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## OPTIMAL CONTROL OF AN ELECTRICAL DRIVE SYSTEM WITH CURRENT CONTROLLED INDUCTION MOTOR

BY

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**Abstract.** The paper deals with an optimal control problem from the energetic point of view of the transient state of an electrical drive system with induction motor. A connection between the minimum energy problem and the maximum torque one is indicated. A detailed solution for a current controlled system is presented for the minimum energy control. Simulation results are indicated.

**Key words:** electrical drives; induction motor; current control; optimal control; copper losses minimization.

### 1. Introduction

The paper deals with an optimal control (Anderson & Moore, 2007; Athans & Falb, 2007) of an electrical drive system with induction motor. This problem is important because it is possible to ensure a significant energy saving, or other advantages, depending on the adopted criterion. There are numerous studies dedicated to this problem, for different types of motors, control strategies, criteria, or used methods (for instance, we mention (Cavallaro *et al.*, 2002; Kurihara & Rahman, 2004; Boțan *et al.*, 2007; Mesemanolis *et al.*, 2012; Yang & Zou, 2013), but many other papers can be indicated). The optimization is appreciated as a main direction of the developing of the electrical drive systems in the future (Lorenz, 2006).

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The optimization problems can be formulated referring to steady state (Dong & Ojo, 2006; Kim, 2017; Boțan, 1984; Kusko & Galler, 1983) or to the dynamic operation of the drive system. The problems formulated in the first case impose minimization of the copper and iron losses power. Also, the maximum torque (per ampere or per volt) problem is considered. It should be noticed that the torque is imposed in steady state by the load and, in fact, the problem is to ensure an imposed torque with a minimum stator current (or voltage), similar with the minimum losses problem.

The optimal control of electrical drive refers to the transient period and different problems are formulated in this case. A first one is the minimum energy (not minimum power as in steady state) problem, with different variants (Abrahamsen *et al.*, 1998; Ta & Yori, 2001; Abrahamsen *et al.*, 2001). The minimization of the energy consumption is practically equivalent with the minimization of the energy losses. Only copper losses are usually considered, because they significantly overcame the iron losses in the transient operation and the solution is simpler in this approach. The minimization is performed having in view the system moving equation and the formula of the electromagnetic torque for the used motor. Another problem is the maximization of the torque. If this problem is formulated in condition of an imposed maximal copper losses (heating), it is in connection with the previous one (losses minimization).

If the same problem is formulated in the condition of the stator current limitation, a connection with the minimum time problem (Chang & Byung, 1997; Vega *et al.*, 2006) can be obtained. This last problem imposes to establish the control which ensures the fastest achieving of the desired speed in condition of limitation of the control variable. Such a problem is of interest especially for the positioning systems.

The control of an electrical drive must be chosen so that to obtain a small energy losses and an acceptable behaviour of the system. The demands and conditions for different applications are not the same and therefore, one can formulate different optimal control problems for an electrical drive system. In addition to the mentioned aspects, the type of the terminal conditions, including the terminal time, imply different solutions control (Anderson & Moore, 2007; Athans & Falb, 2007).

The paper indicates the connection between the minimum energy and maximum torque problems and develops a solution for the first one. The problem is discussed for the current control in a rotor flux oriented structure of an induction motor (Kim, 2017; Kelemen & Imecs, 1989; Boldea & Nasar, 1992).

## 2. Problems Formulation

The moving equation of the electrical drive system is

$$\dot{\omega}(t) = f_1(t) = \frac{1}{J}[m(t) - m_r(t)], t \in [0, T], \quad (1)$$

where:  $\omega$  is the rotational speed,  $J$  – the system inertia,  $m(t)$  – the electromagnetic torque and  $m_r(t)$  – the load torque. The torque of an induction motor can be expressed as

$$m(t) = c_1(i_{1q}i_{2d} - i_{1d}i_{2q}), \quad (2)$$

where the indices 1 and 2 refers to the stator and rotor, respectively, the indices  $d$  and  $q$  refers to the currents components in a d-q adequate reference frame and  $c_1$  is a motor constant. The problem can be solved for this general case, but the solution is more complicated. Therefore, one will refer directly to a rotor flux oriented control, when the torque can be written as

$$m(t) = c\psi_{2d}(t)i_{1q}(t), \quad (3)$$

where:  $\psi_{2d}(t)$  is the direct component of the rotor flux (the quadratic component is zero) and  $c$  – a constant.

The initial and final speed

$$\omega(0) = 0 \text{ and } \omega(T) = \omega_f, \quad (4)$$

are imposed and it results from (1):

$$\omega_f = \omega(T) = \frac{T}{J}(\bar{m} - \bar{m}_r), \quad (5)$$

where  $\bar{m} = \frac{1}{T} \int_0^T m(t)dt$  and  $\bar{m}_r = \frac{1}{T} \int_0^T m_r(t)dt$  denote the mean values on the optimization interval  $[0, T]$  for the electromagnetic torque and for load torque, respectively.

The equation for the direct rotor flux component is

$$\dot{\psi}_{2d} = f_2(t) = -\frac{1}{T_r}\psi_{2d}(t) + \frac{L_m}{T_r}i_{1d}(t), \quad (6)$$

where  $T_r = L_r/r_2$  is the rotor time constant,  $L_r$  and  $L_m$  are the rotor and mutual inductances and  $r_2$  is the rotor resistance.

The control variables for the drive system are  $i_{1d}$  and  $i_{1q}$ . The state variables are the speed  $\omega(t)$ , the flux  $\psi_{2d}$  and the rotor current components. The last ones can be expressed as functions of other variables, so that one can consider only the first two mentioned state variables. The differential equations for the state variables are (1), (3) and (6).

The performance index of the optimal energy control corresponds to

$$I = \frac{1}{2} \int_0^T p(t)dt, \quad (7)$$

with

$$p(t) = r_1 i_{1d}^2(t) + r_1 i_{1q}^2(t) + r_2 i_{2d}^2(t) + r_2 i_{2q}^2(t). \quad (8)$$

**Problem P1:** find the control variables  $i_{1d}$  and  $i_{1q}$  which transfer the system described by (1) and (6) from the initial to final conditions (4), so that the performance index (7) to be minimized.

In every control problem, the command variables have to be chosen so that the motor torque ensures the reaching of the final conditions (5):

$$\int_0^T \psi_{2d}(t) i_{1q}(t) dt = \frac{J\omega_f}{cT} + \frac{\overline{m_r}}{c}. \quad (9)$$

In the optimal control case, there is only one variant that ensures the minimum of (7). The optimal control can be obtained only if data referring to the system are known, namely the mean value  $\overline{m_r}$  of the load torque is beforehand known (at least the shape of variation on  $[0, T]$  must be known and the magnitude of the torque has to be estimated at the beginning of the optimal process). This condition can be satisfied in many applications.

The formulated problem imposes to minimize a functional (the energy losses (7)) and to ensure an imposed value for another functional (the torque (9)). Therefore, the P1 optimal control problem is an isoperimetric one. In such problems (Anderson & Moore, 2007), it is possible to change the role of the functional and to formulate the

**Problem P2:** find the control variables  $i_{1d}$  and  $i_{1q}$  which maximizes the torque (9) if the energy losses (7) has an imposed value on the interval  $[0, T]$ .

The Hamiltonian for the first problem is

$$H = \frac{1}{2} p(t) + \lambda_1(t) \left[ \frac{c}{J} \psi_{2d}(t) i_{1q}(t) - \frac{1}{J} \overline{m_r} \right] + \lambda_2(t) f_2(t), \quad (10)$$

with  $f_2(t)$  and  $p(t)$  given by (6) and (8) and  $\lambda_1$  and  $\lambda_2$  are the co-state variables.

For the second problem, the Hamiltonian is

$$H' = \psi_{2d}(t) i_{1q}(t) + \lambda_1'(t) p(t) + \lambda_2'(t) f_2(t), \quad (10')$$

Both variants have the similar main terms and this fact leads to similar solutions in this cases. Of course, supplementary differences can appear because of the terminal conditions. The formulated problems can be solved in similar manner. In the sequel, only the P1 problem will be considered.

### 3. Main Results

The problem is solved based on Hamiltonian equations:

$$\frac{\partial H}{\partial i_{1d}} = 0 \quad (11)$$

$$\frac{\partial H}{\partial i_{1q}} = 0 \quad (12)$$

$$-\dot{\lambda}_1 = \frac{\partial H}{\partial \omega} \quad (13)$$

$$-\dot{\lambda}_2 = \frac{\partial H}{\partial \psi_{2d}} \quad (14)$$

where  $H(\cdot)$  is given by (10).

The system equations (1) and (6) are attached to the above equations.

It results from (10) and (13)  $d\lambda_1 / dt = 0$  and thus  $\lambda_1(t) = \text{constant}$ .

If the general expression (2) of the motor torque is used, the obtained solution has a complicated form. A simpler form (3) is preferably, especially because it refers to a modern and very used control technique. A supplementary significant simplification is obtained if the electromagnetic transient process is neglected, taking into account that this process has a fast decrease and affects in a small extent the electromechanical evolution. This case will be firstly analyzed in the sequel and then will be considered the ensemble of the transient evolution.

### 3.1. The Constant Variables Case

If the variation of the flux is neglected,  $i_{2d} = 0$  and since

$$i_{2q} = -\frac{L_m}{L_r} i_{1q} \quad (15)$$

the necessary conditions of optimality (11) and (12) lead to

$$r_1 i_{1d} + \lambda_1 \frac{L_m c}{J} i_{1q} = 0, \quad (16)$$

$$r_e i_{1q} + \lambda_1 \frac{L_m c}{J} i_{1d} = 0, \quad (17)$$

with

$$r_e = r_1 + r_2 \frac{L_m^2}{L_r^2}. \quad (18)$$

It results from (16) and (17):

$$r_e i_{1q}^2 = r_1 i_{1d}^2, \quad (19)$$

$$\lambda_1 = -\frac{J}{cL_m} \sqrt{r_1 r_e}. \quad (20)$$

The last equations indicate that constant values for  $i_{1d}$  and  $i_{1q}$  can be solution for the problem. The performance index (7) can be written in this case:

$$I = \frac{T}{2} (r_1 i_{1d}^2 + r_e i_{1q}^2). \quad (21)$$

The transfer time T results from (5)

$$T = \frac{\omega_f J}{m - m_r}, \quad (22)$$

with  $\bar{m} = m = \text{const.}$  Replacing (22) in (21), one obtains

$$I = \frac{\omega_f J}{m - m_r} r_e i_{1q}^2 \quad (23)$$

The condition of optimality becomes

$$\frac{\partial I}{\partial i_{1q}} = 0 \quad \text{or} \quad 2(m - \bar{m}_r) - cL_m i_{1q} i_{1d} = 0 \quad (24)$$

$$2(m - \bar{m}_r) - c\Psi_{2d} i_{1q} = 0$$

and therefore

$$m^* = 2\bar{m}_r \quad (25)$$

The obtained result indicates that the minimum of cooper losses is

achieved if the electromagnetic torque is double of the mean value of the load torque. This result was also indicated by authors (Boțan *et al.*, 2007) for different motor types in the same conditions of neglecting of the flux variation. It should also be noticed the equality (19) of the losses produced by the  $d$  and  $q$  components of the stator currents, the first one including the influence of the rotor losses.

The optimal control variables can be obtained from (19) and (25):

$$i_{1d}^2 = \frac{2\bar{m}_r}{cL_m} \sqrt{\frac{r_e}{r_1}}, \quad i_{1q}^2 = \frac{2\bar{m}_r}{cL_m} \sqrt{\frac{r_1}{r_e}}. \quad (26)$$

In this period, the rotor flux is given by the constant value

$$\psi_{2d} = L_m i_{1d} \quad (27)$$

The relations (22) and (25) show that the transfer time  $T$  increases very much when the load torque is small. In such cases it is possible to be necessary to introduce a limitation for  $T$ . Also, (19) indicates that  $i_{1d}$  is bigger than  $i_{1q}$  and then the flux can overcome its nominal value. Therefore, in certain applications it will be necessary to introduce a limitation of the reactive stator component. The limitations mentioned here are not considered in the paper.

### 3.2. The Complete Transient Process

Although the transient electromagnetic and electromechanical processes are held together, it is possible to separate these processes, in order to simplify the obtaining of the solution. In this respect, one will consider in the following that the electromagnetic process is ended at the moment  $T_1$  and the flux and currents remain unchanged after this moment. This moment was chosen  $T_1 = 4T_r$ . By comparison with the previous case, the final value of the speed  $\omega_f$  is obtained at the moment  $T' > T$ , because the mean value of the motor torque is smaller on the initial interval  $[0, T_1]$ .

If one denotes with  $\bar{m}_1$ ,  $\bar{m}_2 = m_2 = \text{const.}$  and  $\bar{m}_{r1}$ ,  $\bar{m}_{r2}$  the mean values of the electromagnetic and of the load torque on the intervals  $[0, T_1]$  and  $[T_1, T']$ , respectively, the final speed can be expressed as

$$\begin{aligned} \omega_f = \omega(T') &= \omega(T_1) + \frac{1}{J} \int_{T_1}^{T'} [m_2(t) - m_{r2}(t)] dt = \\ &= \frac{T_1}{J} (\bar{m}_1 - \bar{m}_{r1}) + \frac{T' - T_1}{J} (\bar{m}_2 - \bar{m}_{r2}). \end{aligned} \quad (28)$$

In order to simplify the notations, one will consider a constant load torque. Comparing (5) and (28), it results

$$T' = T + T_1 \frac{\bar{m}_2 - \bar{m}_1}{\bar{m}_2 - \bar{m}_{r2}}.$$

*Remark 1:* The moving begins only if  $m(t) > m_r$ ; since  $\Psi_{2d}(0) = 0$ , the moving will start at a moment  $t_p$ . If  $\bar{m}_1$  is interpreted as the mean value on the interval  $[t_p, T_1]$ , the previous reasoning remain valid and the above formula becomes

$$T' = T + T_1 \frac{\bar{m}_2 - \bar{m}_1}{\bar{m}_2 - \bar{m}_r} + t_p \frac{\bar{m}_1 - m_r}{m_2 - m_r}. \quad (29)$$

If the two processes are separated, the performance index (7) can be computed as

$$I = I_1 + I_2 = \int_0^{T_1} p_1(t) dt + \int_{T_1}^{T'} p_0(t) dt, \quad (30)$$

where  $p_v(t)$  and  $p_0(t)$  are the power losses on the respective intervals. On the second interval, all variables are constant, having the values indicated in the section 3.1 and will be denoted with  $i_{1d0}$ ,  $i_{1q0}$ ,  $\Psi_{2d0}$  and

$$I_2 = (T' - T_1) r_e i_{1q0}^2. \quad (31)$$

Since the speed does not influence the flux and the co-state variable  $\lambda_1$  is constant (having the same value (20)), only the equations (6) and (14) remain from optimality conditions. The last one leads to

$$-\dot{\lambda}_2(t) = \lambda_1 \frac{c}{J} i_{1q}(t) - \lambda_2(t) \frac{1}{T_r} + r_2 \frac{\partial i_{2d}^2(\psi_{2d})}{\partial \psi_{2d}}. \quad (32)$$

In order to obtain a simpler form of relations, the above equation was established neglecting the influence of the current component  $i_{2d}$ .

From the necessary conditions (11) and (12), one obtains:

$$i_{1q}(t) = -\lambda_1 \frac{c}{J r_e} \psi_{2d}(t), \quad (33)$$

$$i_{1d}(t) = -\frac{L_m}{T_r} \frac{1}{r_1} \lambda_2(t). \quad (34)$$



Introducing these values in (6) and (32), and using (20), it results:

$$\dot{\psi}_{2d}(t) = -\frac{1}{T_r} \psi_{2d}(t) - a \lambda_2(t), \quad (35)$$

$$\dot{\lambda}_2(t) = b \psi_{2d}(t) + \frac{1}{T_r} \lambda_2(t), \quad (36)$$

with

$$a = \frac{L_m^2}{T_r^2 r_1}, b_1 = \frac{1}{T_r L_r} - \frac{r_1}{L_m^2}, b_2 = \frac{1}{T_r} \left[ 1 - \frac{L_m^2}{L_r r_1} \right]. \quad (37)$$

If one extracts  $\lambda_2(t)$  from (35), computes the derivative of this function and introduces these expressions in (36), it results a second order differential equation for  $\Psi_{2d}(t)$ . Of course it is possible to obtain an analytical solution to this equation but the coefficients and the exponents have a complicated form, which does not allow a simple analysis of the results. Therefore one prefers to simplify the problem, considering a first order equation in the form

$$\ddot{\Psi}_{2d}(t) = -\frac{1}{T_r} [\Psi_{2d}(t) - \Psi_{2d0}]. \quad (38)$$

The final value  $\Psi_{2d}(T_1) = \Psi_{2d0}$  corresponds to the value of the second interval of the optimization (Section 3.1). It results the solution

$$\Psi_{2d}(t) = (1 - e^{t/T_r}) \Psi_{2d0}. \quad (39)$$

The current  $i_{1q}$  is obtained from (33):

$$i_{1q}(t) = (1 - e^{t/T_r}) i_{1q0}. \quad (40)$$

The imposed final condition for  $\Psi_{2d}$  can be achieved if the current  $i_{1d}$  has a variation which is in concordance with the equation (6). It results

$$i_{1d}(t) = i_{1d0} = \frac{\Psi_{2d0}}{L_m} \quad (41)$$

This value has to be commuted to  $i_{1d0}$  at the moment  $T_1$ .

The evolution of the variables is indicated in Fig.1. The idealized variation of the speed, described in the Section 3.1 is indicated with  $\omega$ , having a linear increase up to  $\omega_f$ . The optimal variation of the speed is denoted with  $\omega$

and it is zero up to  $t_p$ , then it has a small acceleration and has the same acceleration as  $\omega$  after  $T_1$ . The current  $i_{1q}$  and the flux  $\Psi_{2d}$  have similar variations and can be represented by the same curve with appropriate scale.

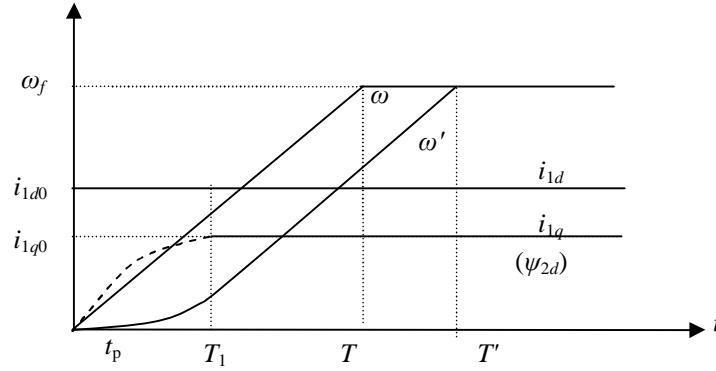


Fig. 1 – Optimal system behaviour.

### 3.3. Comparison Between Certain Control Variants

In the sequel, one will present and analyze three control variants on the interval  $[0, T_1]$ .

**Variante (a):** the optimal control established in the Section 3.2.

The flux and currents variations are indicated in (39), (40), (41). The moment  $t_p$  when the moving begins is given by the condition

$$c\psi_{2d}(t_p)i_{1q}(t_p) = m_r \quad (42)$$

It results from (39) and (40)

$$c(1 - e^{-t/T_r})^2 \psi_{2d0} i_{1q0} = m_r. \quad (43)$$

Taking into account that

$$m^* = c\psi_{2d0}i_{1q0} \quad (44)$$

and optimal torque  $m^*$  satisfies (25), one obtains

$$(1 - e^{-t/T_r})^2 = 1/2, \quad (45)$$

where  $\beta = t_p/T_1$ . Finally, it results  $\beta = 0.19$ .

Having in view the same formulas (39) and (40), the mean value of the torque on the interval  $[t_p, T_1]$  is

$$\overline{m}_1 = \frac{1}{T_1 - t_p} \int_{t_p}^{T_1} c\Psi_{2d0} i_{1q0} (1 - e^{-t/T_r})^2 dt \quad (46)$$

and one obtains

$$\overline{m}_1 = 0.72m^* \quad (47)$$

In this conditions, the final moment resulted from (29) is

$$T' = T + 0.82T_1 \quad (48)$$

**Variante (b):** the control currents are constant, with the values  $i_{1d0}, i_{1q0}$  established in the Section 3.1; the flux is imposed by the current  $i_{1d0}$  as in (6) and it is

$$\Psi_{2d}(t) = (1 - e^{-t/T_r})\Psi_{2d0}. \quad (49)$$

with

$$\Psi_{2d0} = L_m i_{1d0}. \quad (50)$$

The developed torque is

$$m_1(t) = c\Psi_{2d}(t)i_{1q0} = c\Psi_{2d0}i_{1q0}(1 - e^{-t/T_r}) = m^*(1 - e^{-t/T_r}) \quad (51)$$

In this conditions, the moving begins at the moment  $t_p = \beta T_1$ , with  $\beta = 0.175$ . The mean value of the torque on the interval  $[t_p, T_1]$  is

$$\overline{m}_1 = \frac{1}{T_1 - t_p} \int_{t_p}^{T_1} m_1(t) dt = 0.85m^* \quad (52)$$

The final moment given by (29) results

$$T' = T + 0.425T_1 \quad (53)$$

**Variante (c):** it is similar with the variante (a), but the component  $i_{1d}$  is beforehand established, so that the motor starts with an established flux  $\Psi_{2d0}$  and only the current  $i_{1q}(t)$  varies on the interval  $[0, T_1]$ . The variations are similar with variante (b), but the total energy losses depend on time of the establishing of the current  $i_{1d}$ .

In the sequel, only references to the first two variants will be made.

### Energy losses

The copper energy losses are in directly concordance with the performance index, which can be computed with (30). The second component is given by (31), with the remark that the value of the final moment  $T'$  depends on the adopted control variante. The first component of the index has four terms, depending on the rotor and stator current components, as it results from (8). The

last term can be directly associated with the first one, depending on  $i_{1q}(t)$ , as it results from (15). The dependence on the current  $i_{2d}(t)$  can be expressed as function on  $i_{1d}(t)$  and  $\Psi_{2d}(t)$ :

$$i_{2d}(t) = \frac{1}{L_r} \Psi_{2d}(t) - \frac{L_m}{L_r} i_{1d}(t). \quad (54)$$

The variations in the optimal control (variant (a) discussed above) of the flux and of the stator current components are indicated in (39), (40), (41), with the relation between the final values given by (20) and (50). The flux variation is imposed by the fixed  $i_{1d0}$ , having the form (49). A similar form has  $i_{1q}(t)$  if one does not impose a constant value, or an optimal variation. Finally, the index  $I_2$  can be expressed in terms of  $i_{1d0}$  and  $i_{1q0}$ . For a simpler comparison between the variants, the final value can be expressed only in dependence of one current component, using (20).

*Variant (a)*: taking into account the above mentioned aspects, the index  $I_1$  is:

$$I_{1a} = \frac{1}{2} \int_0^{T_1} \left\{ r_e i_{1q0}^2 (1 - e^{-t/T_r})^2 + r_1 i_{1d0}^2 + \frac{r_2}{L_r^2} [\Psi_{2d0} (1 - e^{-t/T_r})^2 - L_m i_{1d0}]^2 \right\} dt. \quad (55)$$

Performing the computing, one obtains finally:

$$I_{1a} = (0.85 + 0.45 \frac{r_e}{r_1}) w, \quad w = r_e i_{1q0}^2 T_r. \quad (56)$$

*Variant (b)*: in this case,

$$I_{1b} = \frac{1}{2} \int_0^{T_1} \left\{ r_e i_{1q0}^2 + r_1 i_{1d0}^2 + \frac{r_2}{L_m^2} [(1 - e^{-t/T_r}) \Psi_{2d0} + i_{1d0}]^2 \right\} dt. \quad (57)$$

The computing leads to

$$I_{1b} = r_e i_{1q0}^2 T_1 + 1.48 \left( \frac{r_e}{r_1} - 1 \right) r_e i_{1q0}^2 T_1. \quad (58)$$

For usual values of the parameters, one can approximate  $r_e/r_1 \cong 2$  and thus

$$I_{1a} \cong 1.75 w; \quad I_{1b} \cong 3.96 w; \quad w = r_e i_{1q0}^2 T_r. \quad (59)$$

The index  $I_2$  on the final interval  $[T_1, T]$  is (31) and is computed for the final time  $T$  given by (48) or (53) for the two variants. It results:

$$I_{2a} = w(T - 0.18T_1) / T_r; \quad I_{2b} = w(T - 0.575T_1) / T_r. \quad (60)$$

The total value of the performance index is  $I = I_1 + I_2$ .

If, for instance,  $T = 5T_1 = 20T_r$ ,  $I_a = 21.43$  w and  $I_b = 22.78$  w and the relative increasing is  $\delta = (I_b - I_a)/I_a = 0.063$ . If  $T = 15T_1 = 60T_r$ ,  $I_a = 61.03$  w and  $I_b = 62.79$  w and the relative increasing is  $\delta = (I_b - I_a)/I_a = 0.029$ .

*Remark 2:* in order to simplify the implementation and to reduce the duration of the process, the optimal control can be modified on the initial interval of the electromagnetic transient process. This modification can be obtained, for instance, maintaining constant the two control currents. This variant was presented with more details and, in comparison with optimal control, it is noted that:

- the energy losses increase, but not very much (between 2% and 7%);
- the total time of the transfer decrease about similar percentages.

These results were expected, because the duration of the electromagnetic process is significantly smaller as of the whole one, but in certain applications, it is important to know what the modifications are if a simpler implementation is adopted.

#### 4. Simulation Results

Certain simulation tests were performed in connection with the studied problem for a induction motor with the data: the rated values – 1.5 kW, 230 V, 3.8 A, 10 Nm, 50 Hz; parameters –  $L_s = 0.28$  H,  $L_m = 0.265$  H,  $L_r = 0.28$  H,  $R_s = 3.4 \Omega$ ,  $R_r = 3.2 \Omega$ . Some results are indicated in the figures presented below. Fig.2 presents the variation of the speed for different electromagnetic torque, comparing with the load torque. The variation of the optimal torque for a 5 Nm load torque is presented in Fig. 3. In the same case, the variations of the speed and of the stator and rotor losses are indicated in Figs. 4 and 5.

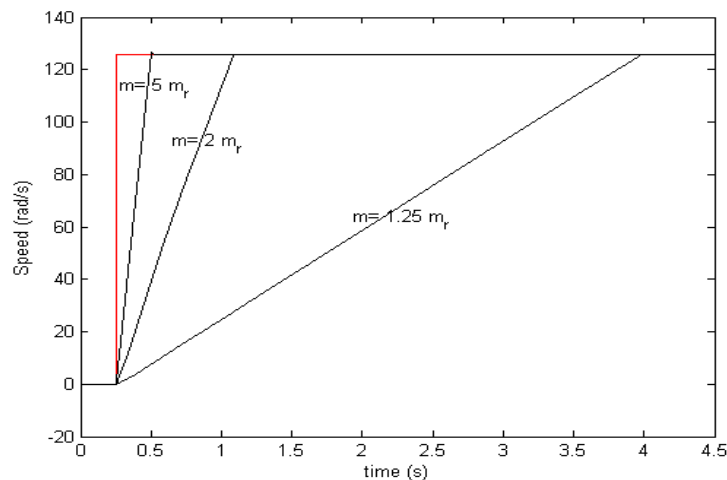


Fig. 2 – Behaviour of the drive system with induction motor for optimal and non-optimal control.

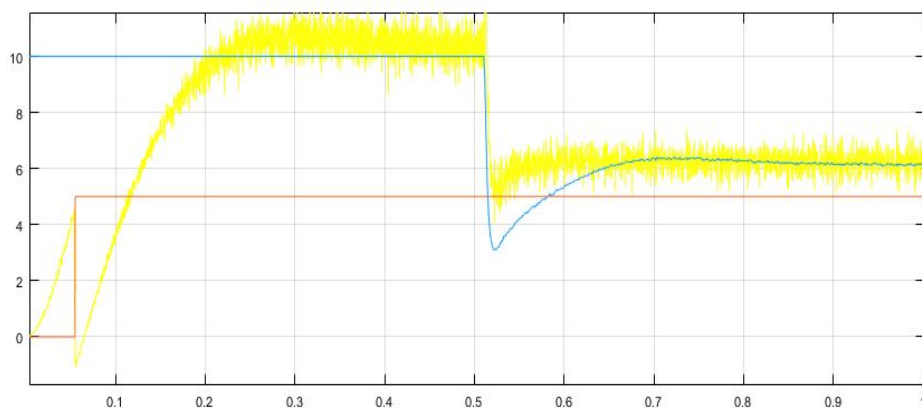


Fig. 3 – Variation of the optimal torque.

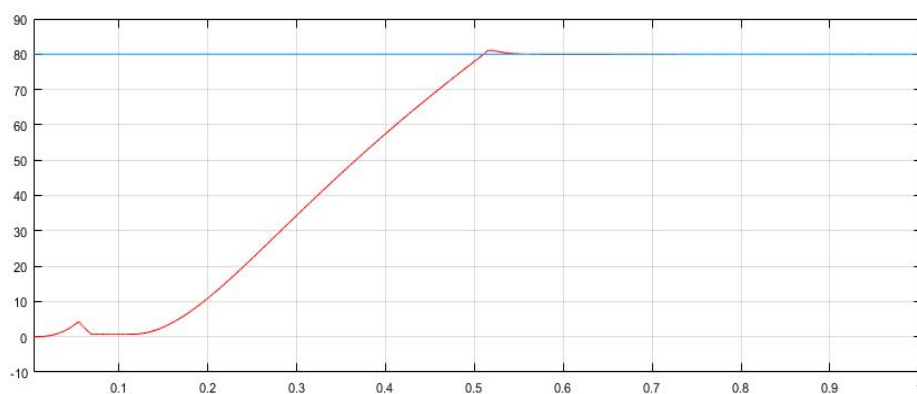


Fig. 4 – Variation of the speed.

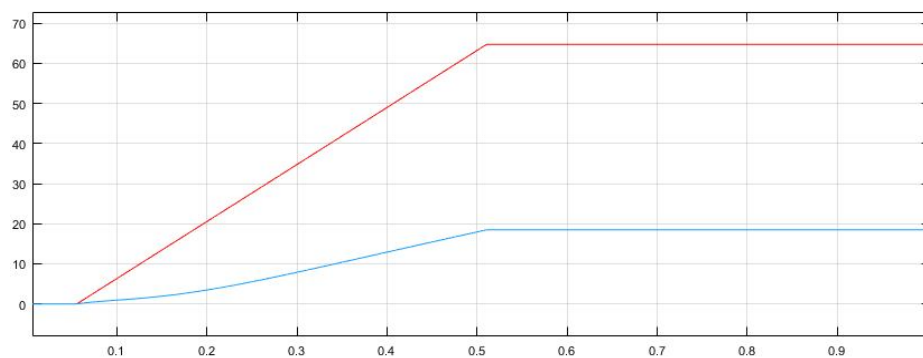


Fig. 5 – Variations of the stator (red colour) and rotor losses.

## 5. Conclusions

A minimum losses problem for an electrical drive system with a current controlled induction motor is presented. A connection with the maximum torque control is indicated. A rotor flux oriented structure is considered. The study is performed with the assumption of neglecting of the transient process of the flux and also in the case when this process is considered.

A comparison of different variants indicates that a simplified control algorithm, with constant current control variables leads to a behaviour which does not differ too much by the exact optimal control. Simulation results are presented.

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#### CONTROLUL OPTIMAL AL UNUI SISTEM DE ACȚIONARE ELECTRICĂ CU MOTOR DE INDUCȚIE CONTROLAT ÎN CURENT

(Rezumat)

Lucrarea se referă la o problemă optimă de control din punct de vedere energetic al regimului tranzitoriu pentru un sistem electric de acționare cu motor de inducție. Este prezentată o conexiune între problema energiei minime și cuplul maxim. O soluție detaliată pentru un sistem controlat în curent este descrisă pentru controlul energiei minime. Sunt prezentate rezultatele simulărilor.