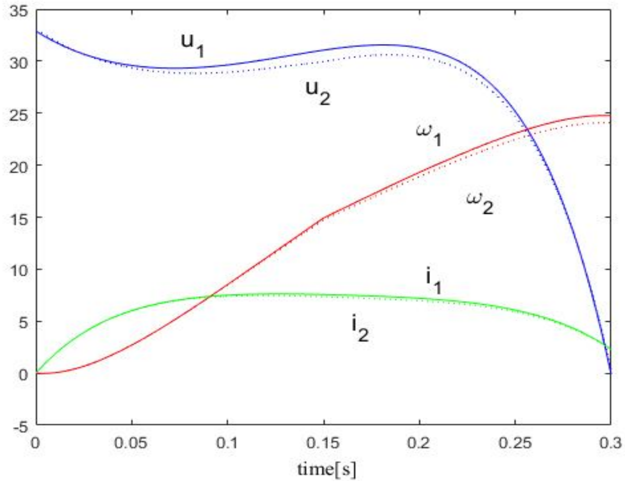

Fig. 4 – Erroneous estimation of the load torque – the same mean value (true: 0.4; 1.5 Nm; estimated: 0.7; 1.2 Nm; switching moment at 0.15 s).

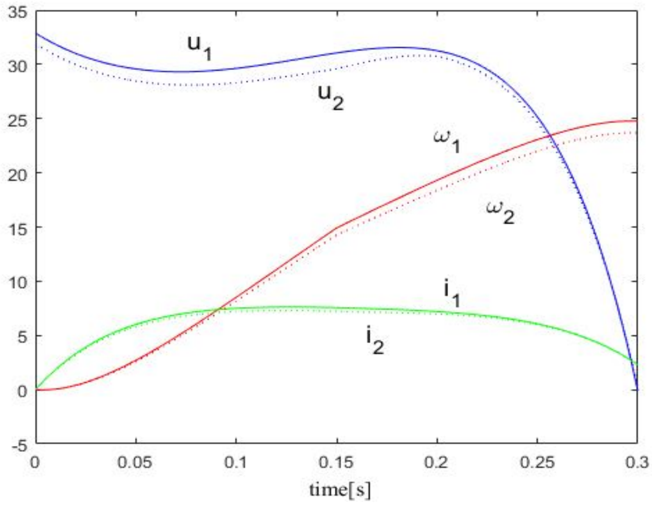

Fig. 5 – The case of erroneous estimation of the switching moment (0.2s instead of 0.15s); m=0.4; 1.5Nm.

4. Conclusions

The paper studies an optimal model following problem for a voltage controlled drive system. For the case when the exogenous variables are beforehand known, or one knows its shape, and the amplitude is estimated at the beginning of the optimization process, a convenient algorithm is presented.

The problem is studied as a linear-quadratic optimal one with finite final time and free end point. A way to use a time-invariant controller is indicated. This controller computes a usual constant feedback component and a corrective component, which depends on the initial state, the desired value of the final state and the disturbance.

The behaviour of the optimal drive system for step variations of the load torque in the transient period is studied and the influences of the erroneous estimations are analyzed.

The simulation tests show the effectiveness of the proposed control system.

REFERENCES

Anderson B.D.O, Moore J.B., *Optimal Control. Linear Quadratic Methods,* Prentice Hall, 1990.

Athans M., Falb P.L., *Optimal Control*, Mc Graw Hill, New York, 1966.

Botan C., *On the Solution of the Riccati Differential Matrix Equation,* Symp. on Computationa System Analysis, Elsever Science Publ, Berlin, 141- 146 (1992).

Botan C., Onea A., *An Optimal Model-Following Problem for Linear Systems*, in Progress in System and Robot Analysis and Control Design, Springer-Verlag, London Limited, 117-126, 1999.

Botan C., Ostafi F., *Optimal Control of Perturbed Linear Systems*, Proc. of 16th Internat. Conf. on System Theory, Control and Computing, Sinaia, Romania, CDROM, 2012.

Botan C., Rățoi M., Horga V., *Optimal Control of Electrical Drives with Induction Motors for Variable Torques*, 13th Internat. Power Electronics and Motion Control Conference, Poznan, Poland, CDROM, 2008.

Cavallaro C., DiTommaso A.O., Miceli R., Raciti A., Galluzzo G.R., Trapanese M., *Efficiency Enhancement of Permanent-Magnet Synchronous Motor Drives by Online Loss Minimization Approaches*, IEEE Trans. on Ind. Electronics, **52**, *4*, 1153-1160 (2002).

Kurihara K., Rahman M.A., *High-Efficiency Line-Start Interior Permanent-Magnet Synchronous Motors*, IEEE Trans. on Ind. Applic., **40**, *3*, 789-796 (2004).

Li B., Yu F., *Optimal Model Following Control of Four-Wheel Active Steering Vehicle*, Intenat. Conf. on Information and Automation, Macau, China, 881-886, 2009.

Lorenz R.D., *Future Motor Drive Technology Issues and their Evolution*, Proc. of 12th Int. Power Electronics and Motion Control Conf., Portoroz, Slovenia, CD-ROM, 2006.

Sheta M.A., Agraval V., Nataraj P.S.V., *A New Energy Optimal Control Scheme for a Separately Excited DC Motor Based Incremental Motion Drive,* Internat. J. of Automation and Computing, **6**, *3*, 267-274 (2009).

Shibasaki H., Yusof R., Ishida Y., *A Design Method of a Model-following Control System*, Internat. J. of Control, Automation, and Systems, **13**, *4*, 843-852 (2015).

Takami H., *Design of Optimal Control Servo-Controller for Current Control in a Permanent Magnet Synchronous Motor*, Control Theory and Applications, **149**, *6*, 564-572 (2006).

Tyler J.S., *The Characteristics of Model-Following Systems, as Synthesised by Optimal Control*, IEEE Trans. on Automatic Control, **AC-9**, *4*, 485-498 (1964).

Xu W.J., *Permanent Magnet Synchronous Motor with Linear Quadratic Speed Controller*, Elsevier, Energy Procedia, **14**, 364-369 (2012).

Yang X. Q., Zou K.F., *A Simulation of an Electrical Servo Drive Based on Optimal Control*, Applied Mechanics and Materials, **380-384**, 476-479 (2013).

PROBLEMA URMĂRIRII OPTIMALE A MODELULUI PENTRU UN SISTEM DE ACȚIONARE ELECTRICĂ CONTROLAT ÎN TENSIUNE

(Rezumat)

Se studiază urmărirea optimală a modelului pentru o problemă cu timp final finit şi stare finală liberă. Se prezintă o metodă de reducere a consumului de energie şi de obținere a unei comportări adecvate a sistemului de acționare, impunând ca acesta să aibă o dinamică asemănătoare cu a unui model convenabil ales.