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STRING STABILITY ANALYSIS OF ADAPTIVE CRUISE CONTROL VEHICLE PLATOONS

BY

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Abstract. The actual worldwide transportation infrastructure has a strong need of solutions for increasing its capacity. One solution could be represented by the vehicle organization in platoons. A well-known advanced driver assistance (ADAS) technology that can be used for that is the adaptive cruise control system (ACC). The ACC equipped vehicles are able to maintain certain spacing with respect to the position and velocity of the vehicle in front, but they are subject to string instability which causes the amplification of the oscillations due to speed changes towards the tail of the platoon. In this paper a string stability analysis of ACC vehicle platoons based on a heuristic method of the authors is presented. A platoon implemented with distance-based ACC control structures that use a linear quadratic regulator with a double integrator is considered. The simulation results were obtained after the considered platoon was modelled and simulated in Matlab/Simulink.

Keywords: vehicle platooning; string stability; adaptive cruise control; transfer function of spacing errors; time headway.

1. Introduction

A well-known problem of the present civilization is the continuously increasing number of vehicles that conducts to very crowded roads together

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with an increased pollution of the environment. A lot of solutions are researched and developed all over the world for the solving of this humanity's problems. One of them can be represented by the implementation of intelligent transportation systems (ITS) in the form of vehicle platoons using an advanced driver assistance (ADAS) technology called adaptive cruise control (ACC) that allows to the equipped vehicles to maintain a certain distance to the vehicle in front depending on their speed and a pre-selected time headway. This is the first commercial implementation of ADAS systems on a large scale for passenger vehicles and represents the first logical step to the progressive development of the automated highway systems (AHS) (Xiao & Gao, 2011). The need of using the ADAS systems on highways is coming from the attempt to reduce or eliminate the human error from some traffic situations that can conduct to car accidents. These have a great potential to reduce the human stress, the travelling costs and the environmental pollution (Tigadi *et al.*, 2016).

The ACC can also be viewed as an autonomous control system that is designed to work with good performances when uncertainties are present in the system and in the environment for long periods and must be capable of compensating the significant system errors without any external intervention (Antsaklis *et al.*, 1990). This is using radar sensors, an electronic control unit and an appropriate software that is processing the sensor data and provides the necessary output (acceleration/speed) to follow the vehicle in front under safety conditions.

Automated vehicles travelling in platoons are subject to string instability which causes the amplification of the oscillations due to speed changes towards the tail of the platoon. The relationship between string stability and spacing policies is very important in designing the controller for an ACC system. The constant time headway (CTH) spacing policy is commonly employed by ACC systems because it has the advantage that generates the string stability, but has the disadvantage that it results in larger steady-state spacing, which increases the platoon length (Omae *et al.*, 2014).

The aim of this paper is to analyze the string stability of an ACC platoon that contains vehicles whose controllers are based on a linear quadratic regulator with double integrator (LQI^2R) using an authors' heuristic method (Lazăr, 2018; Lazăr & Ţigănaşu, 2019). The obtaining of some small inter-vehicle distances is in focus using the proposed algorithm this being a measure for the performance of the designed controllers. The weak string stability condition from (Lazăr & Ţigănaşu, 2019) will be considered to determine if a platoon configuration is stable. Another proof of the string stability could be represented by the fact that the control errors are decreasing towards the tail of the platoon.

The paper is organized as follows. Section **2** presents the ACC vehicle control algorithm together with the vehicle model. In section **3** the string stability analysis heuristic method is described. Section **4** focuses on some simulation results and the conclusions are mentioned in section **5**.

2. ACC Vehicles Control Structure

The vehicle platoon considered in this paper is composed of one leader controlled by a classical cruise control system (CC) and a set of followers that are equipped with ACC systems to be able to maintain the distance to the vehicle in front depending on their speed. The reference for the ACC control structures is the desired distance d_i^* defined using the constant time headway policy:

$$d_i^* = d_{i0} + h_i v_i \,, \tag{1}$$

where: d_{i0} is the desired distance at standstill, h_i – the desired headway time and v_i – the measured velocity of vehicle i.

Considering the measured distance d_i that is a data received from a radar sensor and the distance reference d_i^*, the control error for the distance-based ACC system has the following expression (Ulsoy *et al.*, 2012):

$$e_i = d_i^* - d_i = d_i^* - (p_{i-1} - L_{i-1} - p_i) \,. \tag{2}$$

The control structure from Fig. 1 was obtained using the Eq. (2) together with Eq. (1), a simplified model of radar device and a simple vehicle dynamics model from (Ulsoy *et al.*, 2012):

$$\dot{v}_i = -\left(1/\tau_v\right)v_i + \left(K_v/\tau_v\right)u_i \,, \tag{3}$$

where: u_i is the control command, v_i – the i-th vehicle velocity, K_v – an amplification factor and τ_v – a time constant.

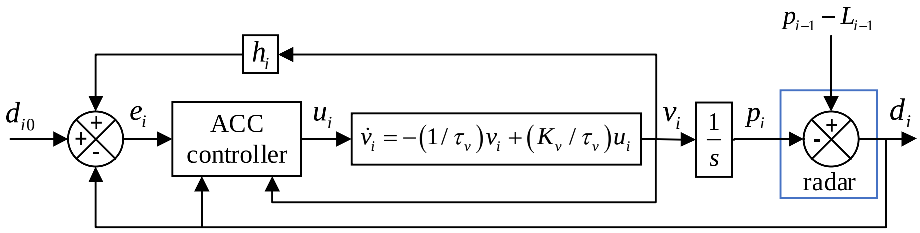

Fig. 1 – ACC system based on distance.

The position p_{i-1} of the vehicle in front is introduced by the radar as a disturbance in the controlled system because it is not directly measured, and because of that two integrators are needed to be introduced into the error channel in order to compensate its negative effect. In this situation the extended state model described by the following expressions results:

$$\begin{cases} \dot{x}_1 = \dot{d}_i = v_{i-1} - v_i \\ \dot{x}_2 = \dot{v}_i = -\left(1/\tau_v\right)v_i + \left(K_v/\tau_v\right)u_i \\ \dot{x}_3 = e_i \\ \dot{x}_4 = x_3 \\ y_i = d_i \end{cases} \Rightarrow \begin{cases} \dot{x}_i = A_i x_i + b_i u_i \\ y_i = c_i^T x_i \end{cases} \tag{4}$$

The introduction of the two integrators in the vehicle state model from Eq. (4) conducts to the obtaining of the ACC system for which a linear quadratic regulator with double integrator with the expression below was chosen:

$$u_i = -K_i x_i = k_1 d_i + k_2 v_i + k_3 \int e_i(\tau)\mathrm{d}\tau + k_4 \int \left(\int e_i(\tau)\mathrm{d}\tau \right) \mathrm{d}\tau \ , \tag{5}$$

where the error e_i has the expression from Eq. (2) and the control u_i is obtained by minimizing a quadratic cost function with this form:

$$J(u_i) = \int_0^\infty (x_i^T Q x_i + u_i^T P u_i)\mathrm{d}t \ , \tag{6}$$

where $Q = Q^T \geq 0$ is a weighting matrix and $P > 0$ is a scalar.

3. String Stability of Platoon

Due to the fact that the ACC vehicle platoons are subject to string instability there is a real need of determining a method through which to decide whether a platoon is stable or not. The proposed method from (Lazar, 2018), (Lazăr & Ţigănaşu, 2019) for string stability analysis is based on the transfer function of the spacing errors between adjacent vehicles:

$$G_i(s) = \frac{E_i(s)}{E_{i-1}(s)} \ , \tag{7}$$

where E_i is the Laplace transform of the spacing error e_i.

A weak solution to string stability of interconnected vehicles is described by the relation (Swaroop, 1994):

$$\left\| G_i(s) \right\|_\infty \leq 1 \tag{8}$$

where: $\left\| G_i(s) \right\|_\infty = \max_{\omega} \left| G_i(j\omega) \right|$.

The condition from Eq. (8) guarantees that $\left\| e_i \right\|_2 \leq \left\| e_{i-1} \right\|_2$ which means that the spacing errors will decrease from one vehicle to another until the last vehicle of the platoon. At the same time this could be a proof that the errors don't amplify upstream (Zhou & Peng, 2004; Rajamani & Zhu, 2002).

In order to perform the weak string stability analysis, the heuristic solution from (Lazăr & Ţigănaşu, 2019) that consists of choosing different values for headway time h_i and analysing if the condition from Eq. (8) is fulfilled or not. Starting from 0, h_i is taking values in ascending order and the corresponding Nyquist diagrams $G_i(j\omega)$ are plotted. For the analysis of the generated diagrams for the Eq. (8) must be considered its complex domain form: $\|G_i(j\omega)\|_\infty \leq 1$, where $\omega \geq 0$. If the Nyquist diagrams are placed into the unit radius circle with the center in the origin of the coordinate axes, the Eq. (8) is fulfilled and thus the string is stable for a certain time headway value. The lowest value of h_i for which the Nyquist diagram is enrolled in the circle is of interest.

Considering the constant time headway policy, the spacing error is determined starting from Eq. (2):

$$e_i = d_{i0} + h_i v_i - d_i \tag{9}$$

Using Eq. (9) together with the vehicle model from Eq. (3) and the LQI^{2}R control law from Eq. (5) in (Lazăr & Ţigănaşu, 2019) the following transfer function of spacing errors between adjacent vehicles was obtained:

$$G_i(s) = \frac{E_i(s)}{E_{(i-1)}(s)} = \frac{K_v k_1 s^2 - K_v k_3 s - K_v k_4}{\tau_v s^4 + \left(1 - K_v k_2\right) s^3 + K_v(-k_3 h_i + k_1) s^2 - K_v(k_3 + k_4 h_i) s - K_v k_4}. \tag{10}$$

4. Case Study

The analysis of the string stability was performed in a simulated manner using Matlab/Simulink and considering a platoon composed of one leader, controlled by a CC system, and seven followers equipped with ACC systems. All vehicles in the platoon are identical from building and dynamics point of view: vehicle length is L_i = 4.5 m, the vehicle mass is m = 1,200 kg and the wheel radius is r = 0.43 m (17 inches). The considered standstill distance is d_{i0} = 1 m.

The transfer function of spacing errors was determined using Eq. (10), the vehicle model parameters K_v and τ_v and the matrix K_i determined from the minimization of cost function from Eq. (6), resulting:

$$G_i(s) = \frac{371.4s^2 + 294.1s + 102}{62.4s^4 + 237.5s^3 + (294.16h_i + 371.4)s^2 + (102h_i + 294.1)s + 102}. \tag{11}$$

Considering that for string stability analysis the time headway h_i must be varied from one simulation to another this is remaining variable in the expression of the transfer function from Eq. (11).

The weak stability condition from Eq. (8) was applied for the transfer function $G_i(s)$ increasing h_i from zero to a value that makes the stability condition to be fulfilled. If the Nyquist diagram plotted after a certain simulation is included into the unit radius circle with the center in the coordinate axes origin then the considered platoon configuration is stable.

For h_i = 0 s, according to the diagram from Fig. 2 the considered platoon is unstable because the representation in complex domain of $G_i(s)$ is not included in the circle.

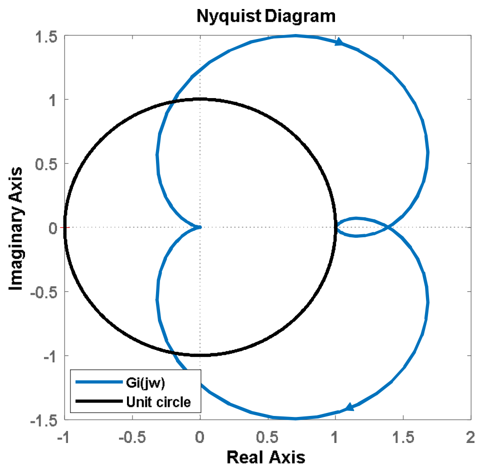

Fig. 2 – Nyquist diagram for h_i = 0 s.

Using the interconnected vehicles platoon simulator from (Lazar *et al.*, 2018), the weak string stability was checked also plotting the spacing errors e_i, together with the inter-vehicle distances d_i with $i=\overline{1,7}$. The platoon dynamics was tested, considering for the leader a null reference in the first 40s to achieve the equilibrium state of the platoon and after that, a step v_{ref} = 30m/s was applied.

For the time headway equal to 0s, according to Eq. (1) the spacing reference becomes constant (d_{i0} = 1 m). In Fig. 3 it can be observed that the ACC control structures are not able to maintain a constant inter-vehicle distance to the vehicle in front, the platoon being very unstable as the amplification of the spacing errors from Fig. 4 shows.

Increasing the time headway with a step of 0.1s the first discovered value for which the string stability condition from Eq. (8) was fulfilled is h_i = = 0.7 s, in Fig. 5 being observed that the Nyquist diagram is placed inside the unit circle.

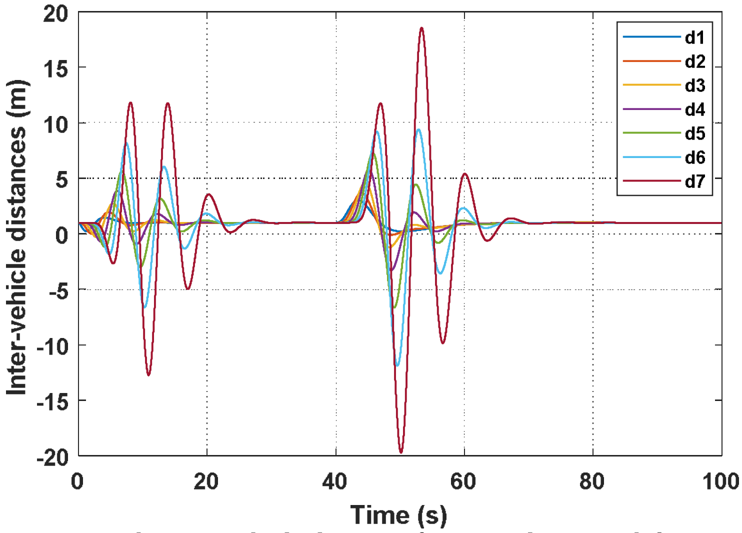

Fig. 3 – The inter-vehicle distances for ACC platoon with h_i = 0 s.

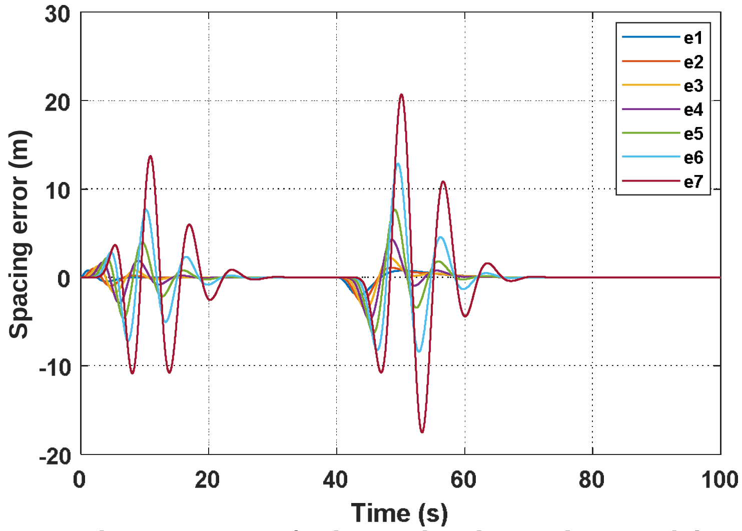

Fig. 4 – The spacing errors for distance-based ACC platoon with h_i = 0 s.

The inter-vehicle distances obtained with h_i = 0.7 s for the ACC control structures and represented in Fig. 6 show a very stable behaviour starting from 1m when the vehicles were in standstill and increasing to 22 m since the leader started to move with 30 m/s. The transient time for an inter-vehicle spacing to reach the stationary regime is around 25 s.

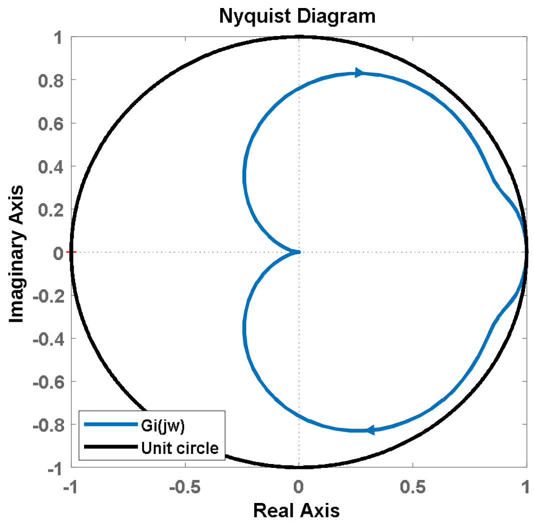

Fig. 5 – Nyquist diagram for h_i = 0.7s.

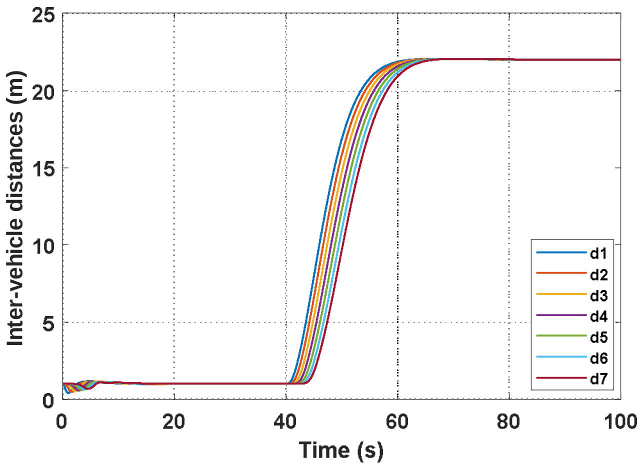

Fig. 6 – The inter-vehicle distances for ACC platoon with h_i = 0.7s.

The stability of the platoon configuration with h_i = 0.7 s is strengthened by the fact that the spacing errors depicted in Fig. 7 decrease starting from the leader until the last vehicle in the platoon. The performance of the considered platoon can be catalogued as good considering that to reach a distance of 22 m the greatest control error is 0.2 m.

Continuing to increase h_i the Nyquist diagram is included better in the circle, but a greater value for the time headway conducts to a greater distance reference for the ACC control structures, according to Eq. (1), and thus to a greater spacing between vehicles in the platoon. In this case in order to obtain stable ACC vehicle platoons with the minimum possible length the smallest value of h_i that assures the string stability is of interest.

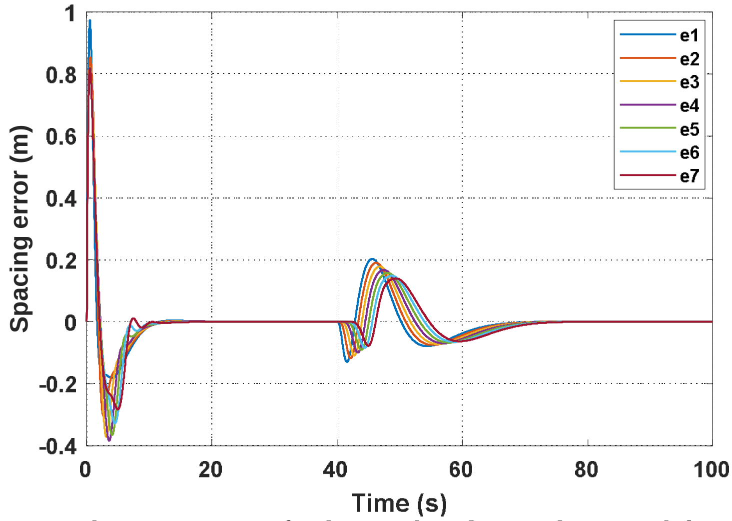

Fig. 7 – The spacing errors for distance-based ACC platoon with h_i = 0.7s.

5. Conclusions

In this paper a string stability heuristic methodology for a platoon composed of ACC vehicles was studied. The vehicles control structures were implemented with a linear quadratic regulator with double integrator. A weak string stability condition taken from literature was used to demonstrate that the configuration of the considered platoon is stable. This condition consisted of the inclusion of some Nyquist diagrams into the unit radius circle with the center in the origin of the coordinate axes. These diagrams represented in the complex domain were plotted for the spacing errors transfer function that has variations depending on the time headway. The smallest value for headway time that fulfilled the string stability condition was of interest. This conducted to smaller inter-vehicle distances together with a smaller platoon length. Another proposed method to establish if a string is stable is represented by the fact that the spacing control errors didn’t amplify upstream. The performed experiments proved that the proposed control algorithm had good results quantifiable in the fact that the vehicles in the platoon were able to maintain relatively small distances between them. The experimental platoon that contained one leader and seven followers was modelled and simulated using Matlab/Simulink. For the actual worldwide infrastructure, the implementation of this type of intelligent transportation system is beneficial due to the overcrowded highways.

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ANALIZA STABILITĂŢII PLUTOANELOR DE VEHICULE ECHIPATE CU SISTEME DE CONTROL ADAPTIV AL VITEZEI DE CROAZIERĂ

(Rezumat)

Infrastructura globală actuală de transport are o nevoie puternică de soluții pentru creşterea capacității acesteia. Una dintre soluții ar putea fi reprezentată de organizarea vehiculelor în plutoane. O tehnologie bine cunoscută pentru asistența avansată a şoferului (ADAS) care ar putea fi folosită în acest scop este sistemul de control adaptiv al vitezei de croazieră (ACC). Vehiculele echipate cu ACC şi grupate în plutoane sunt capabile să mențină o anumită distanță față de poziția vehiculului din față, dar principala problemă este asigurarea stabilității şirului. Proprietatea de stabilitate a şirului asigură atenuarea spre capătul plutonului a erorilor de distanță ce apar la modificarea vitezelor. În această lucrare este prezentată o analiză a stabilității şirului pentru plutoanele formate din vehicule echipate cu ACC bazată pe o metodă euristică a autorilor. Este considerat un pluton implementat cu structuri de control ACC bazate pe distanță care utilizează un regulator liniar pătratic cu dublu integrator şi se analizează stabilitatea şirului folosind pentru definirea distanței dorite metoda intervalului de timp constant. Rezultatele simulării au fost obținute după ce plutonul considerat a fost modelat şi simulat în Matlab/Simulink.